

PREDICTING AND PREVENTING SEGREGATION FOR SMALL, DENSE, IRREGULAR, AND FINE PARTICLES

IPFRI Meeting, June, 2023

RICHARD M. LUEPTOW

with

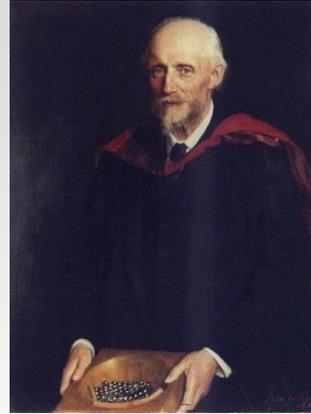
ZHEKAI DENG, YIFEI DUAN, YI FAN, BEN FREIREICH, ALEX FRY, SONG GAO, HANTAO HE, JOHN HECHT,
AUSTIN ISNER, KARL JACOB, LU JING, RYAN P. JONES, JAMES F. KOCH, MADHUSUDHAN KODAM,
JULIO M. OTTINO, CONOR SCHLICK, JÖRG THEUERKAUF, PAUL B. UMBANHOWAR,
VIDYA VIDYAPATI, DHAIRYA VYAS, HONGYI XIAO, MENGQI YU, ZAFIR ZAMAN, QIONG ZHANG, YONGZHI ZHAO

Funded by The Dow Chemical Company, Procter & Gamble Company, and
NSF CMMI-1435065, CBET-1511450, CBET-1929265, CBET-2203703



Northwestern | McCORMICK SCHOOL OF
ENGINEERING

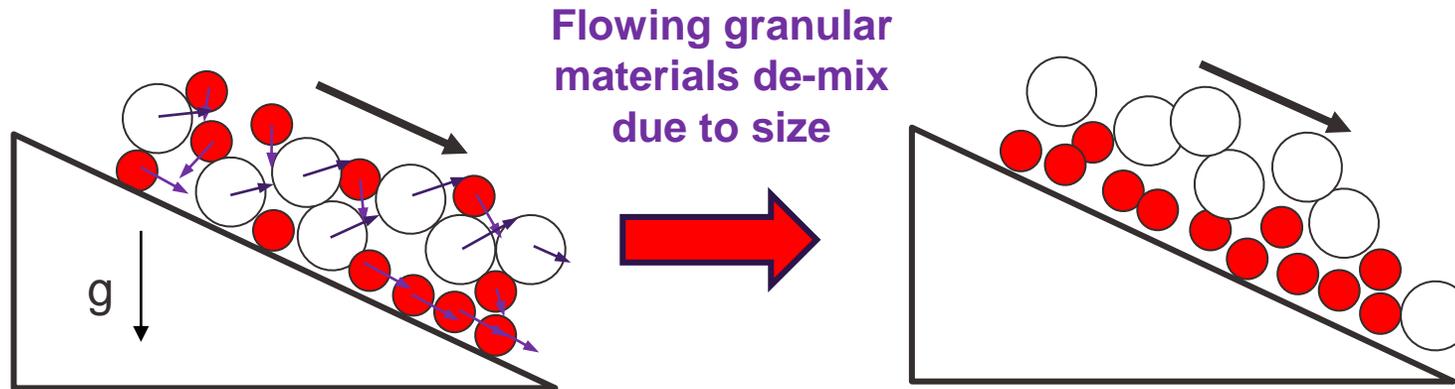
Granular flows



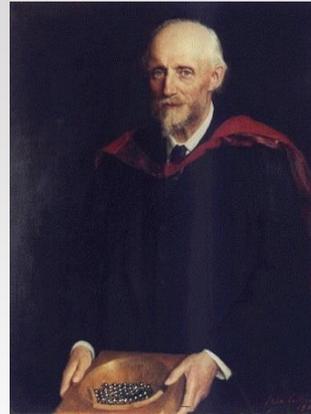
Osborne Reynolds ($Re = \rho VD / \mu$):
“On the sub-mechanics of the universe”, 1902.



“Can we develop a general theory
of ... the **motion of granular
materials?**”



Granular flows



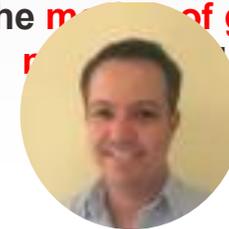
Osborne Reynolds ($Re=\rho VD/\mu$):
“On the sub-mechanics of the universe”, 1902.



Karl Jacob: “Can we better understand granular segregation?” 2010

Yi Fan: “Can we predict granular segregation?” 2014

“Can we develop a general theory of the **m** of granular



John Hecht: “How can we apply this to industrial segregation problems?” 2015

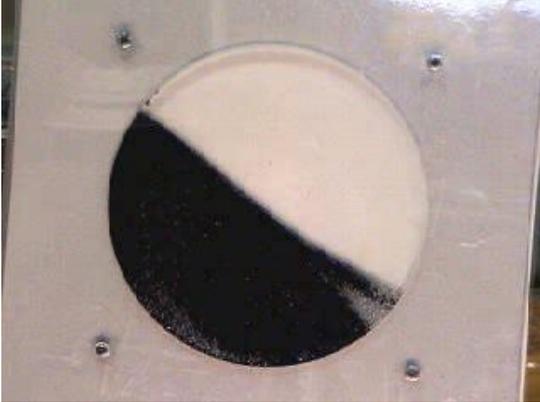


ZHEKAI DENG, YIFEI DUAN, YI FAN, BEN FREIREICH, ALEX FRY, SONG GAO, HANTAO HE, JOHN HECHT, AUSTIN ISNER, KARL JACOB, LU JING, RYAN P. JONES, JAMES F. KOCH, MADHUSUDHAN KODAM, JULIO M. OTTINO, CONOR SCHLICK, JÖRG THEUERKAUF, PAUL B. UMBANHOWAR, VIDYA VIDYAPATI, DHAIRYA VYAS, HONGYI XIAO, MENGQI YU, ZAFIR ZAMAN, QIONG ZHANG, YONGZHI ZHAO

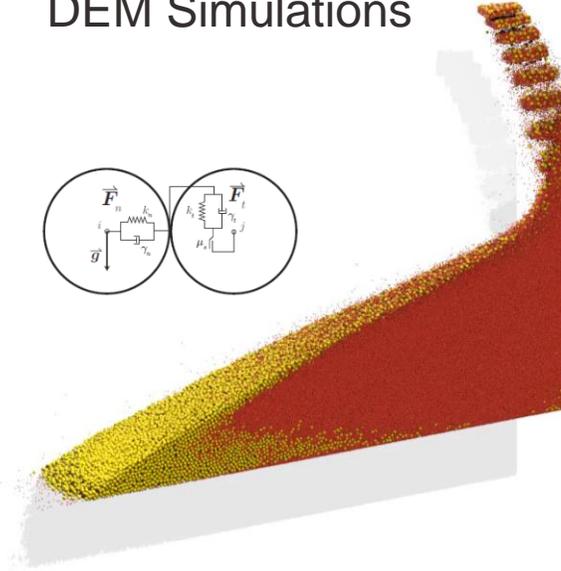


Three tools to study de-mixing / segregation & four questions

Experiments



DEM Simulations

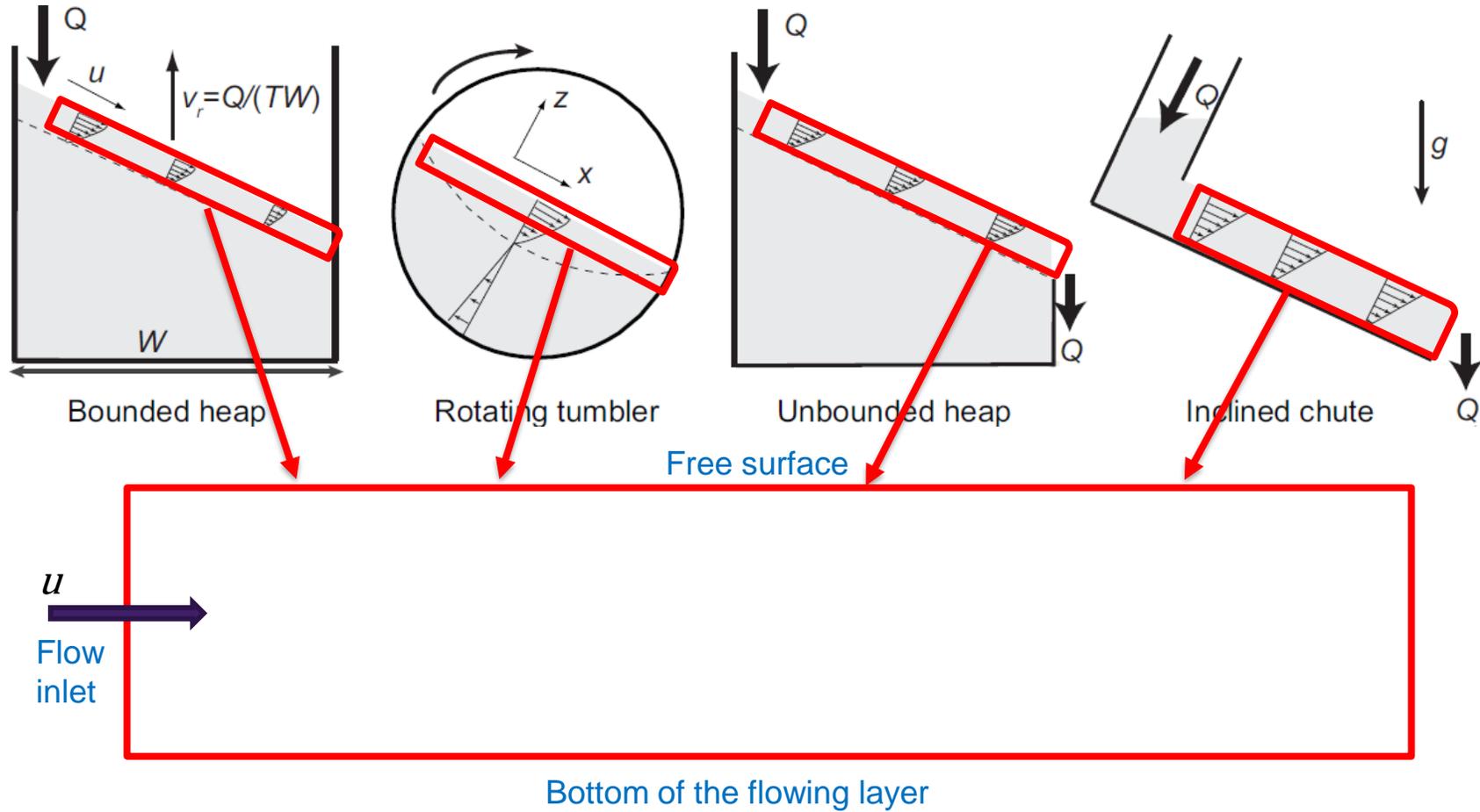


Theoretical Models

$$\frac{\partial c_i}{\partial t} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

- Can we **predict** demixing/segregation?
- Can we **prevent** segregation?
- Can we **understand** segregation?
- Can we understand segregation of **fine particles**?

Theoretical model of segregation

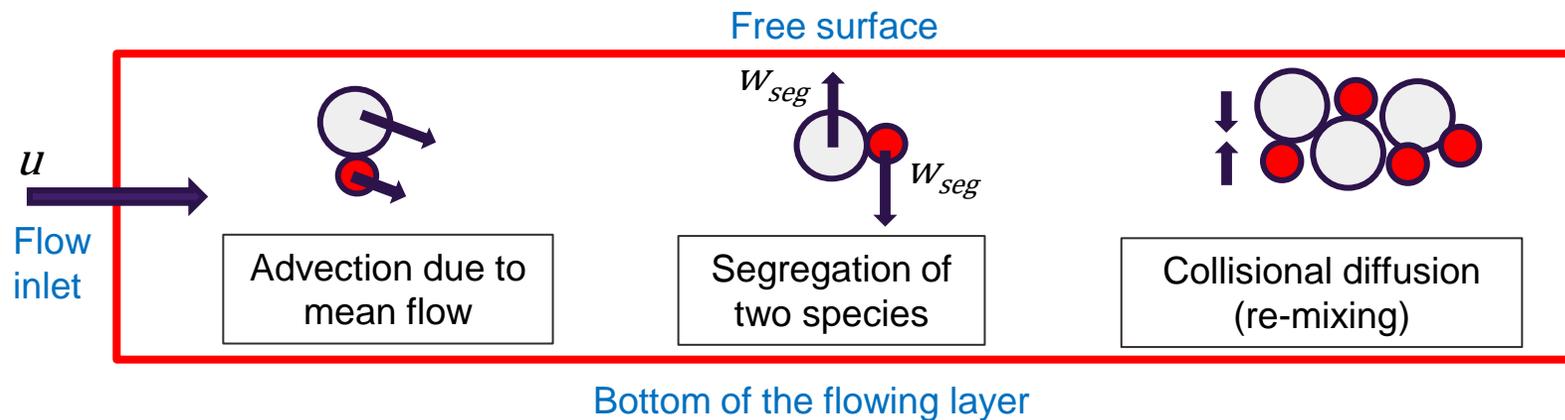


Theoretical model of segregation

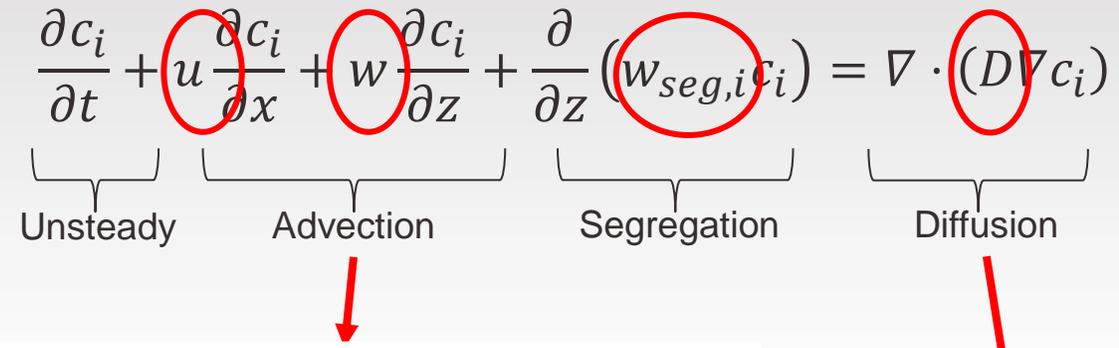
$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

Advection-Diffusion Eq.

- Fan, Schlick, Umbanhowar, Ottino & Lueptow, *JFM* 2014
- Drahn & Bridgwater 1983
- Gray & coworkers 2005-11
- Marks et al 2011
- Kowalski & McElwaine 2013
- May et al 2010

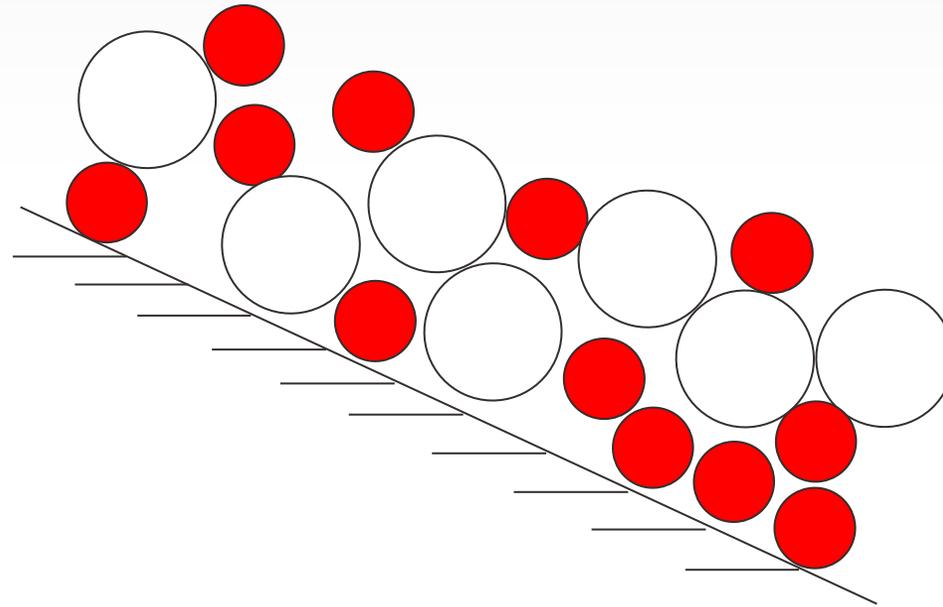


Transport equation for species volume concentration

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$


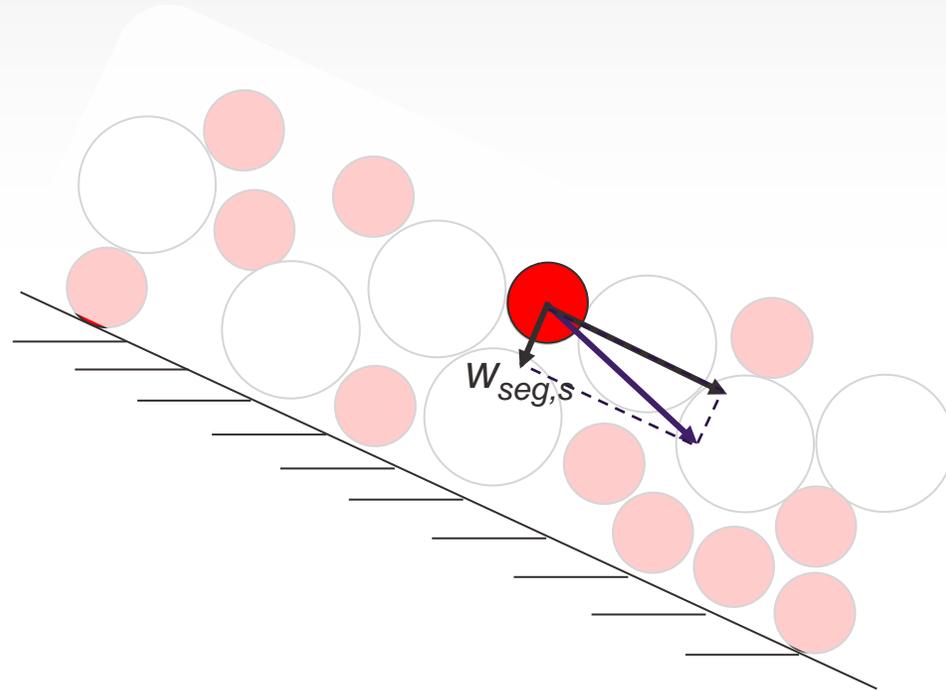
Transport equation for species volume concentration

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$



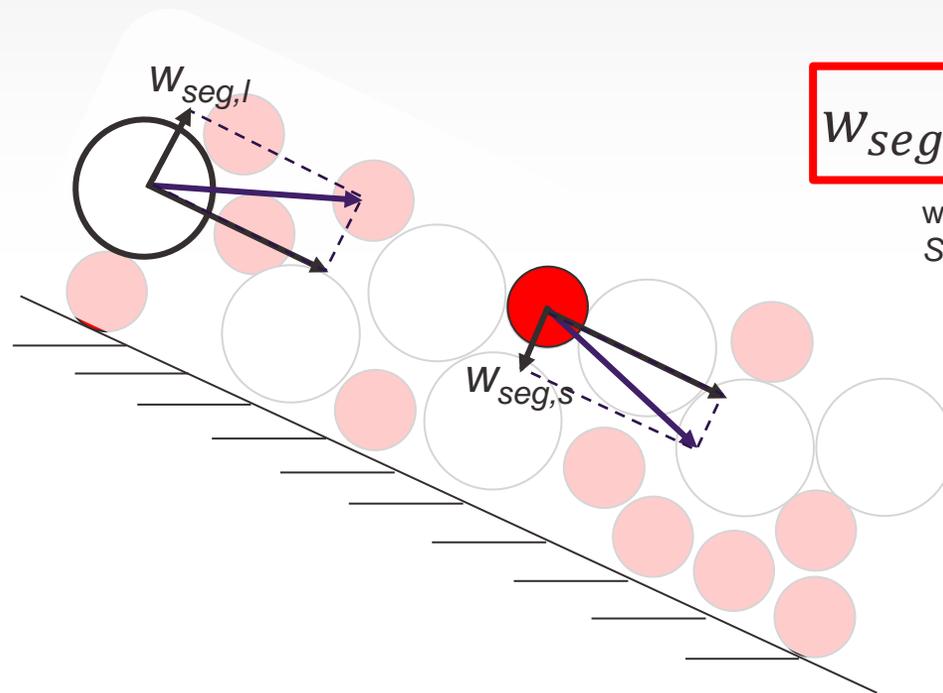
Transport equation for species volume concentration

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$



Transport equation for species volume concentration

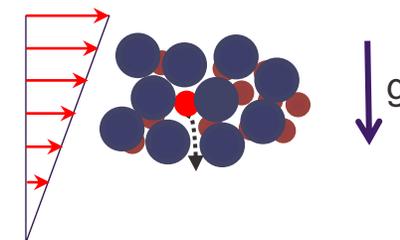
$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$



$$w_{seg,i} = S \dot{\gamma} (1 - c_i)$$

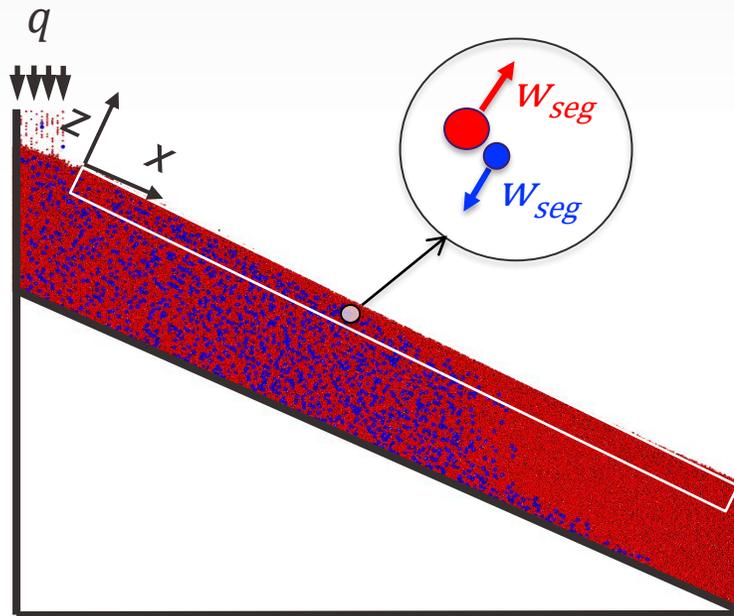
where $\dot{\gamma}$ is the shear rate & S depends on size ratio, $R = d_l / d_s$

Fan, Schlick, Umbanhowar, Ottino & Lueptow, *JFM* 2014
Savage & Lun, *JFM* 1988 (kinetic sieve)



Transport equation for species volume concentration

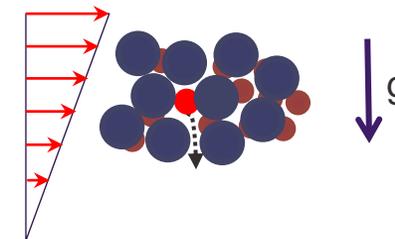
$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$



$$w_{seg,i} = S \dot{\gamma} (1 - c_i)$$

where $\dot{\gamma}$ is the shear rate &
 S depends on size ratio, $R = d_l / d_s$

Fan, Schlick, Umbanhowar, Ottino & Lueptow, JFM 2014
 Savage & Lun, JFM 1988 (kinetic sieve)

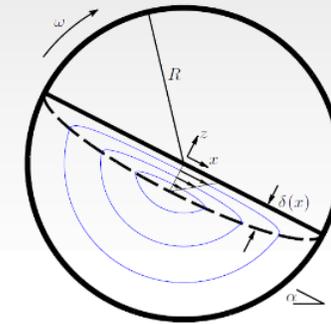


Does the model work?

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

Rotating tumbler (transient)

- u, w from theory
- w_{seg}, D from DEM for heap flow

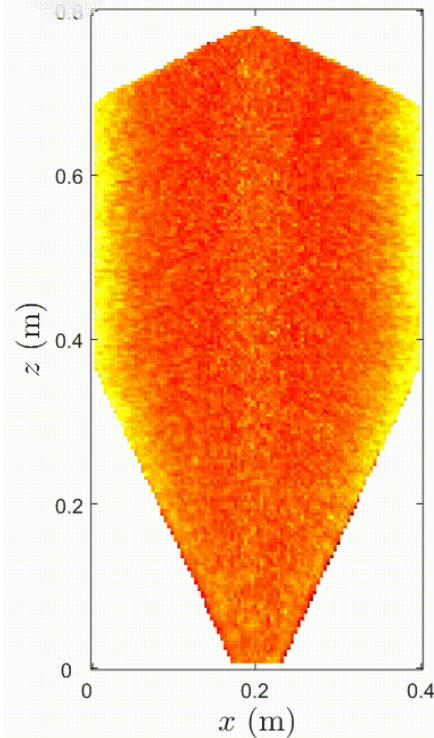


Comparisons with experiments and DEM simulations

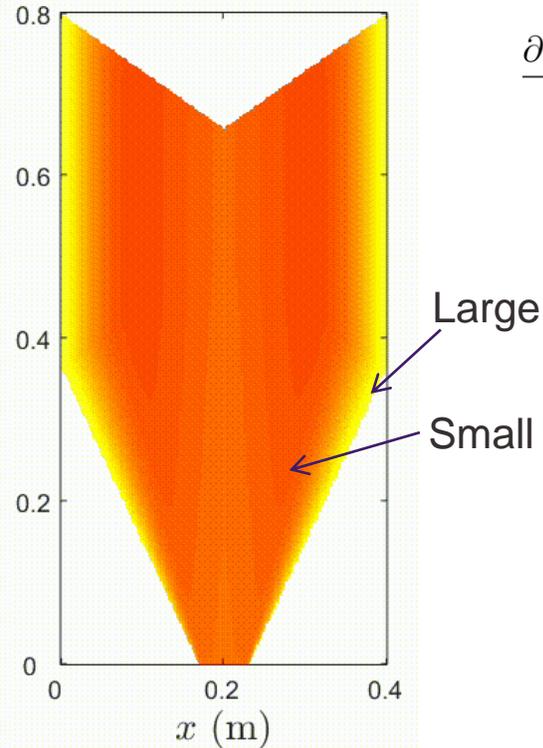
Experiment



DEM Simulation

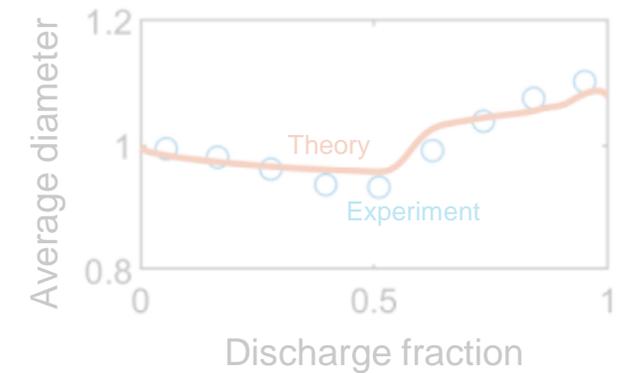


Polydisperse continuum model



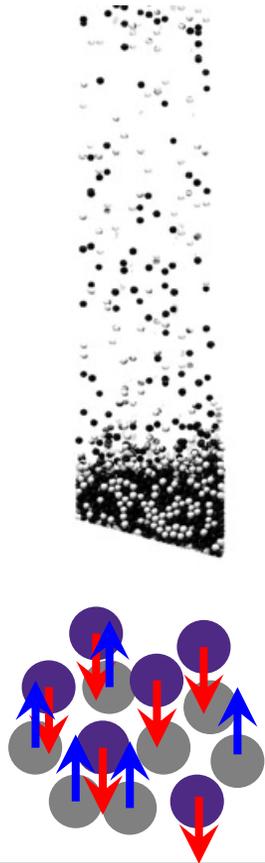
Polydisperse Continuum Model

$$\frac{\partial c(\mathbf{x}, t, \alpha)}{\partial t} + \nabla(\mathbf{u} \cdot c(\mathbf{x}, t, \alpha)) + \frac{\partial}{\partial z} \int_{d_{\min}}^{d_{\max}} S(\alpha, \beta) \dot{\gamma} c(\mathbf{x}, t, \alpha) c(\mathbf{x}, t, \beta) d\beta = \nabla \cdot (D \nabla c(\mathbf{x}, t, \alpha))$$



Xiao, Fan, Jacob, Umbanhowar, Kodam, Koch & Lueptow, *Chem Eng Sci* 2019
 Deng, Fan, Theurkauf, Jacob, Umbanhowar & Lueptow, *Powder Technol* 2020

Density segregation

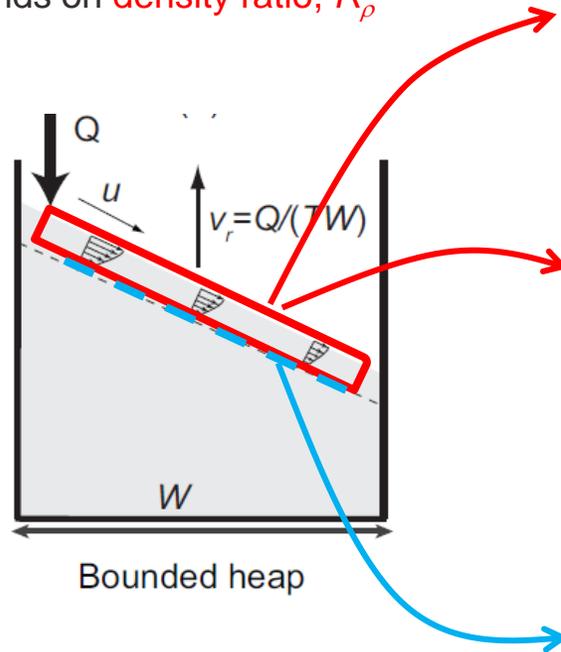


*Xiao, Umbanhowar, Ottino &
Lueptow, Proc Royal Soc A, 2016*

Density segregation

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$

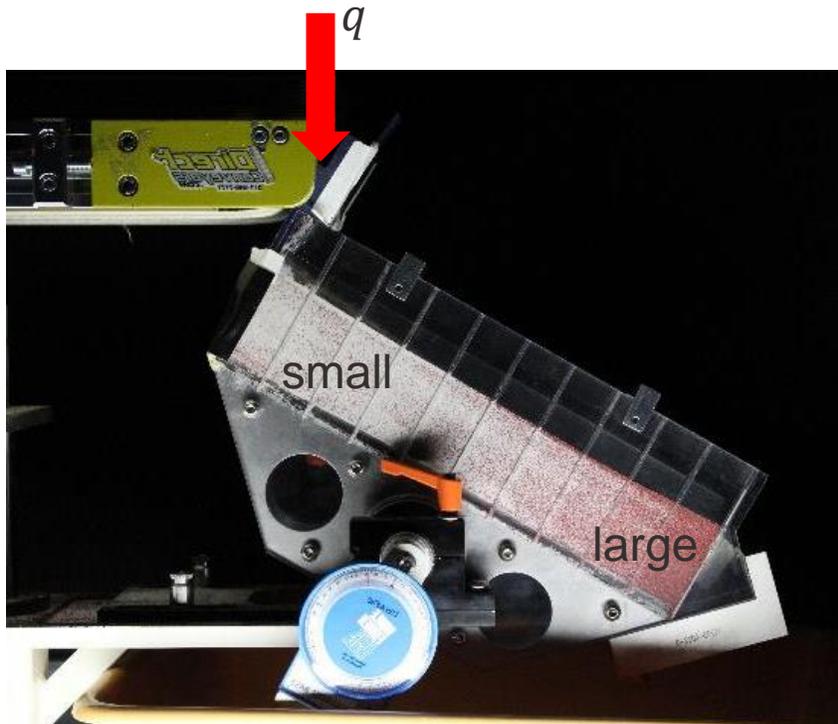
where S depends on density ratio, R_ρ



*Xiao, Umbanhowar, Ottino &
Lueptow, Proc Royal Soc A, 2016*

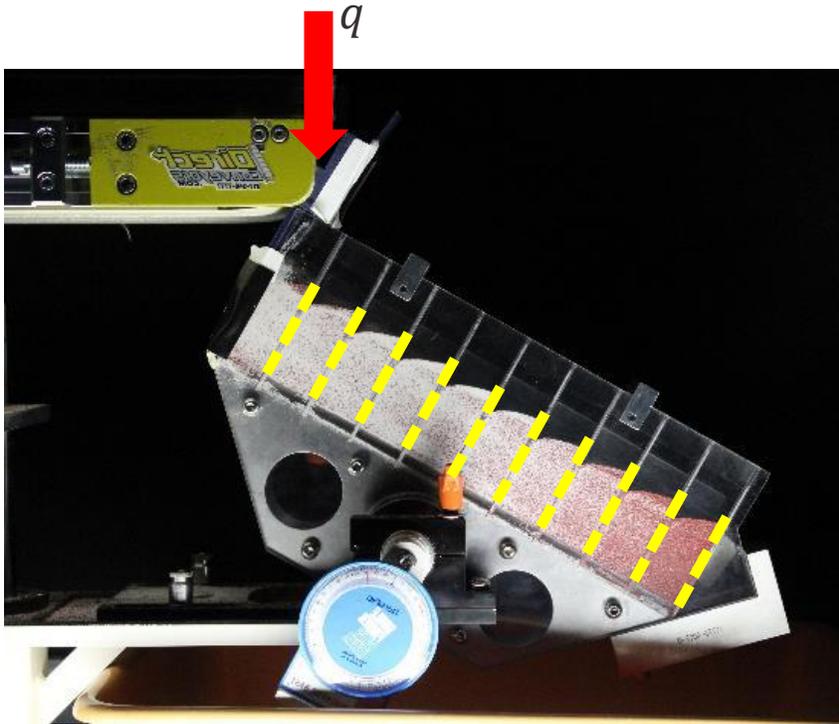
Determining segregation parameter S for actual mixtures

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$



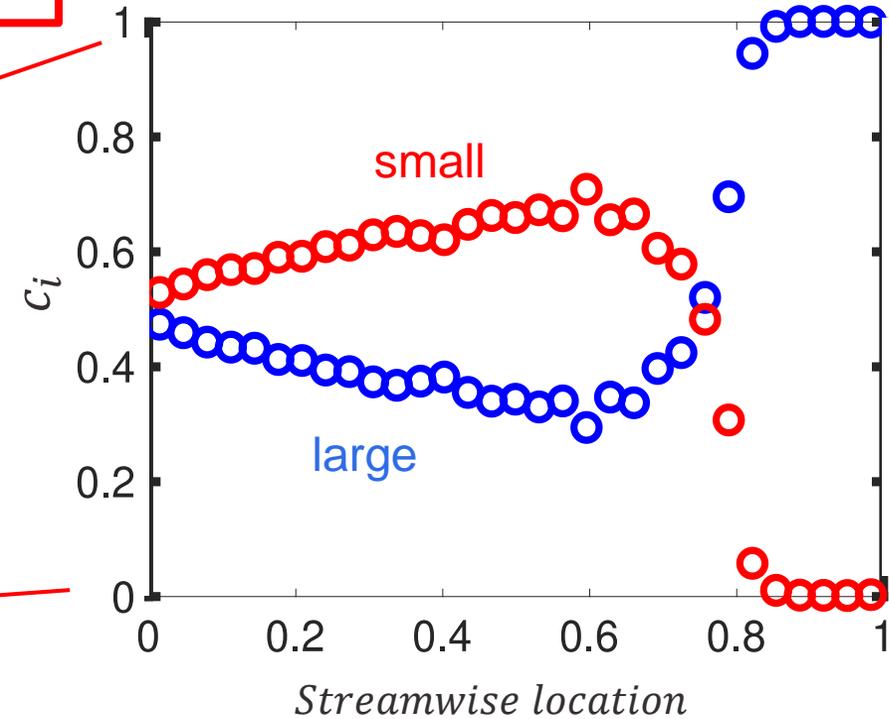
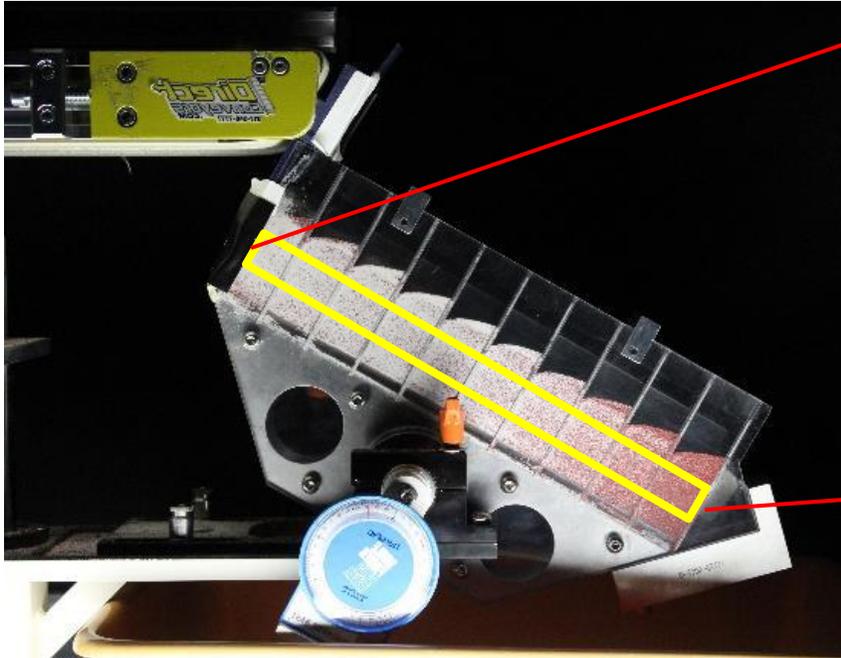
Determining segregation parameter S for actual mixtures

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$



Determining segregation parameter S for actual mixtures

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$

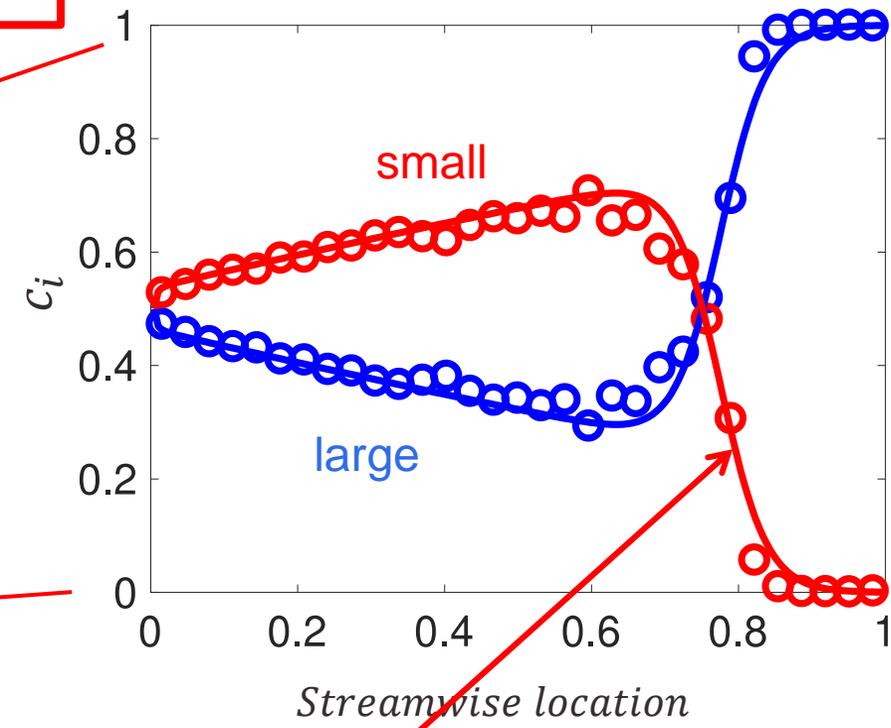
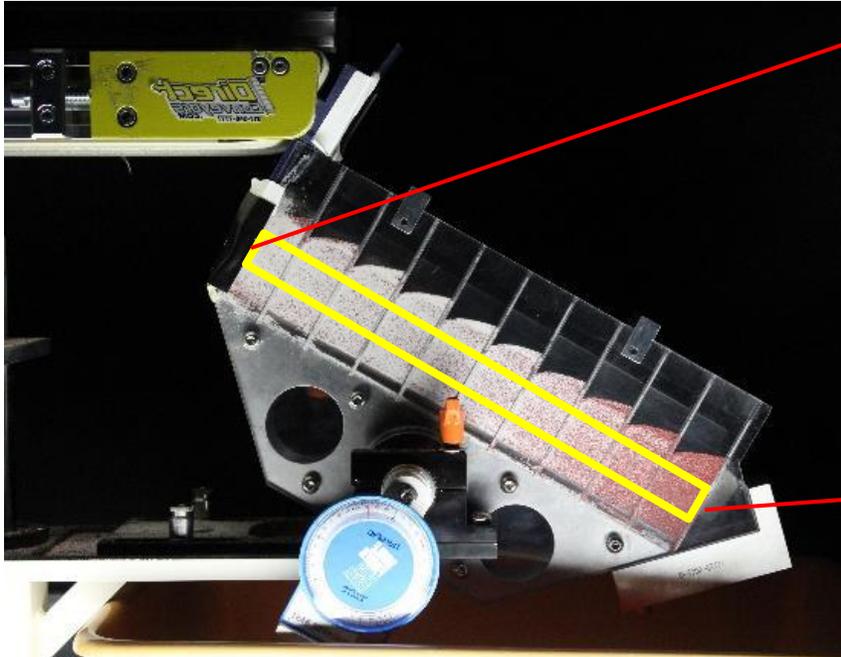


$$\frac{\partial c_i}{\partial t} + \frac{\partial(uc_i)}{\partial x} + \frac{\partial(wc_i)}{\partial z} + S \frac{\partial}{\partial z} [\dot{\gamma}(1 - c_i)c_i] - \left[\frac{\partial}{\partial x} \left(D \frac{\partial c_i}{\partial x} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial c_i}{\partial z} \right) \right] = 0$$

Fry, Vidyapati, Hecht, Umbanhowar, Ottino & Lueptow, Powder Technol 2020a
 Fry, Vidyapati, Hecht, Umbanhowar, Ottino & Lueptow, Powder Technol 2020b

Determining segregation parameter S for actual mixtures

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$



Adjust S to fit data

$$\frac{\partial c_i}{\partial t} + \frac{\partial(uc_i)}{\partial x} + \frac{\partial(wc_i)}{\partial z} - \underbrace{S}_{\text{Adjust } S \text{ to fit data}} \frac{\partial}{\partial z} [\dot{\gamma}(1 - c_i)c_i] - \left[\frac{\partial}{\partial x} \left(D \frac{\partial c_i}{\partial x} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial c_i}{\partial z} \right) \right] = 0$$

Fry, Vidyapati, Hecht, Umbanhowar, Ottino & Lueptow, Powder Technol 2020a
 Fry, Vidyapati, Hecht, Umbanhowar, Ottino & Lueptow, Powder Technol 2020b

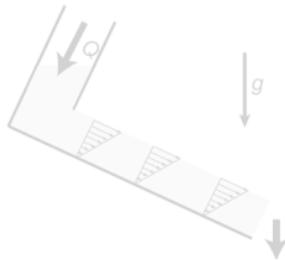
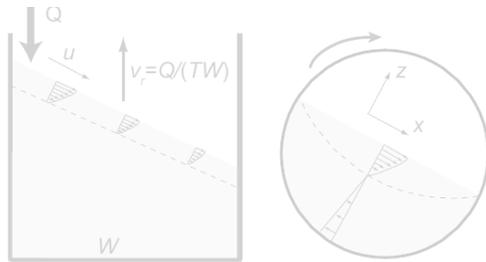
Can we predict demixing/segregation? -- YES

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$

Fan, Schlick,
Umbanhowar, Ottino &
Lueptow, *JFM*, 2014

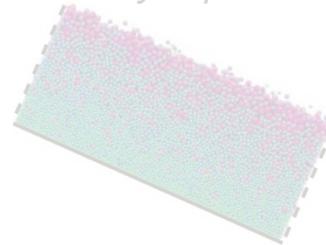
Kinematics and Geometry



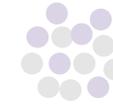
Size Bidisperse



Size Polydisperse

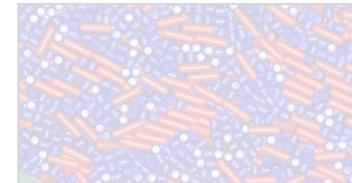


Density Bidisperse



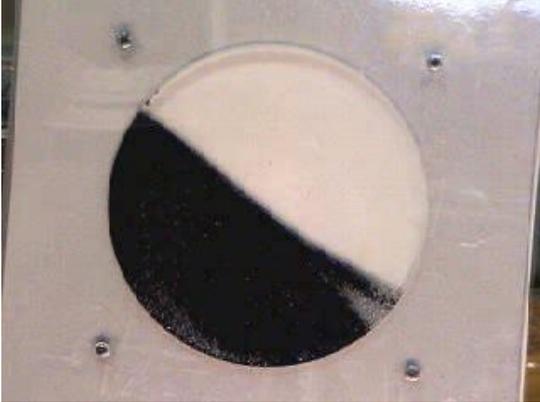
Rod-like Particles

Zhao, Xiao, Umbanhowar &
Lueptow, *AIChE J*, 2018

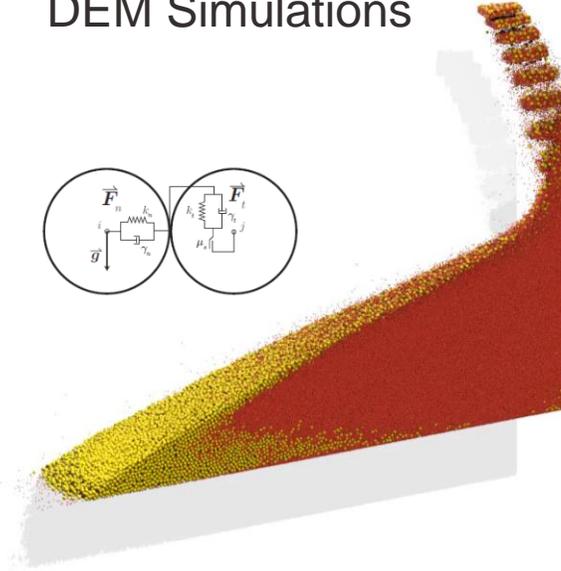


Three tools to study de-mixing / segregation & four questions

Experiments



DEM Simulations

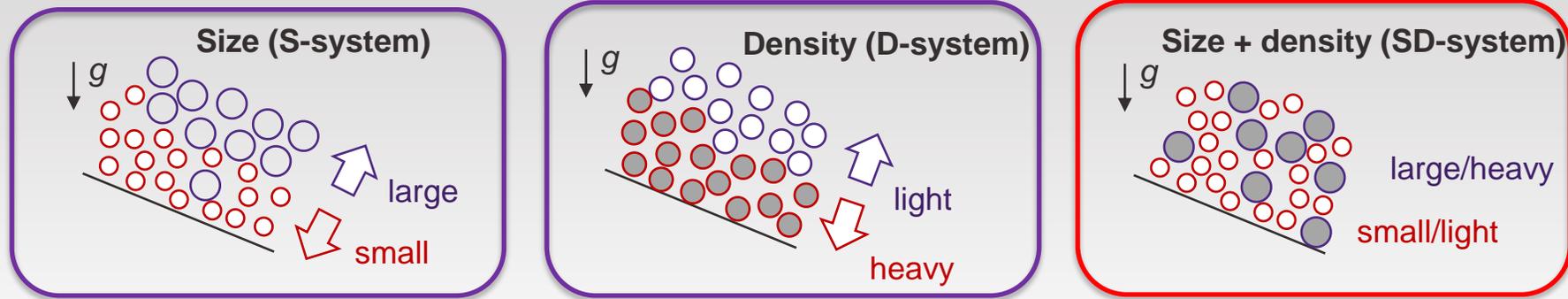


Theoretical Models

$$\frac{\partial c_i}{\partial t} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

- Can we predict demixing/segregation?
- Can we prevent segregation?
- Can we understand segregation?
- Can we understand segregation of fine particles?

Preventing de-mixing/segregation



$$R_d = \frac{d_l}{d_s}$$

$$R_\rho = \frac{\rho_l}{\rho_s}$$

Can we design non-segregating mixtures by balancing effects of size and density?



Can we design non-segregating mixtures by balancing shape effects?

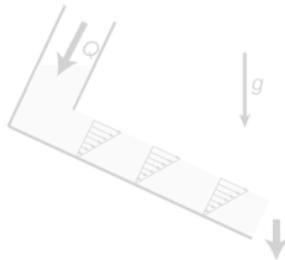
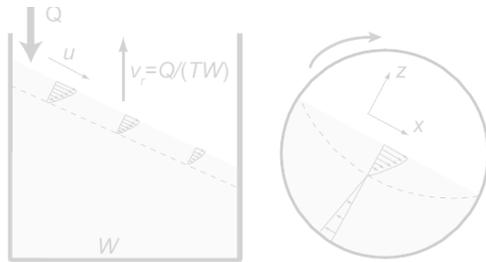
Combined size and density segregation

$$\underbrace{\frac{\partial c_i}{\partial t}}_{\text{Unsteady}} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$

Fan, Schlick,
Umbanhowar, Ottino
& Lueptow, JFM 2014

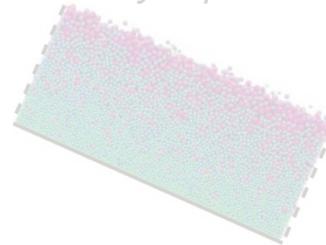
Kinematics and Geometry



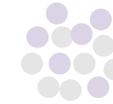
Size Bidisperse



Size Polydisperse

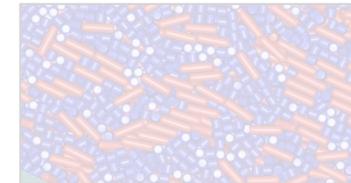


Density Bidisperse

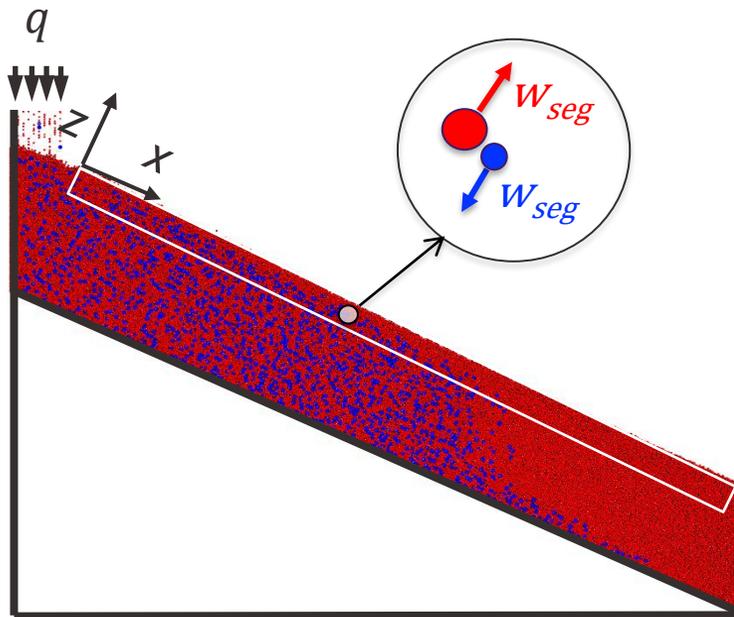


Rod-like Particles

Zhao, Xiao, Umbanhowar &
Lueptow, AIChE J 2018



Combined size and density segregation in bounded heap flow



- 350 simulations with different size ratios R_d and density ratios R_ρ
- Measure segregation velocity w_{seg}
- Wide range of shear rates $\dot{\gamma}$ and particle concentrations c_i

Combined size-density segregation

Large particle feed concentration:

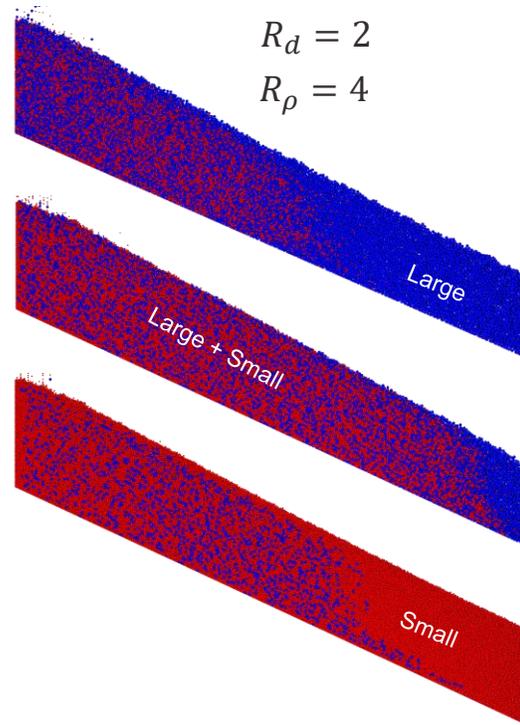
$$\hat{c}_l = 0.8$$

$$R_d = 2$$

$$R_\rho = 4$$

$$\hat{c}_l = 0.5$$

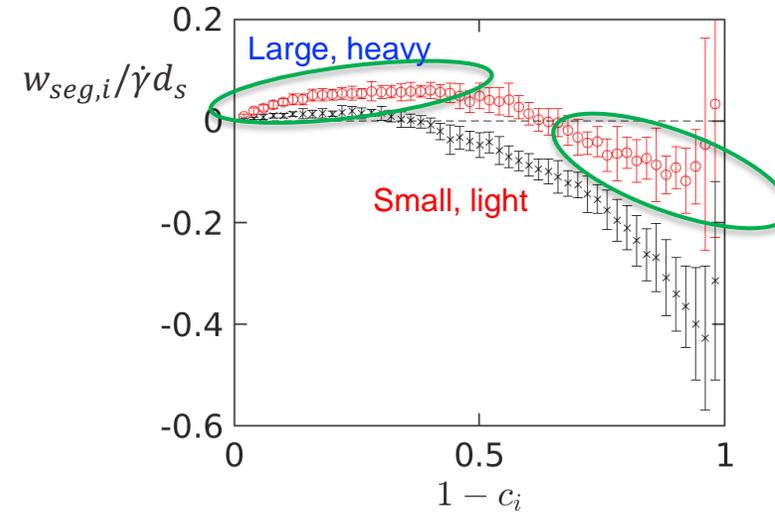
$$\hat{c}_l = 0.2$$



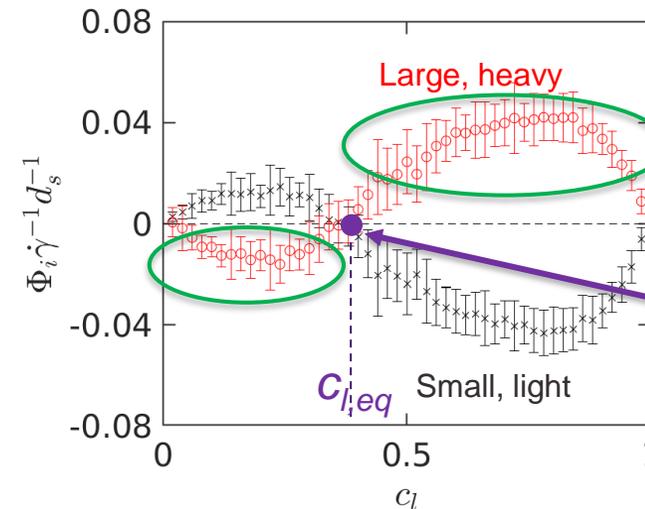
Segregation flux:

$$\Phi_i = w_{seg,i} c_i$$

Normalized segregation velocity

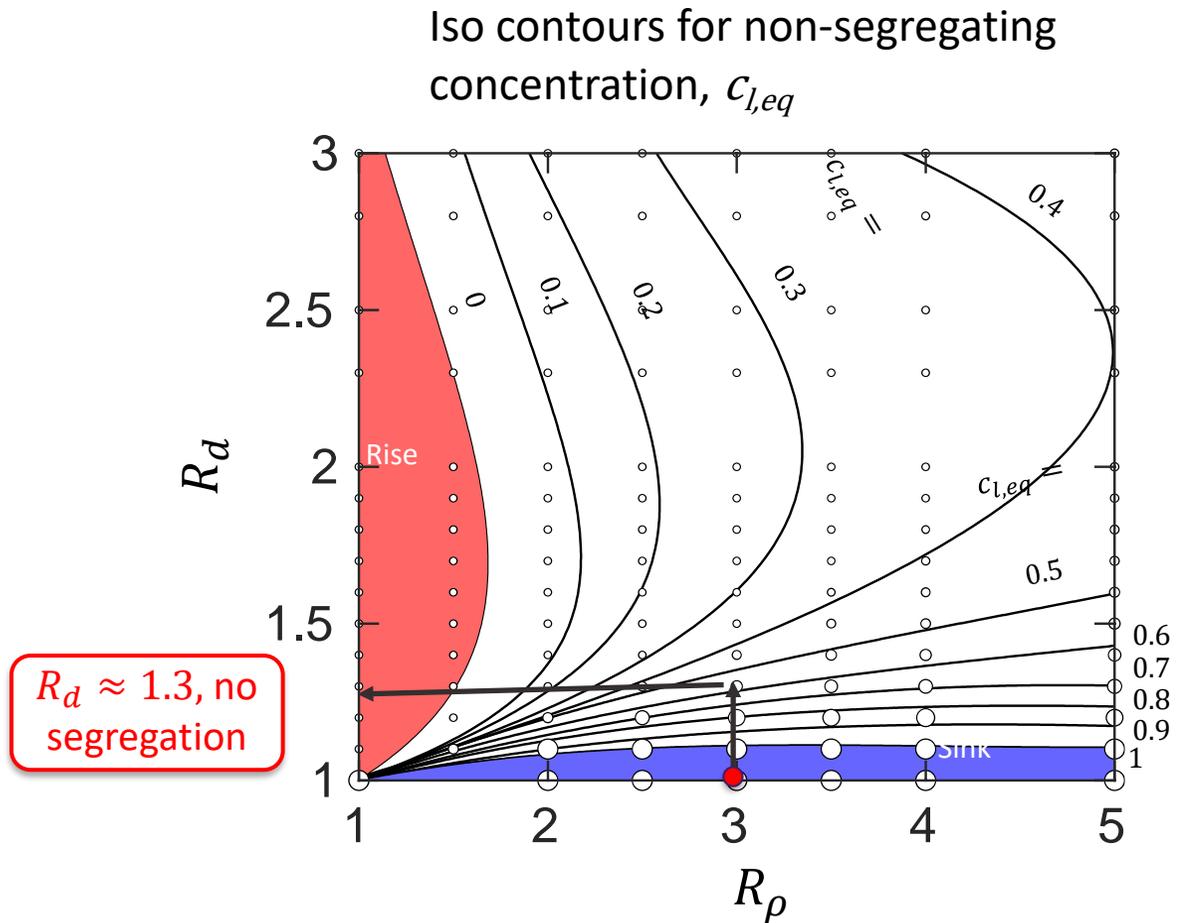


Segregation direction depends on local concentration



No segregation!

Equilibrium phase diagram for non-segregating mixtures



Consider a 60-40 mixture of large steel and small glass particles

$$R_\rho = 3$$

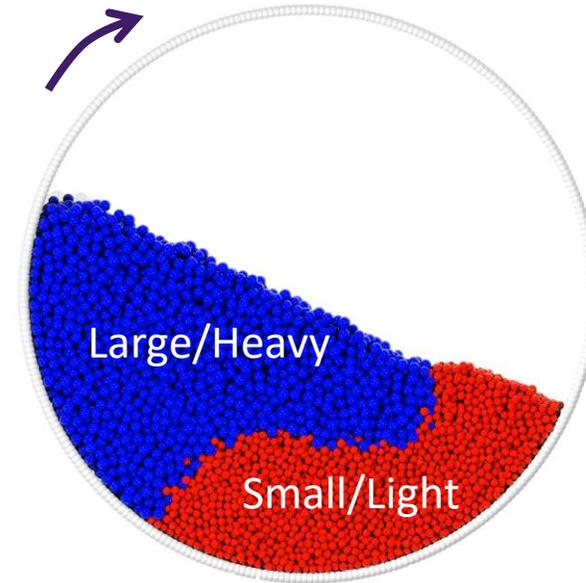
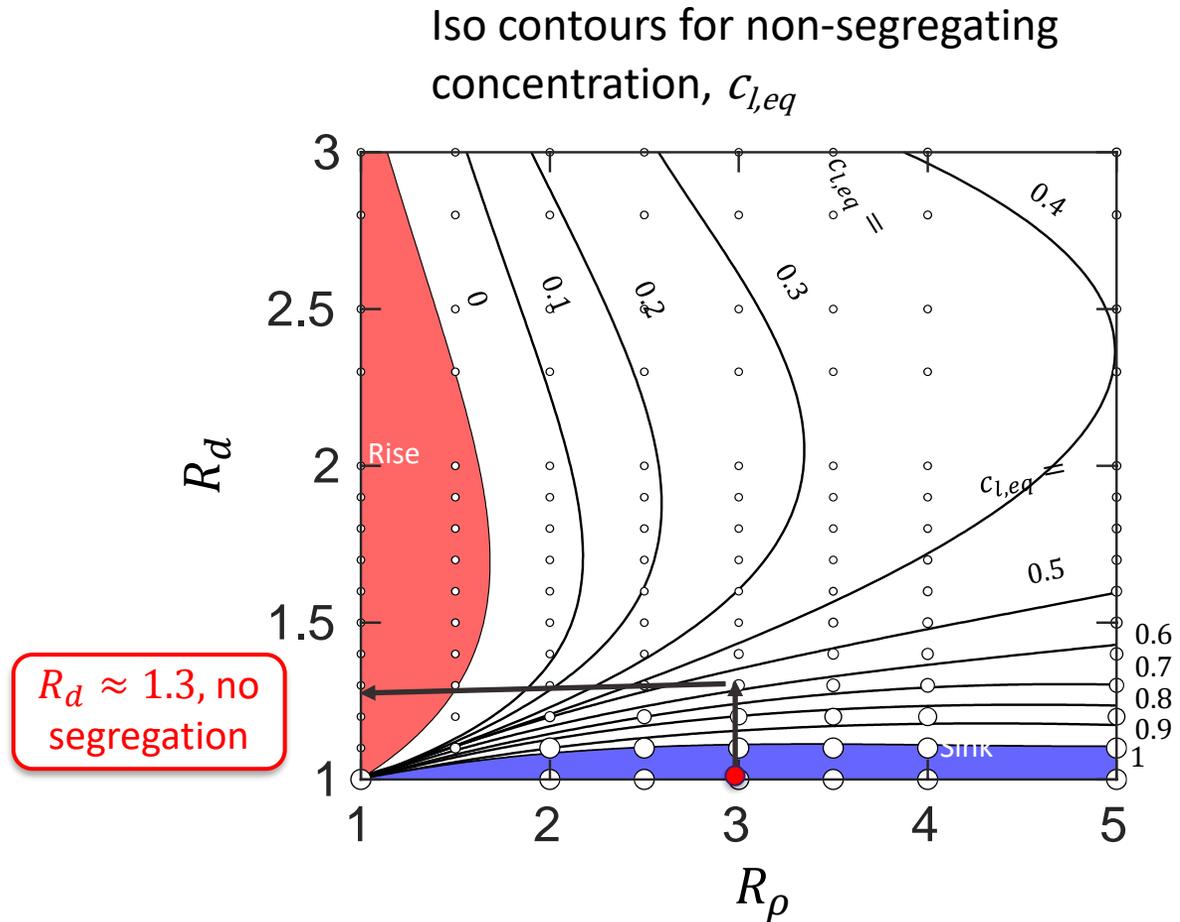
$$c_{l,eq} = 0.6$$

Duan, Umbanhowar, Ottino & Lueptow, JFM, 2021

Duan, Umbanhowar, Ottino & Lueptow, AIChE J, 2023

Equilibrium phase diagram for non-segregating mixtures

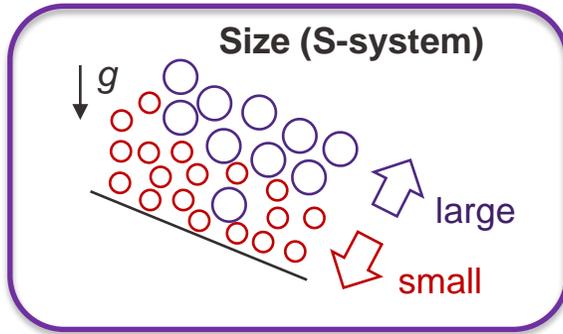
Can we prevent segregation? -- YES



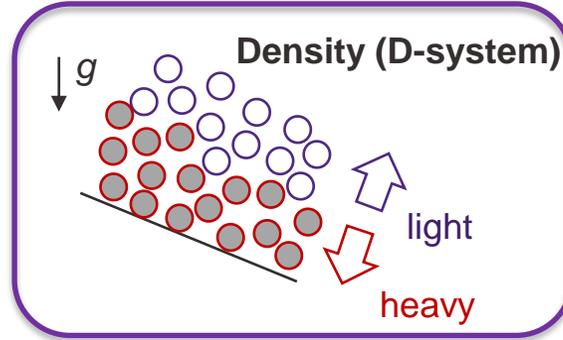
Duan, Umbanhowar, Ottino & Lueptow, JFM, 2021

Duan, Umbanhowar, Ottino & Lueptow, AIChE J, 2023

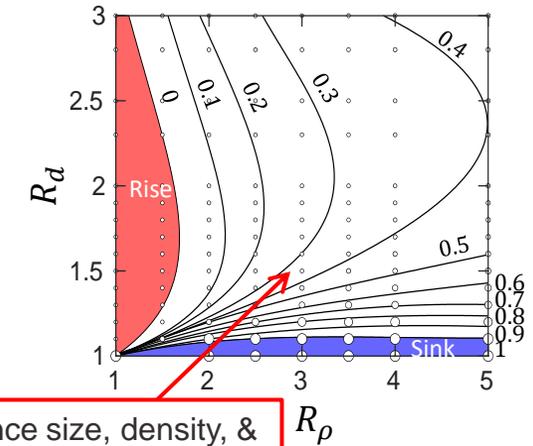
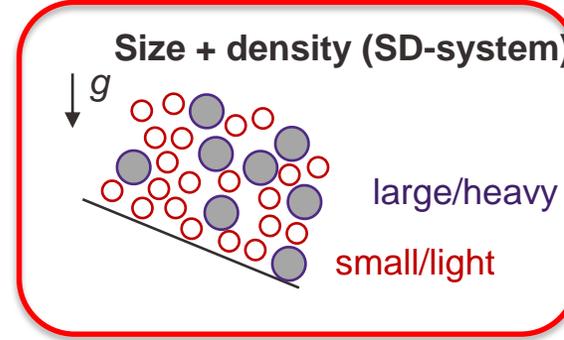
Preventing de-mixing/segregation



$$R_d = \frac{d_l}{d_s}$$



$$R_\rho = \frac{\rho_l}{\rho_s}$$

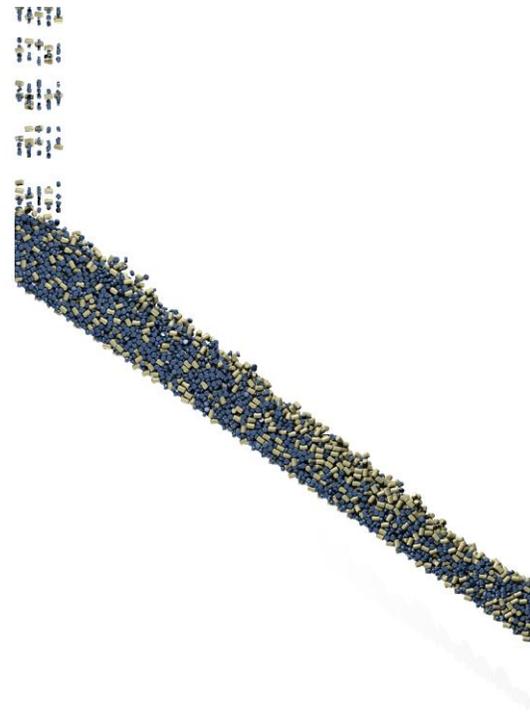
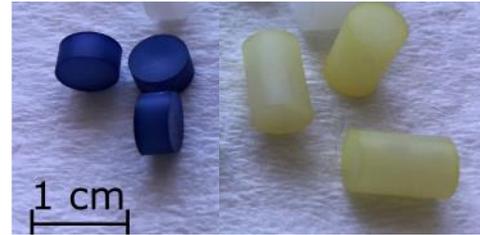


Can we design non-segregating mixtures by balancing effects of size and density? -- YES



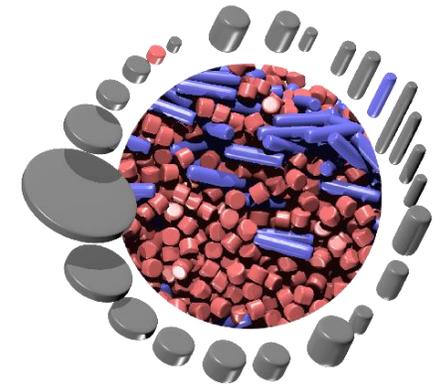
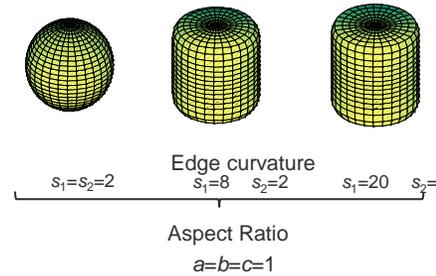
Can we design non-segregating mixtures by balancing shape effects?

Heap flow of non-spherical particles

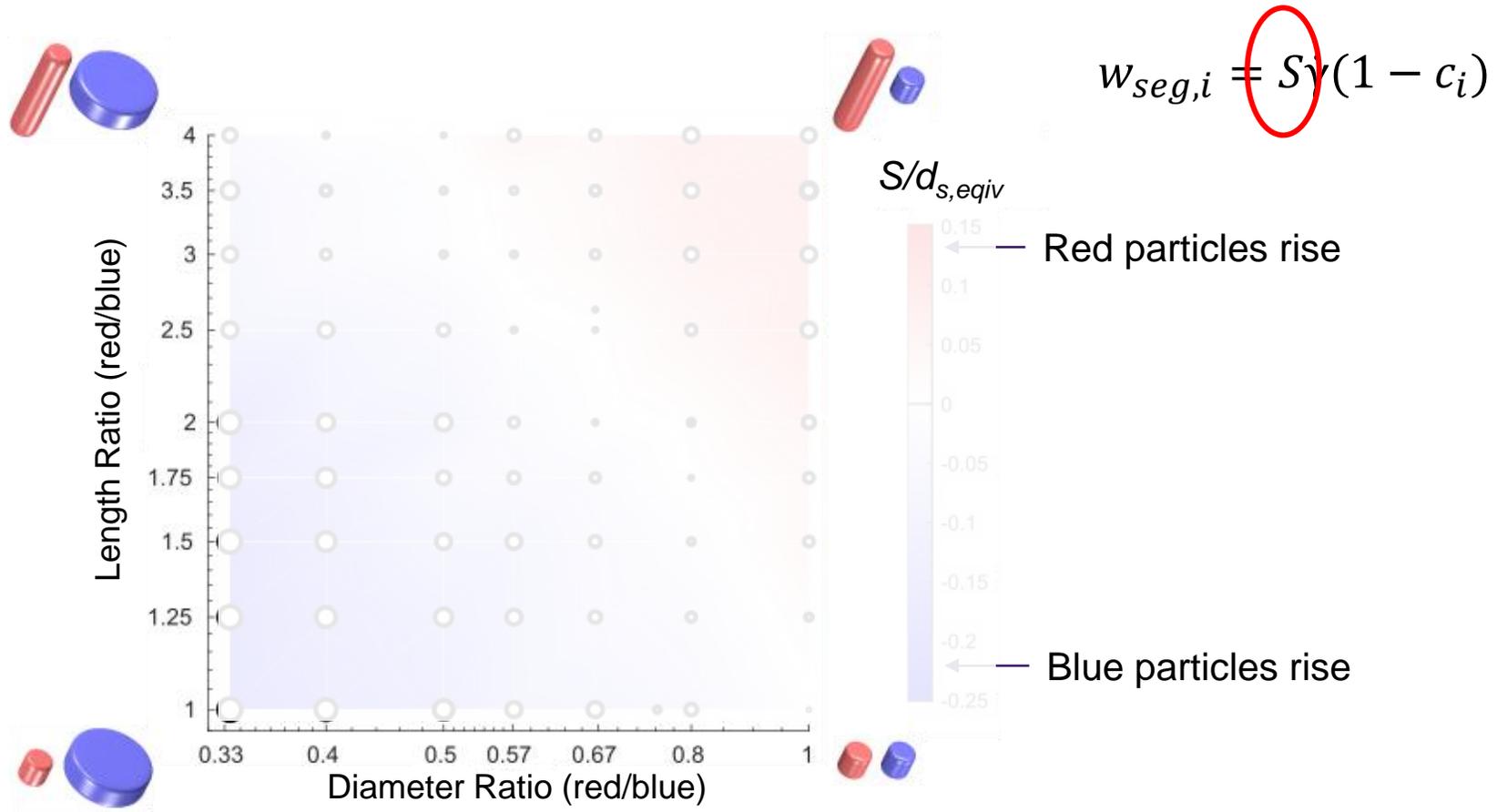


Super-ellipsoids

$$\left(\left| \frac{x}{a} \right|^{s_2} + \left| \frac{y}{b} \right|^{s_2} \right)^{\frac{s_1}{s_2}} + \left| \frac{z}{c} \right|^{s_1} = 1$$



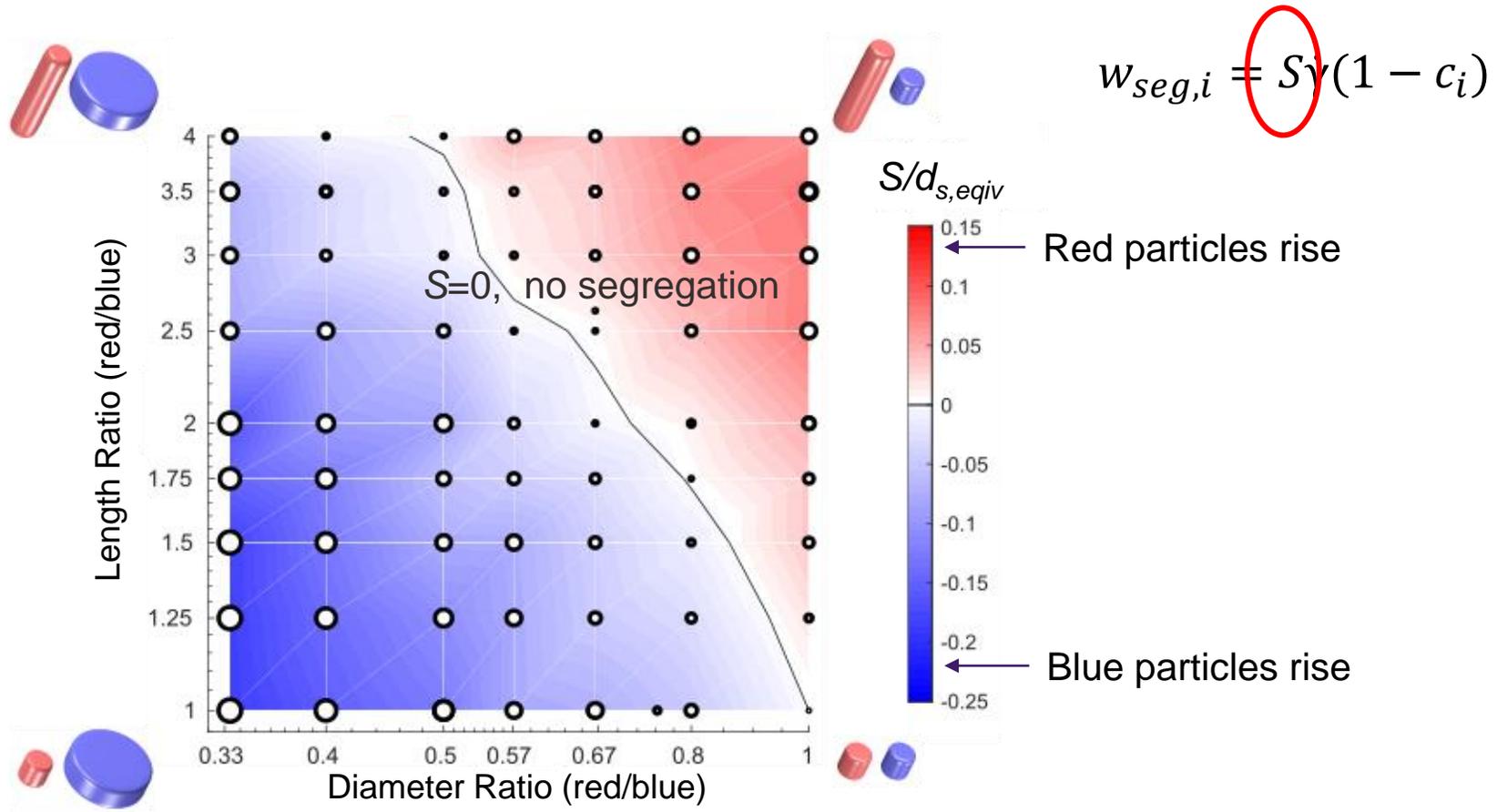
Segregation of disk and rods



$$w_{seg,i} = S\gamma(1 - c_i)$$

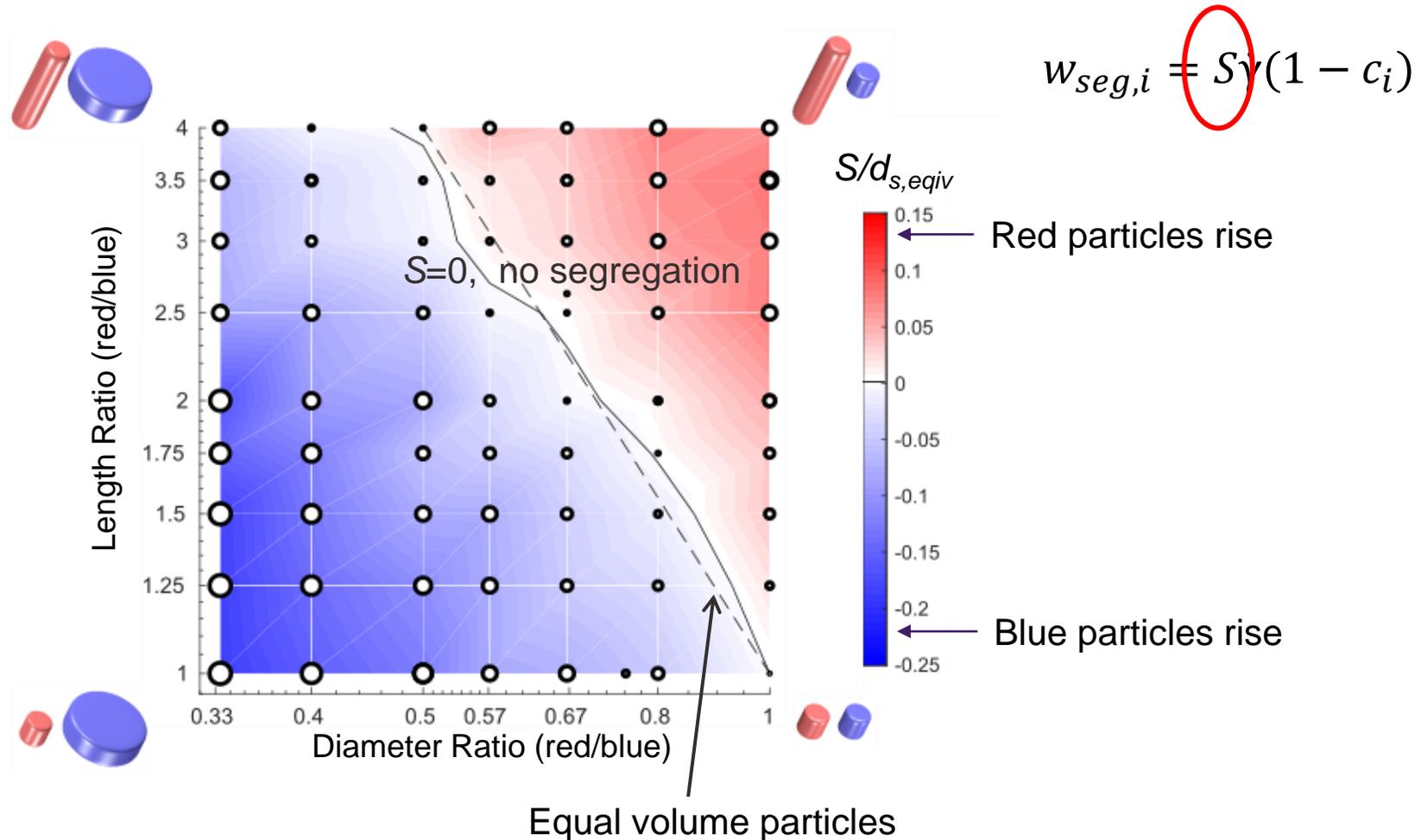
Jones, Ottino, Umbanhowar & Lueptow, *PRRes* 2020
 Jones, Ottino, Umbanhowar & Lueptow, *PRFluids* 2021

Segregation of disk and rods



Jones, Ottino, Umbanhowar & Lueptow, PRRes 2020
 Jones, Ottino, Umbanhowar & Lueptow, PRFluids 2021

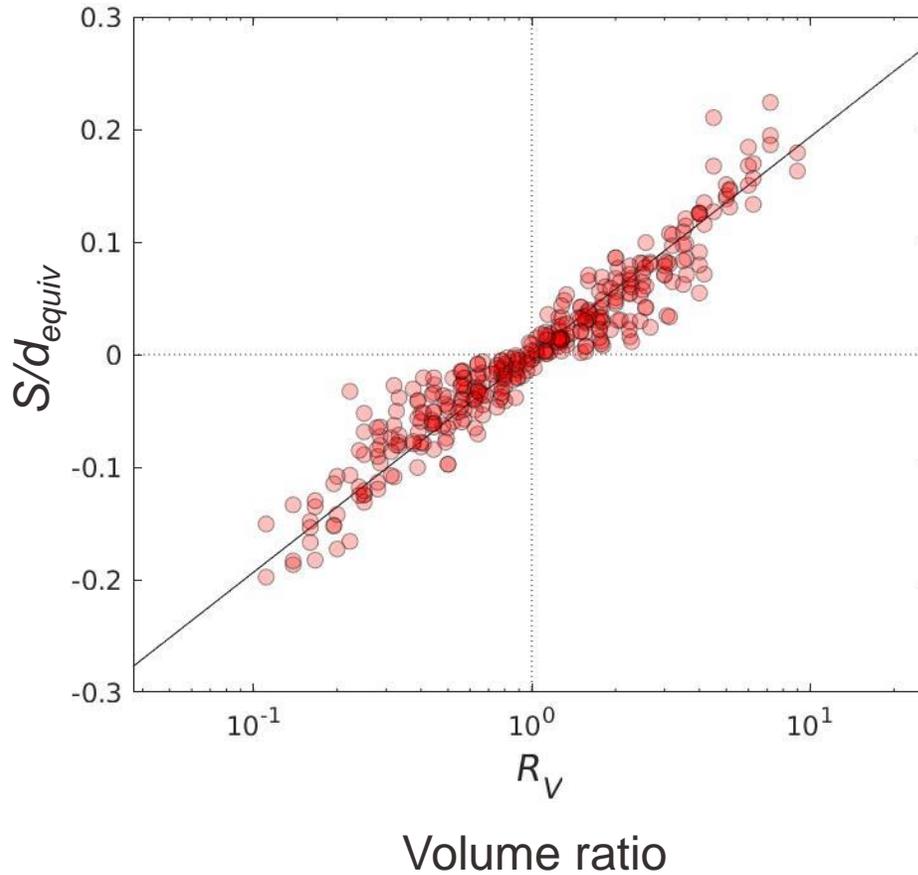
Segregation of disk and rods



Jones, Ottino, Umbanhowar & Lueptow, *PRRes* 2020

Jones, Ottino, Umbanhowar & Lueptow, *PRFluids* 2021

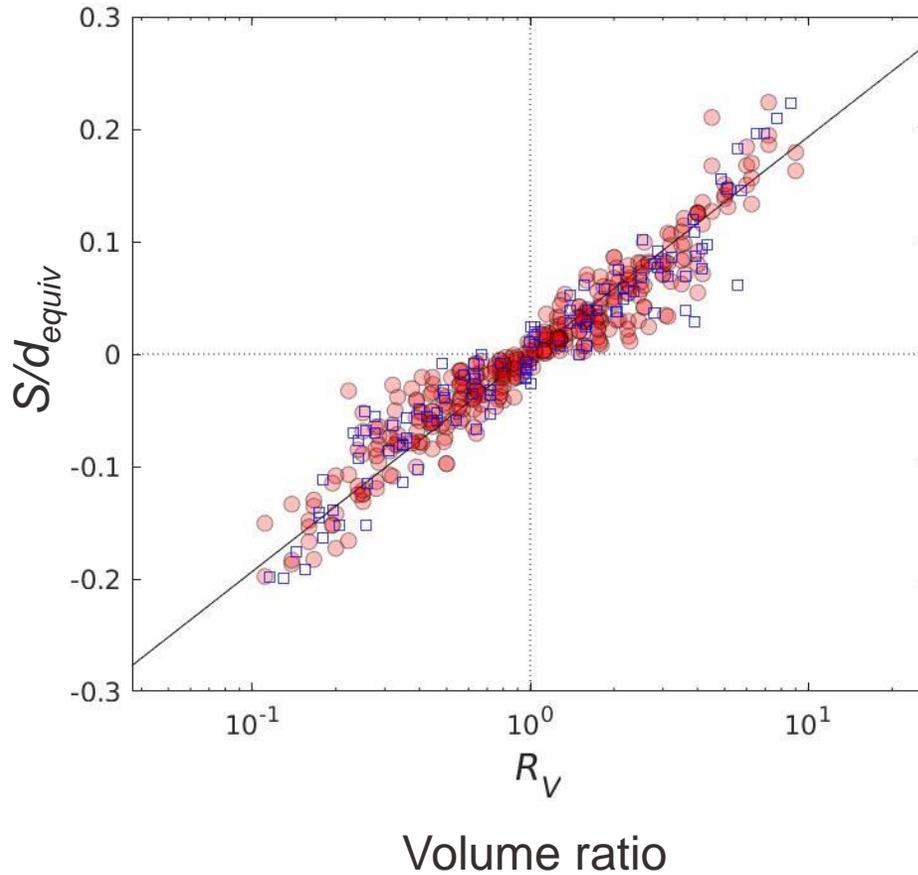
Segregation dependence on volume ratio



$$w_{seg,i} = S\gamma(1 - c_i)$$

○ : cylinder/cylinder 

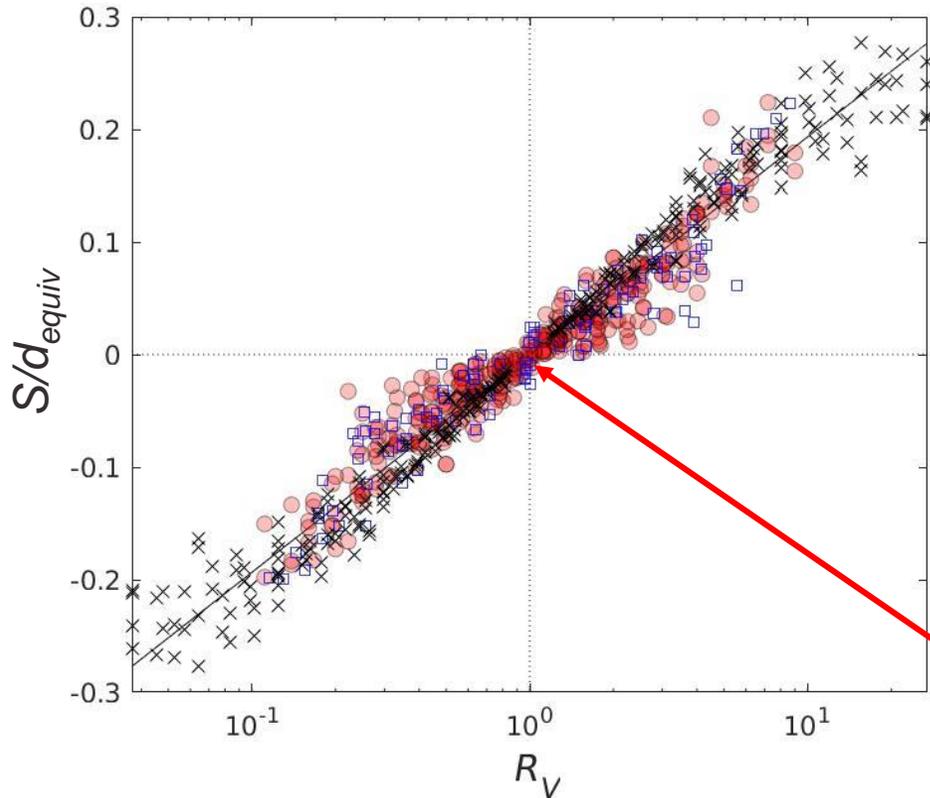
Segregation dependence on volume ratio



$$w_{seg,i} = S\gamma(1 - c_i)$$

- : cylinder/cylinder
 - : sphere/cylinder
-

Segregation dependence on volume ratio



Volume ratio

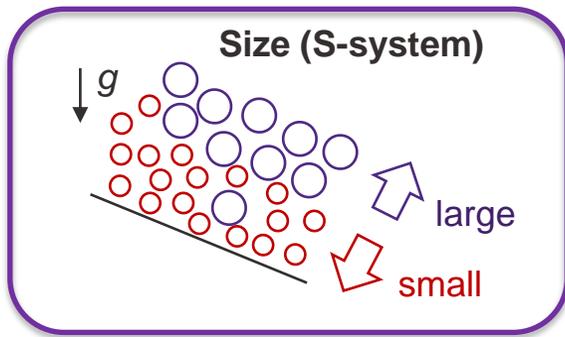
$$w_{seg,i} = S\gamma(1 - c_i)$$

- : cylinder/cylinder
- : sphere/cylinder
- x : sphere/sphere

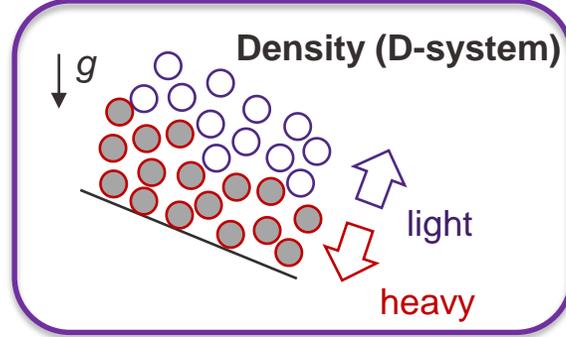
Schlick, Fan, Isner, Umbanhowar, Ottino & Lueptow, *AIChE J* 2015

Equal volume particles results in minimal segregation

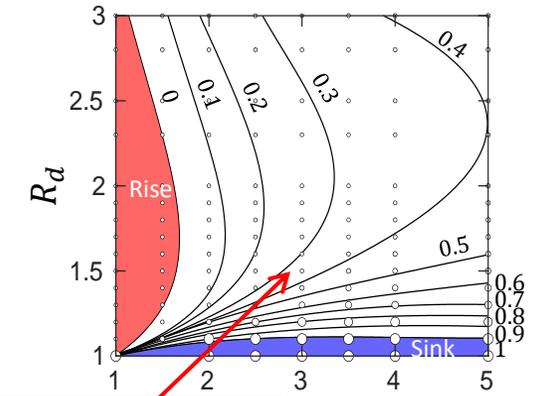
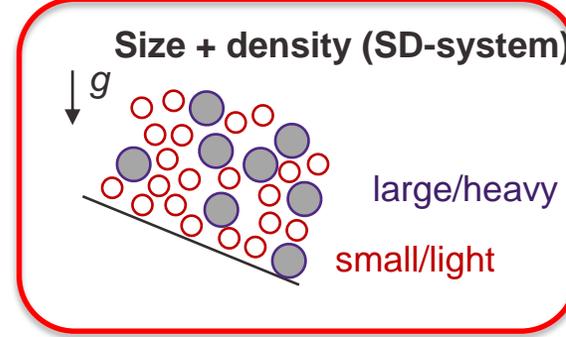
Preventing de-mixing/segregation



$$R_d = \frac{d_l}{d_s}$$



$$R_\rho = \frac{\rho_l}{\rho_s}$$

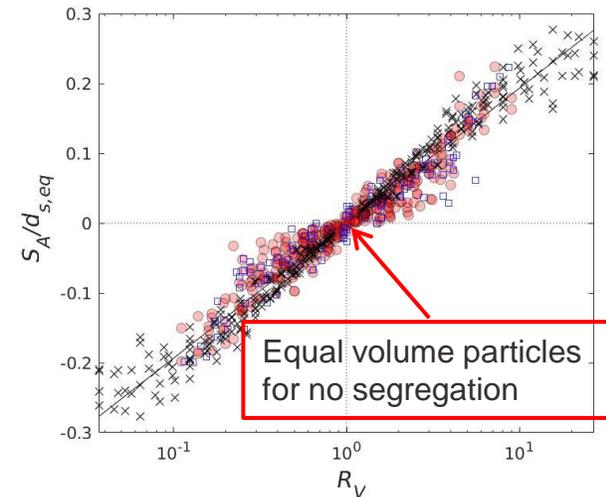


Balance size, density, & concentration for no segregation

Can we design non-segregating mixtures by balancing effects of size and density? -- YES

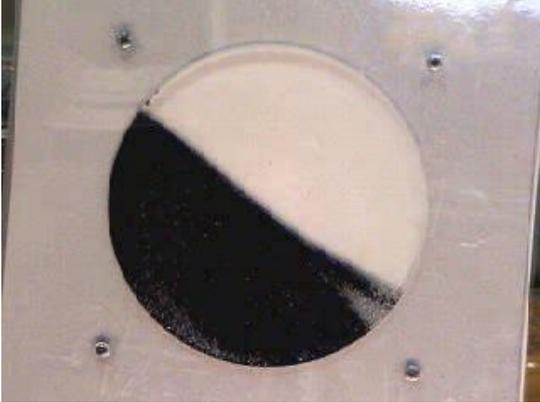


Can we design non-segregating mixtures by balancing shape effects? -- YES

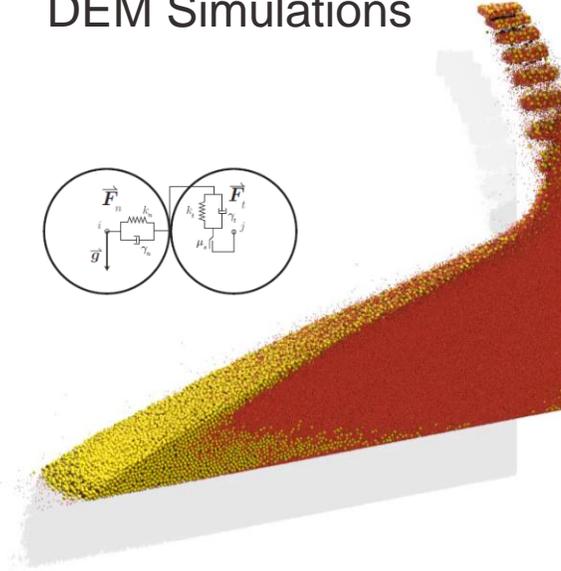


Three tools to study de-mixing / segregation & four questions

Experiments



DEM Simulations



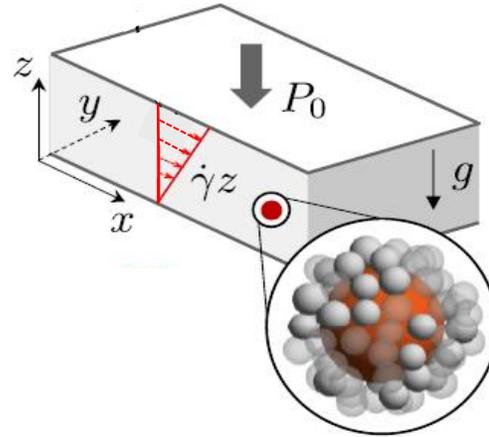
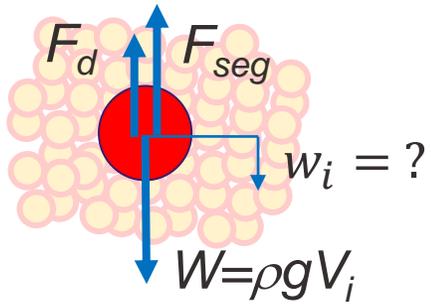
Theoretical Models

$$\frac{\partial c_i}{\partial t} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

- Can we predict demixing/segregation?
- Can we prevent segregation?
- Can we understand segregation?
- Can we understand segregation of fine particles?

Single intruders

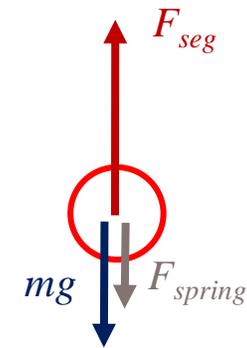
Intruder



Virtual spring to measure force on intruder

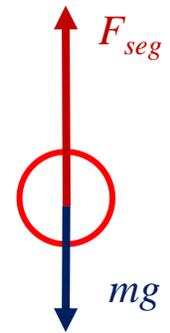
Guillard, Forterre & Pouliquen, *JFM* 2016

$$F_{seg} = mg + F_{spring}$$



Equilibrium

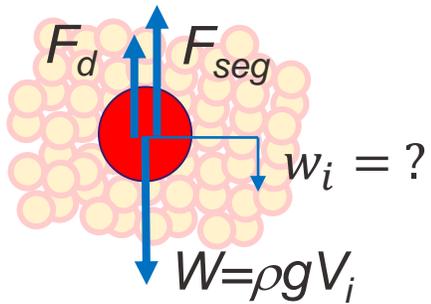
OR



Rise or sink? Velocity?
(no spring)

Single intruder – segregation force

Intruder



$$F_{seg} = f_g(R) \rho g V_i + f_k(R) \frac{p \partial \dot{\gamma}}{\dot{\gamma} \partial z} V_i$$

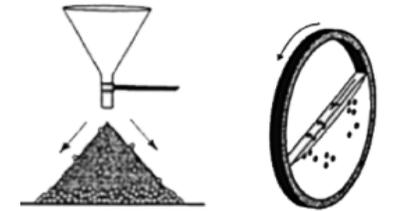
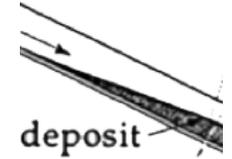
gravity/
buoyancy

kinematics

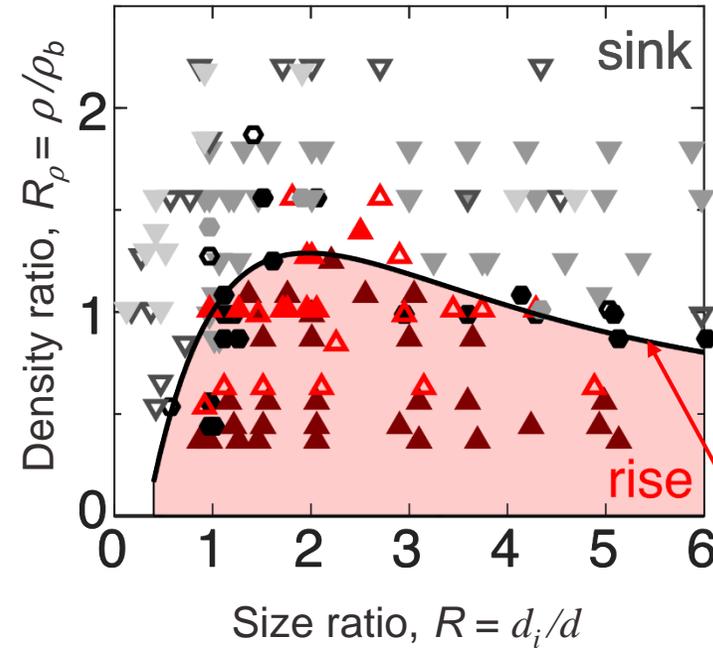
Jing, Ottino, Lueptow & Umbanhowar, PRRes 2020

Jing, Ottino, Lueptow & Umbanhowar, JFM 2021

Experimental intruder data



Felix & Thomas, PRE, 2004

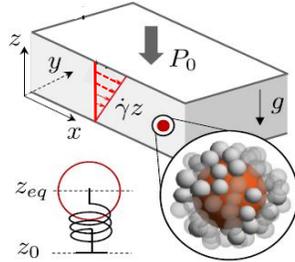
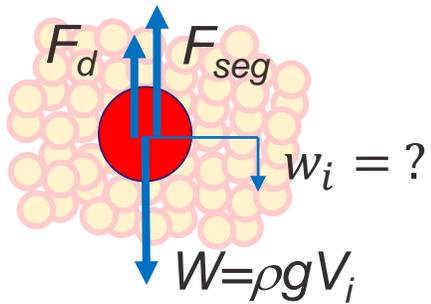


Prediction for
neutral situation

Jing, Ottino, Lueptow & Umbanhowar, PRRes 2020

Single intruder -- drag force (the C_d - Re relation)

Intruder



Stokes-like Drag:

$$F_d = c \pi d_i \eta v_t$$

where $1 \leq c \leq 3$

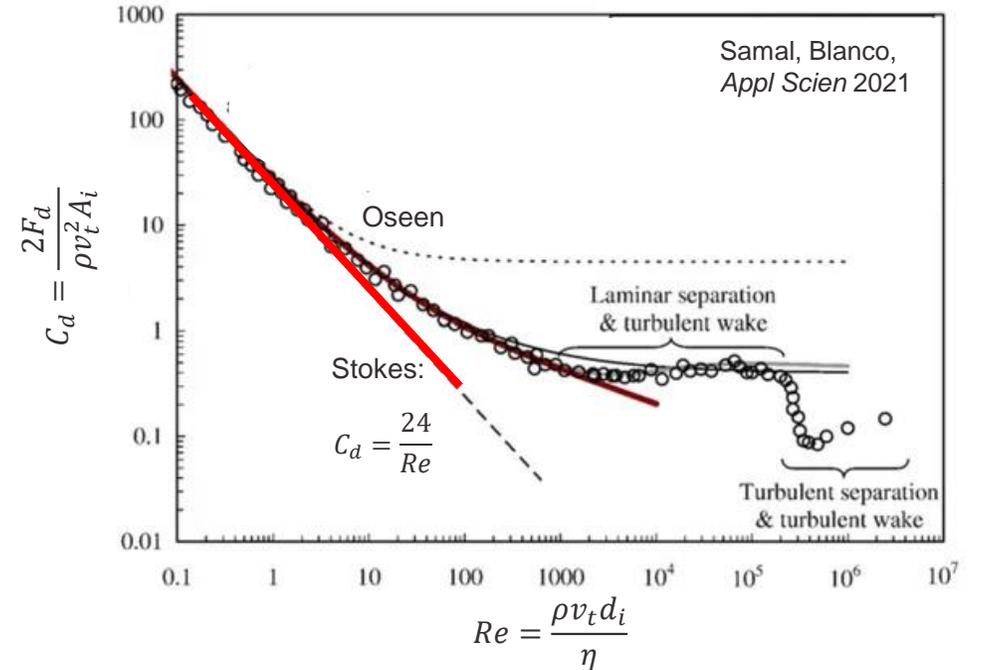
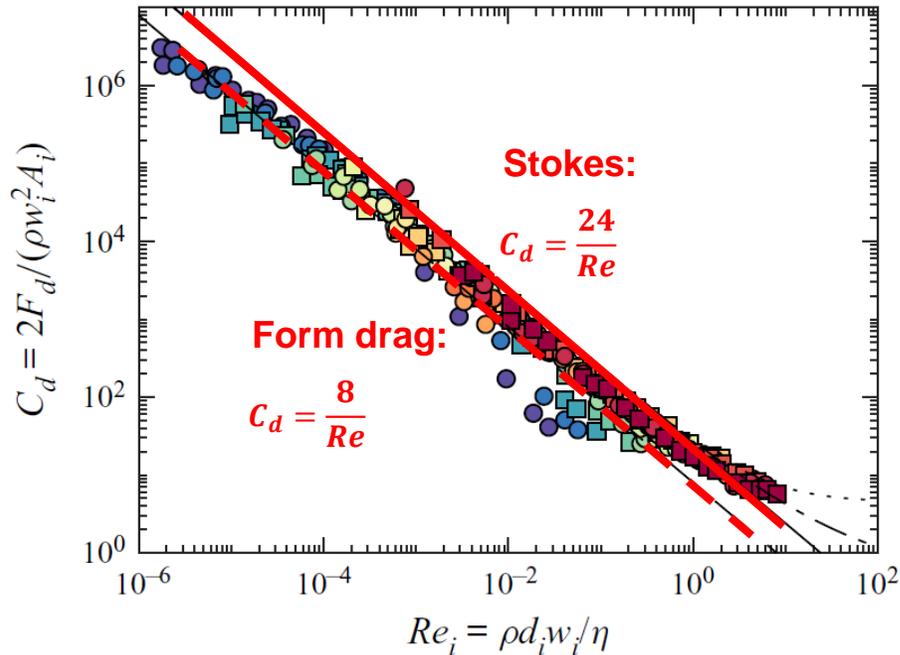


Stokes Drag:

$$F_d = 3\pi d_i \eta v_t$$

$$= 2\pi d_i \eta v_t + \pi d_i \eta v_t$$

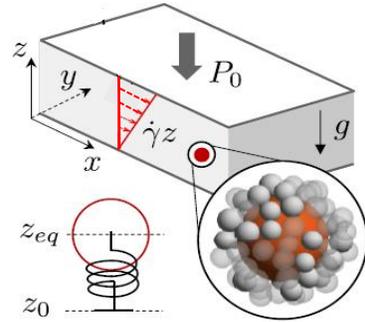
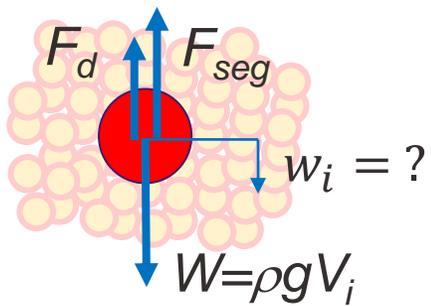
Friction drag Form drag



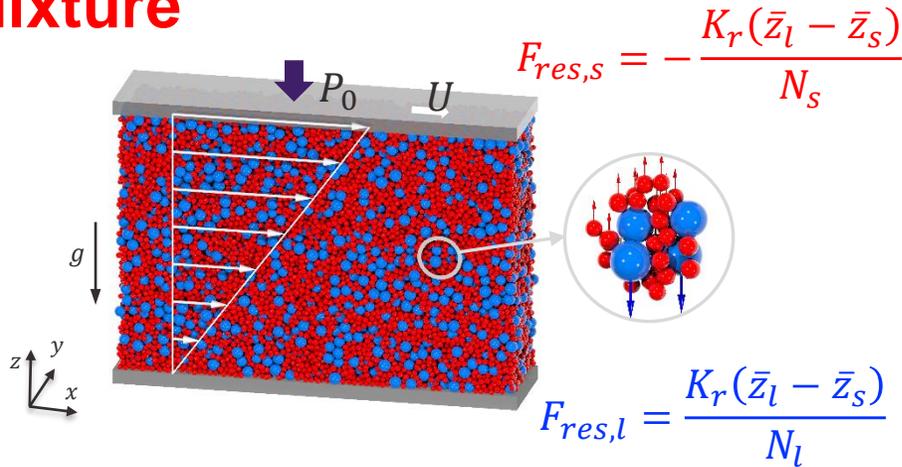
Jing, Ottino, Umbanhowar & Lueptow, JFM 2022

Single intruder extension to mixtures

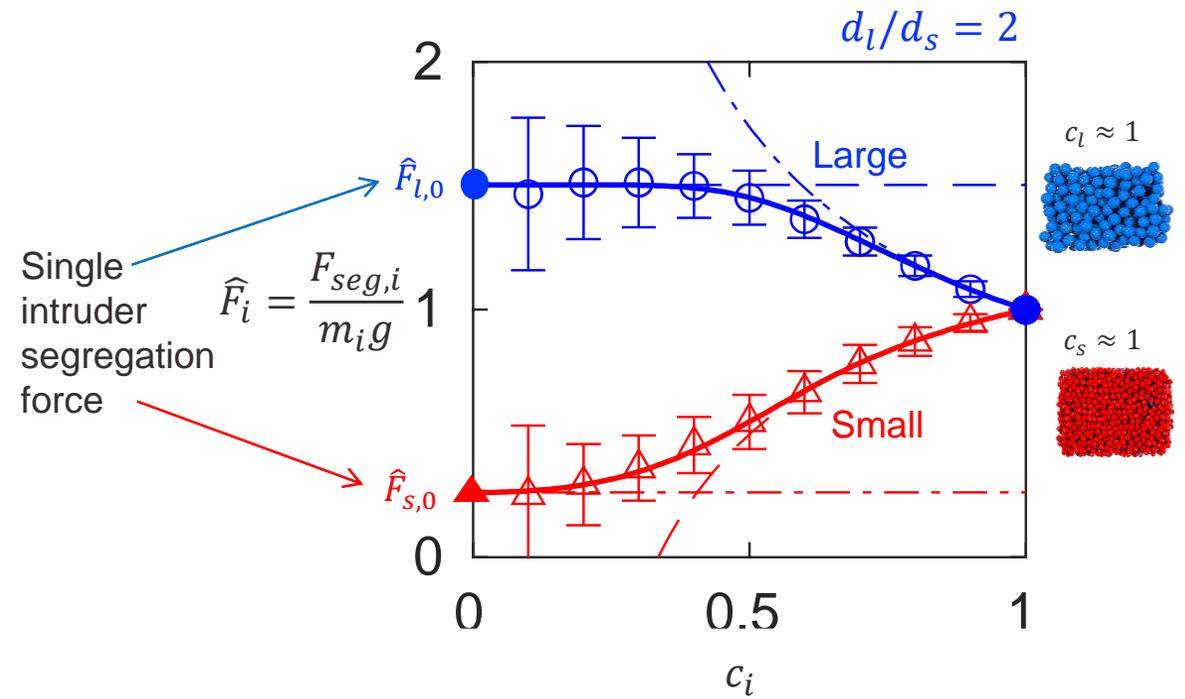
Intruder



Mixture



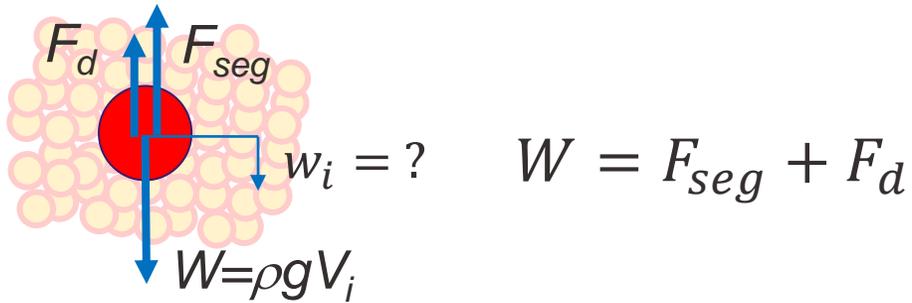
Mixture



Duan, Jing, Umbanhowar, Ottino & Lueptow, JFM 2022

Connecting segregation and drag forces to gravity-driven segregation

Intruder



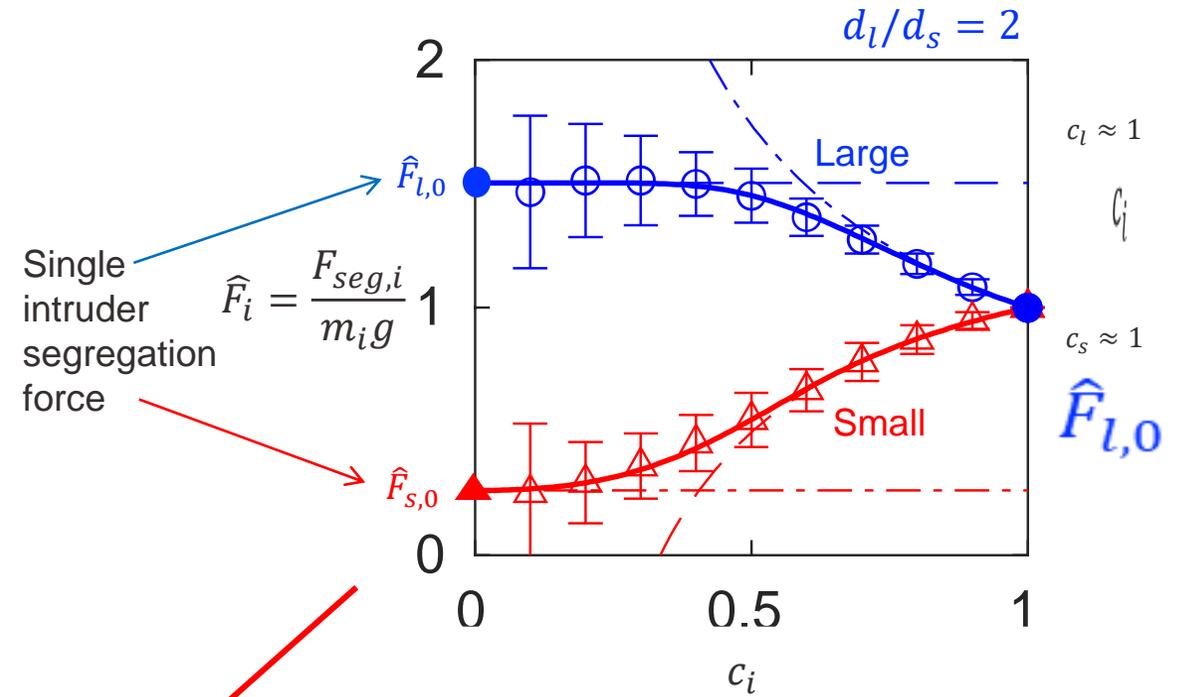
$$F_d = -c\pi\eta d_i w_i$$

Jing, Ottino, Umbanhowar, & Lueptow, JFM 2022

$$F_{seg} = f_g \rho g V_i + f_k \frac{p \partial \dot{\gamma}}{\dot{\gamma} \partial z} V_i$$

Jing, Ottino, Umbanhowar, & Lueptow, JFM 2021

Mixture



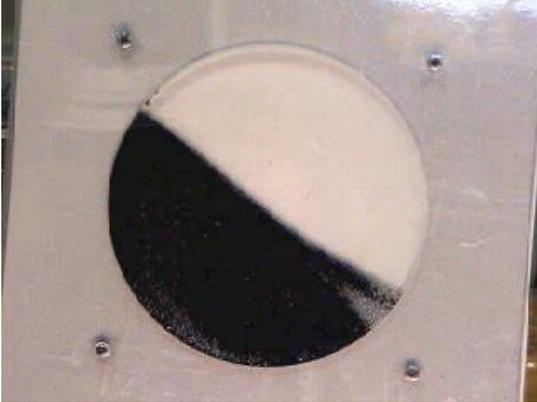
Duan, Jing, Umbanhowar, Ottino & Lueptow, JFM 2022

?

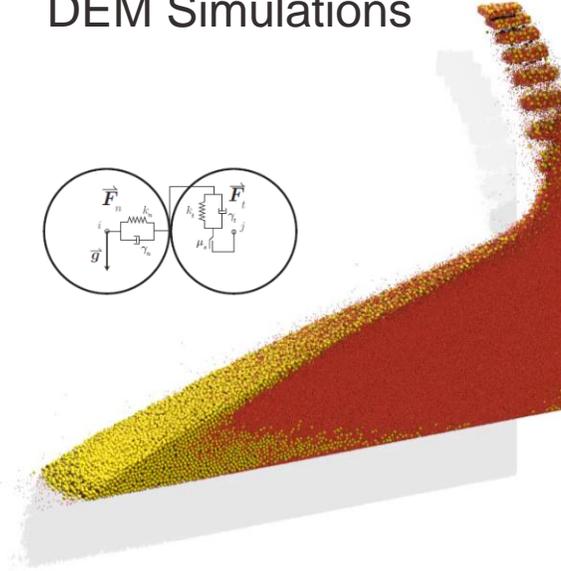
$$\frac{\partial c_i}{\partial t} + \nabla \cdot (\mathbf{u} c_i) + \frac{\partial}{\partial z} (c_i w_{seg,i}) = \nabla \cdot (D \nabla c_i)$$

Three tools to study de-mixing / segregation & four questions

Experiments



DEM Simulations



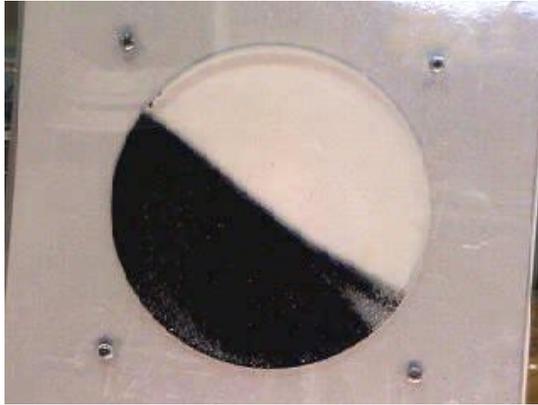
Theoretical Models

$$\frac{\partial c_i}{\partial t} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

- Can we predict demixing/segregation?
- Can we prevent segregation?
- Can we understand segregation?
- Can we understand segregation of **fine particles**?

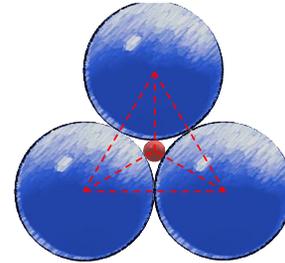
Segregation of fine particles ($R > 3$)

Shear-driven percolation for $R < 3$



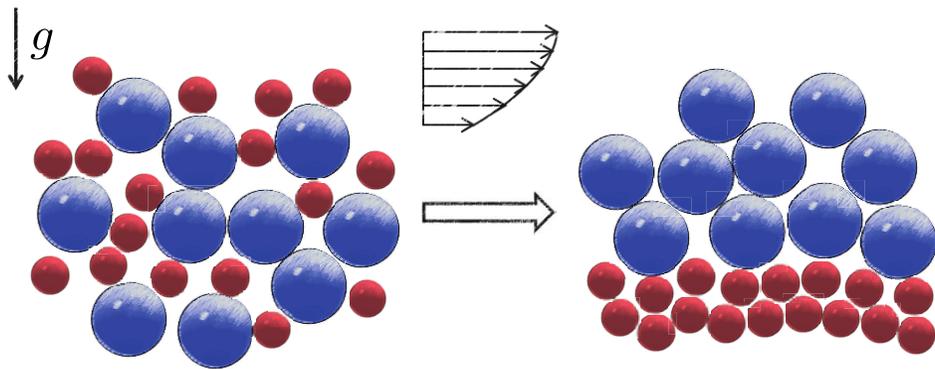
versus

Free-sifting



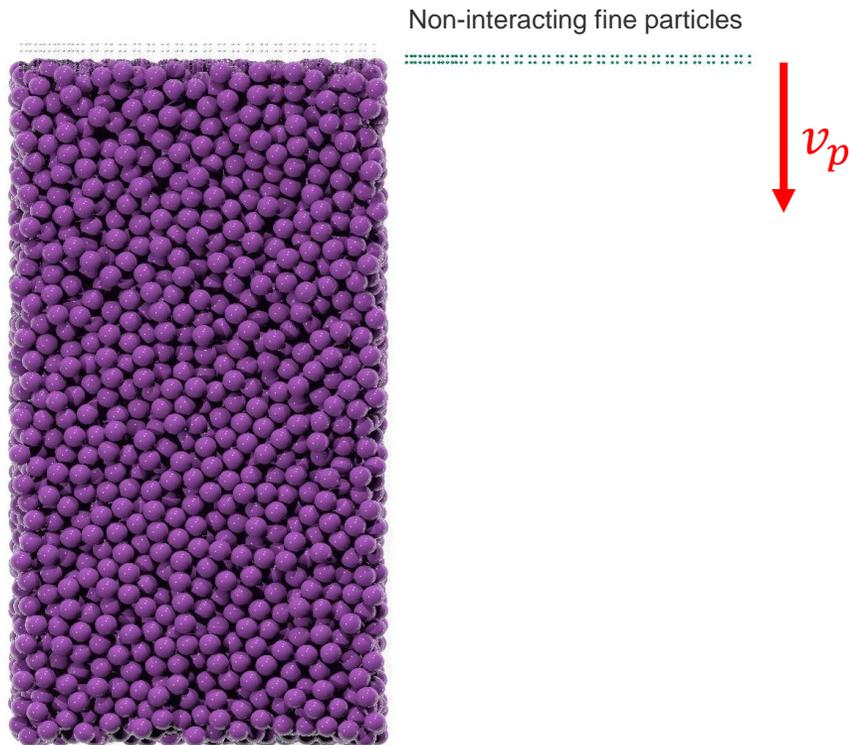
Critical size ratio:

$$R_{cr} = \frac{d_l}{d_s} = \left(\frac{2}{\sqrt{3}} - 1 \right)^{-1} = 6.464$$

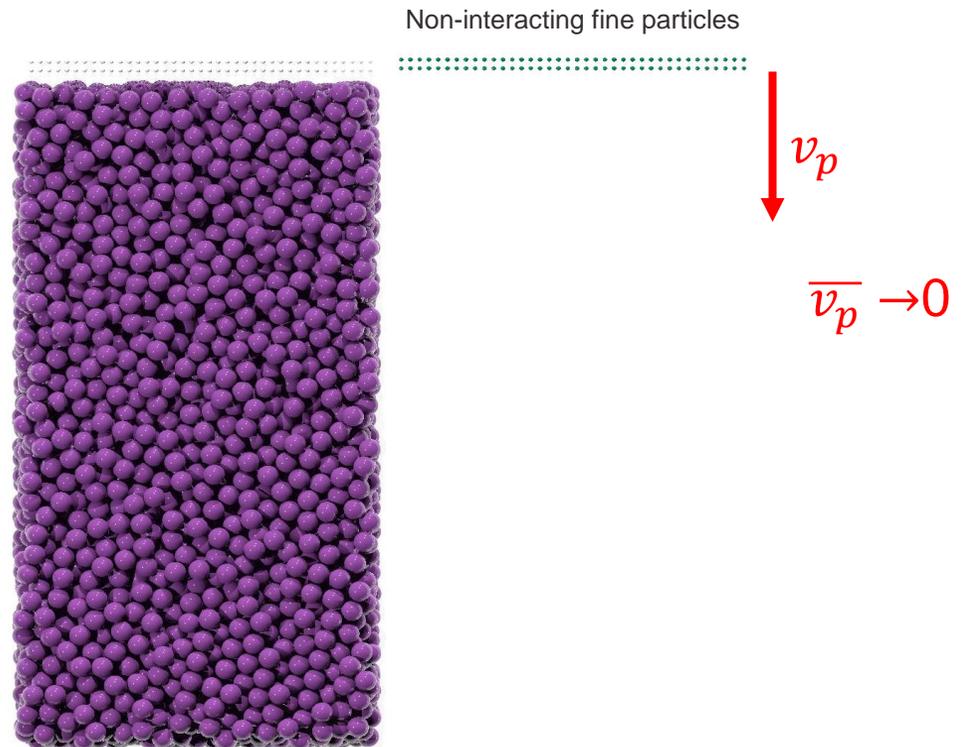


Fine particle percolation in a static bed of large particles: a quick check

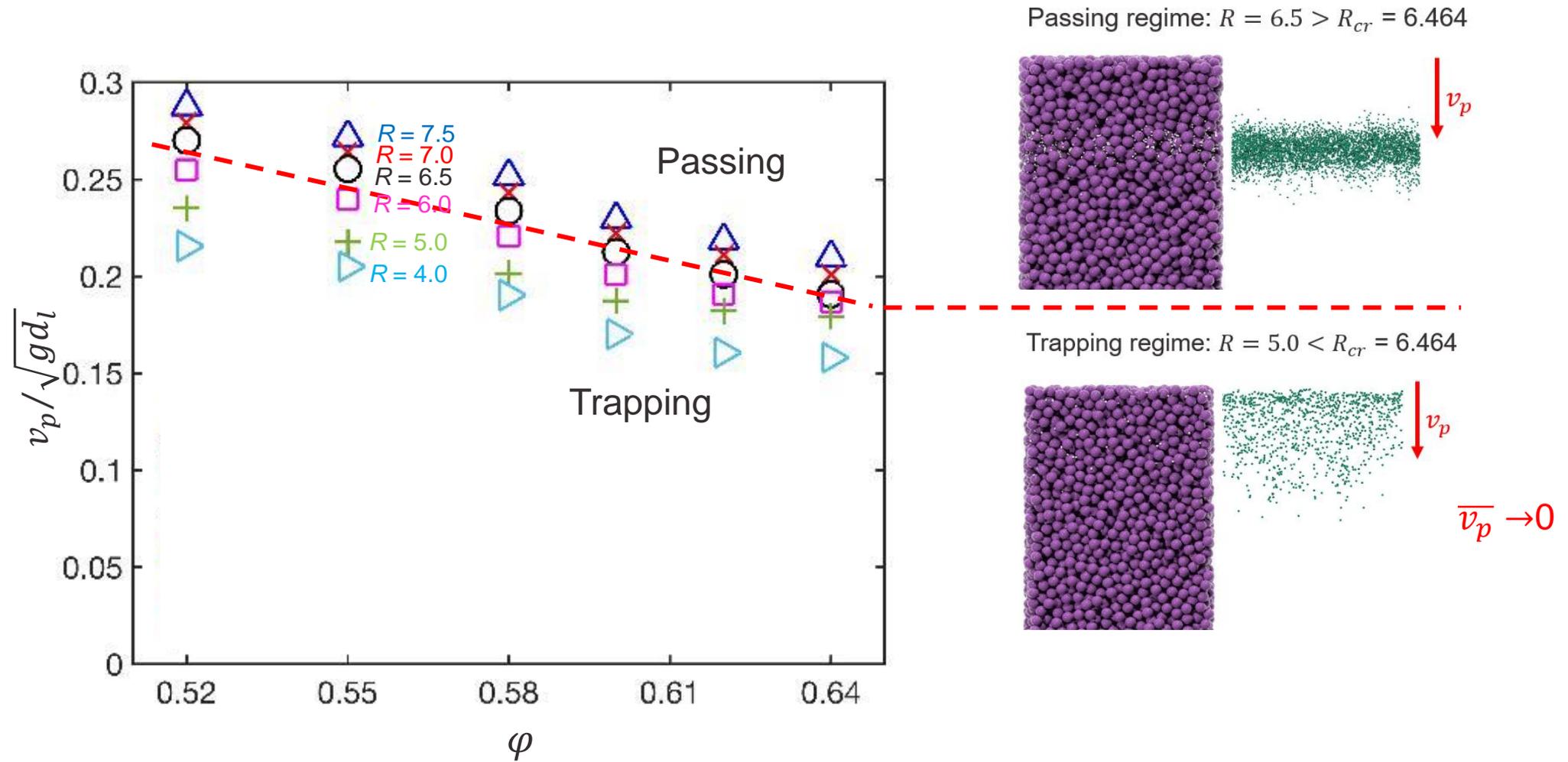
Passing regime: $R = 6.5 > R_{cr} = 6.464$



Trapping regime: $R = 5.0 < R_{cr} = 6.464$

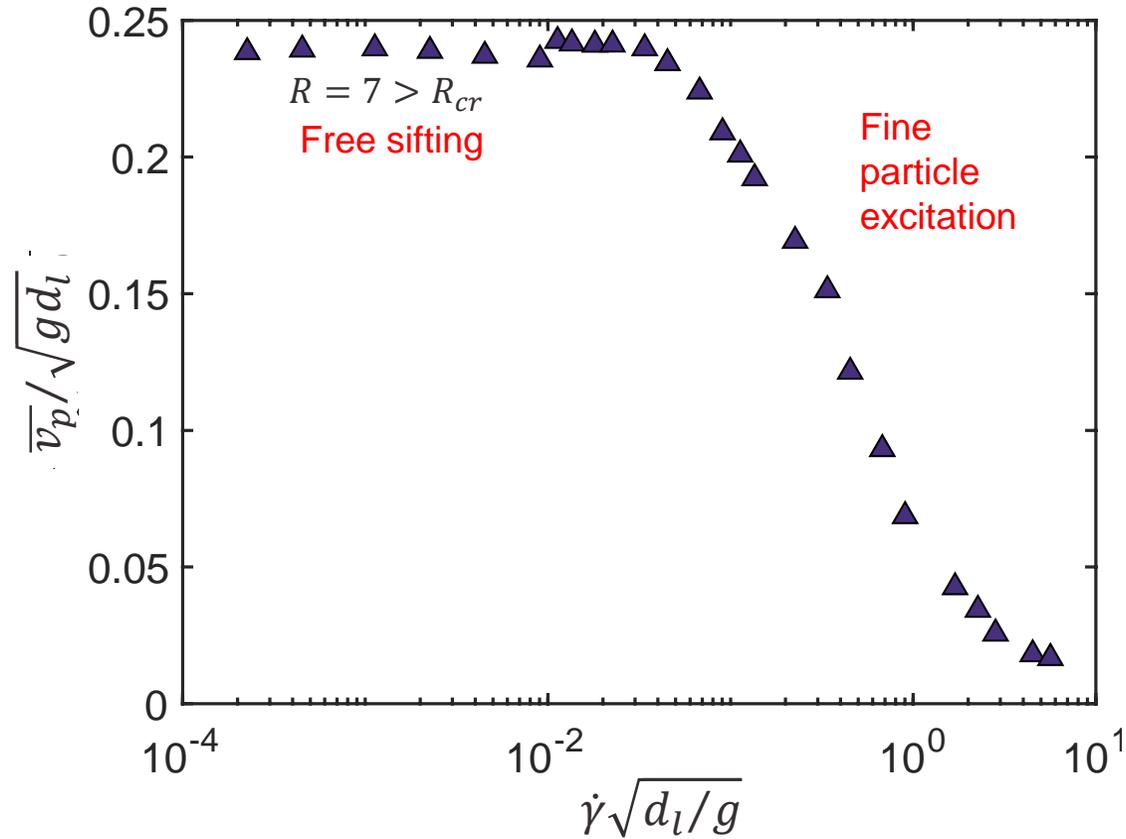


Percolation velocity in a static bed of large particles: trapping and passing regimes

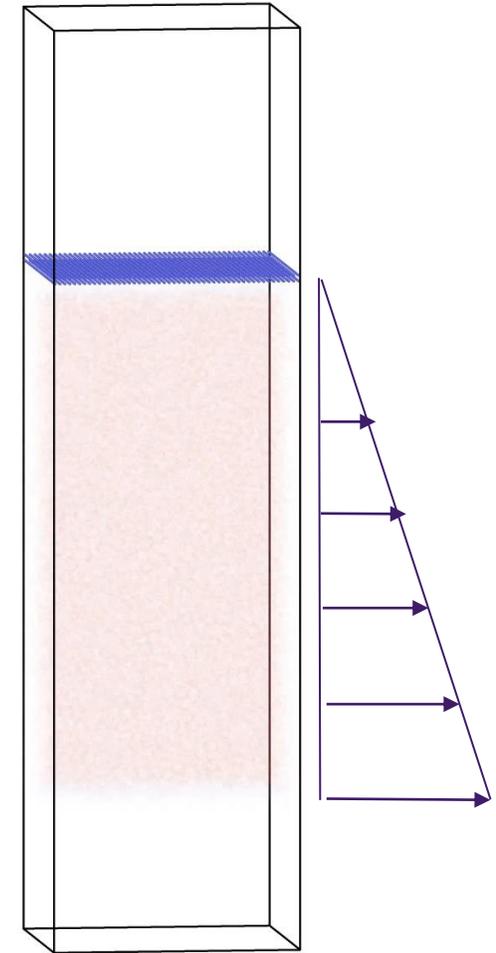


Gao, Ottino, Umbanhowar & Lueptow, PRE 2023

Percolation velocity in a sheared bed of large particles

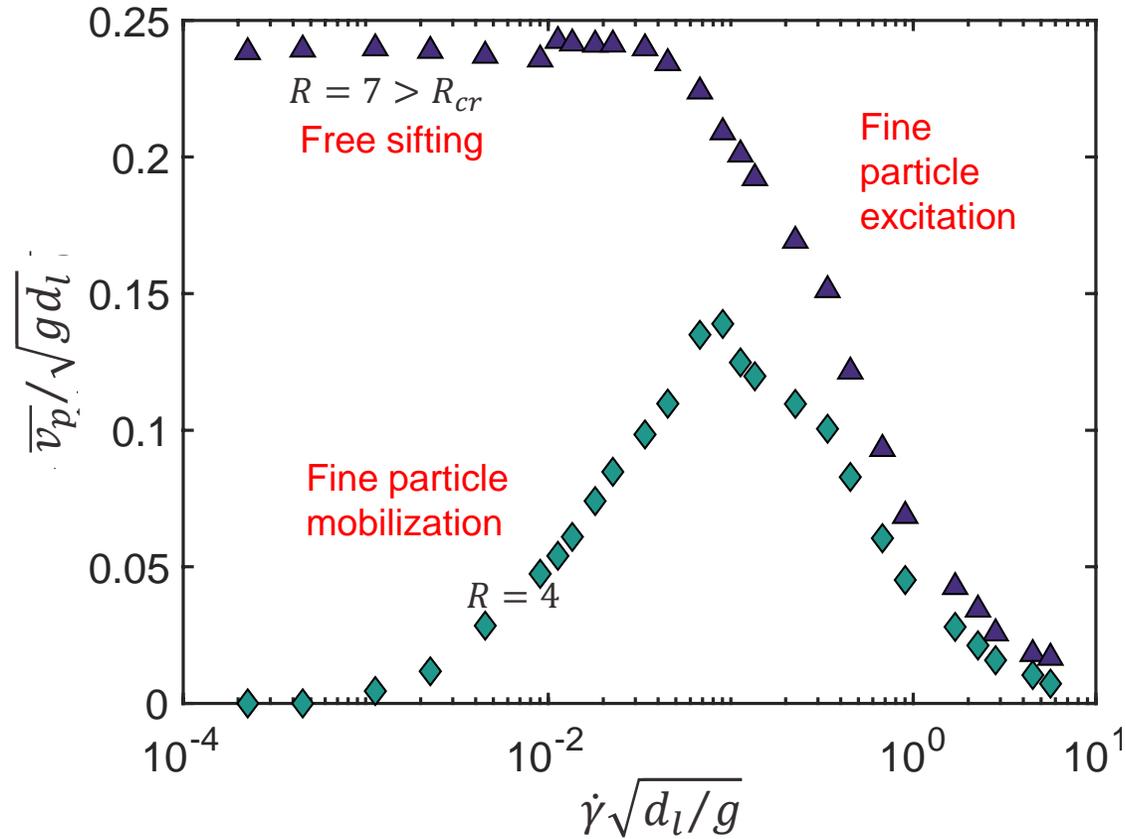


Gao, Ottino, Lueptow & Umbanhowar, in preparation, 2023

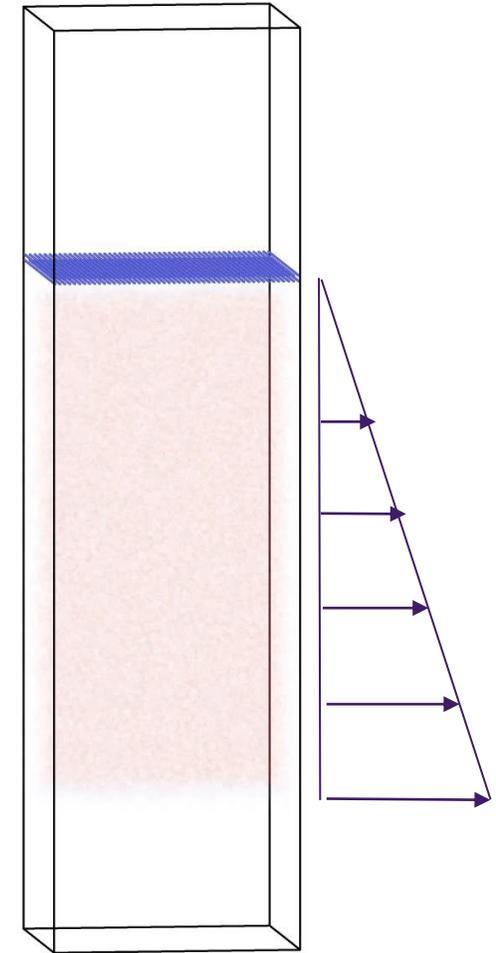


Simulation by Dhairya Vyas

Percolation velocity in a sheared bed of large particles

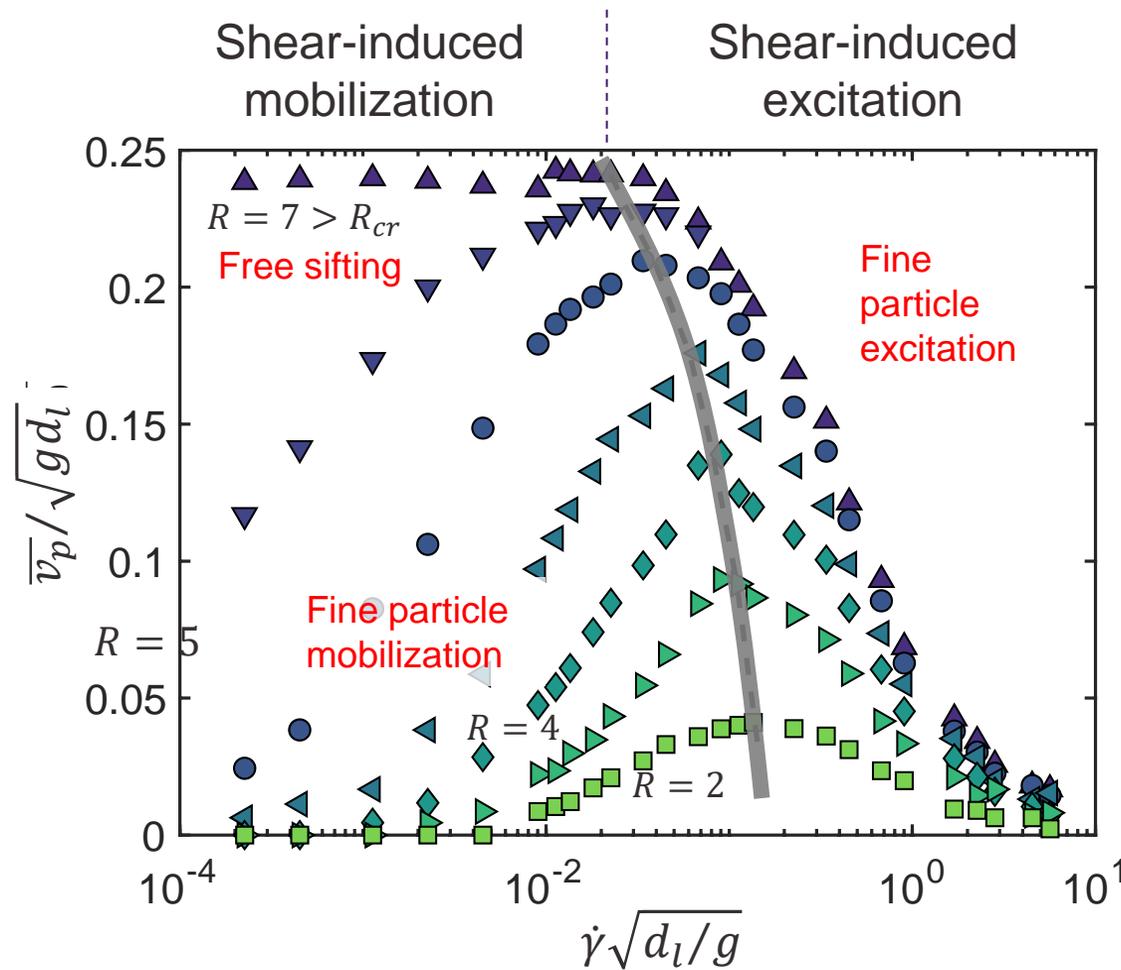


Gao, Ottino, Lueptow & Umbanhowar, in preparation, 2023



Simulation by Dhairya Vyas

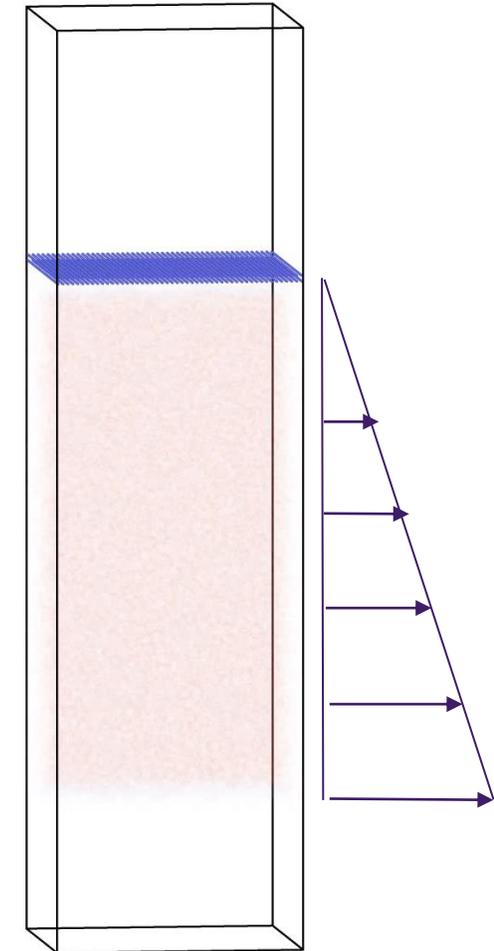
Percolation velocity in a sheared bed of large particles



Gao, Ottino, Lueptow & Umbanhowar, in preparation, 2023

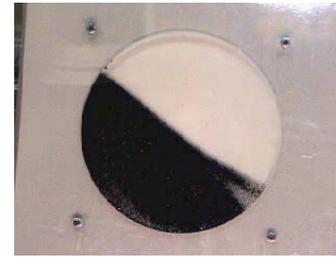
Previous work:

Khola...Wassgren, *Powd Tech*, 2016
Trehwela...Gray, *JFM*, 2021



Simulation by Dhairya Vyas

What's the point?



- Can we predict segregation?
- Can we prevent segregation?
- Can we understand segregation?
- Can we understand segregation of fine particles?

1. Predict segregation

$$\frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z} + \frac{\partial}{\partial z} (w_{seg,i} c_i) = \nabla \cdot (D \nabla c_i)$$

$$w_{seg,i} = S\dot{\gamma}(1 - c_i)$$

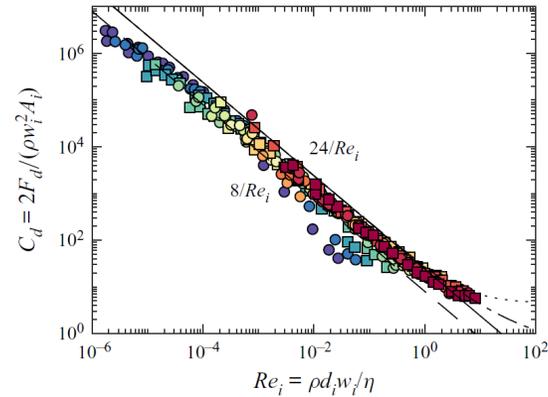
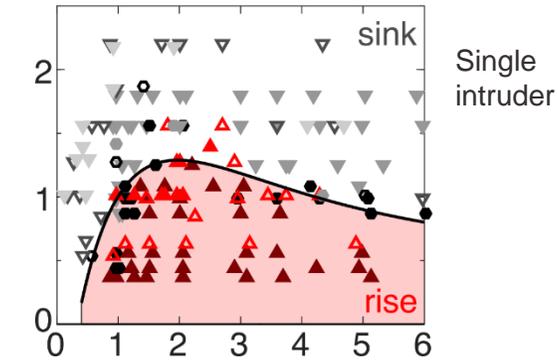
2. Prevent segregation

.....

Equal volume particles
mix regardless of shape



3. Particle level forces

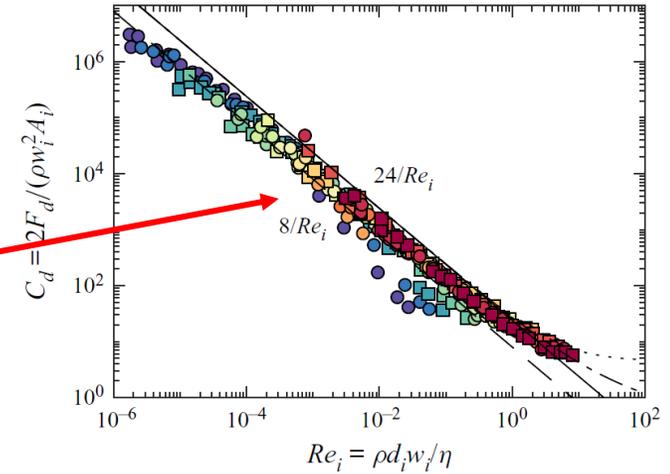


4. Fine particles

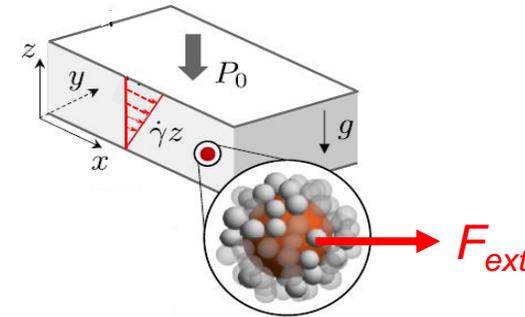
What is next?

- Forces on a particle
 - Can we connect particle forces to segregation velocity?

$$\frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z} + \frac{\partial}{\partial z} (w_{seg,i} c_i) = \nabla \cdot (D \nabla c_i)$$



- Can we measure the lift forces on a particle due to slip or rotation?

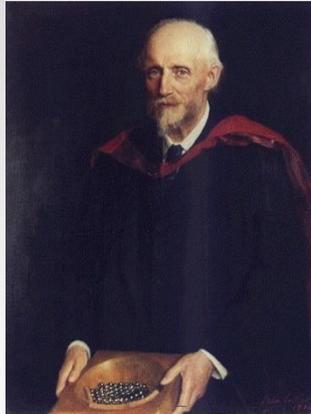


- Free sifting for $R_d > 3$

$$R_d = 6$$

Simulation by Song Gao

Thanks to.....



Osborne Reynolds ($Re=\rho VD/\mu$):
 “On the sub-mechanics of the universe”, 1902.



“Can we develop a general theory of
 the **motion of granular materials?**”



Karl Jacob: “Can we better
 understand granular
 segregation?” 2010



Yi Fan: “Can we predict granular
 segregation?” 2014



John Hecht: “How can we apply
 this to industrial segregation
 problems?” 2015

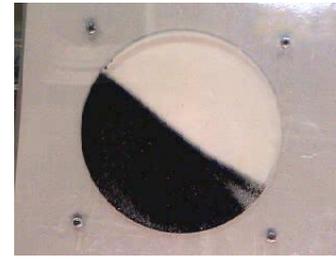


ZHEKAI DENG, YIFEI DUAN, YI FAN, BEN FREIREICH, ALEX FRY, SONG GAO, HANTAO HE, JOHN HECHT,
 AUSTIN ISNER, KARL JACOB, LU JING, RYAN P. JONES, JAMES F. KOCH, MADHUSUDHAN KODAM,
JULIO M. OTTINO, CONOR SCHLICK, JÖRG THEUERKAUF, **PAUL B. UMBANHOWAR**,
 VIDYA VIDYAPATI, HONGYI XIAO, MENGQI YU, ZAFIR ZAMAN,
 QIONG ZHANG, YONGZHI ZHAO



Funded by NSF CMMI-1000469, CBET-1511450, CBET-1929265
 In collaboration with Procter & Gamble and The Dow Chemical Company

Questions?

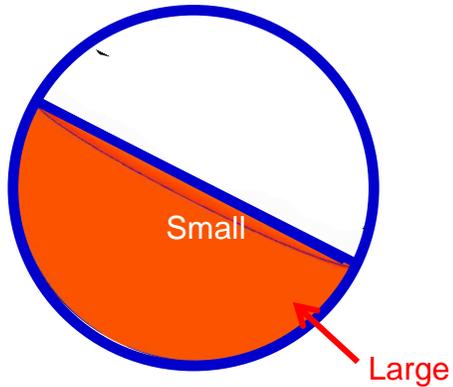


- Can we predict segregation?
- Can we prevent segregation?
- Can we understand segregation?
- Can we understand segregation of fine particles?

1. Predict segregation

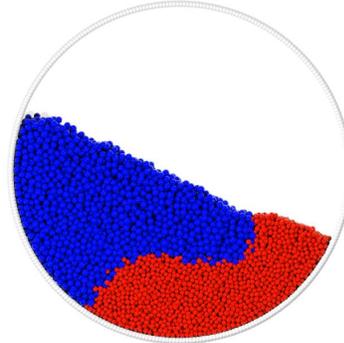
$$\frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z} + \frac{\partial}{\partial z} (w_{seg,i} c_i) = \nabla \cdot (D \nabla c_i)$$

$$w_{seg,i} = S \dot{\gamma} (1 - c_i)$$



2. Prevent segregation

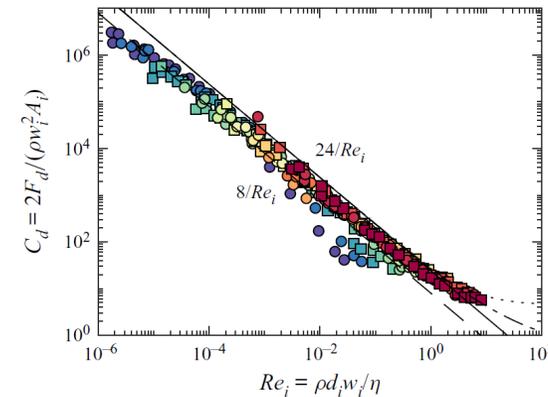
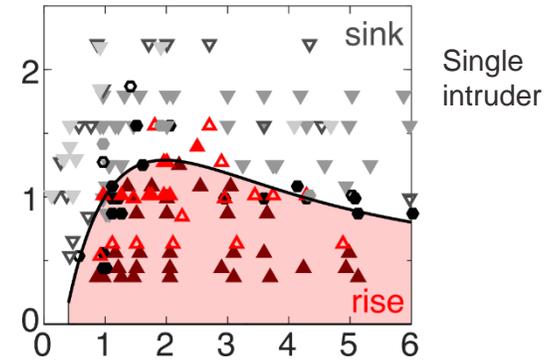
Mixing depends on R_ρ , R_d , and c



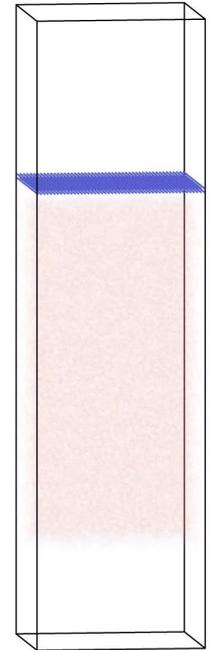
Equal volume particles mix regardless of shape



3. Particle level forces



4. Fine particles



PREDICTING AND PREVENTING SEGREGATION FOR SMALL, DENSE, IRREGULAR, AND FINE PARTICLES

IPFRI Meeting, June, 2023

RICHARD M. LUEPTOW

with

ZHEKAI DENG, YIFEI DUAN, YI FAN, BEN FREIREICH, ALEX FRY, SONG GAO, HANTAO HE, JOHN HECHT,
AUSTIN ISNER, KARL JACOB, LU JING, RYAN P. JONES, JAMES F. KOCH, MADHUSUDHAN KODAM,
JULIO M. OTTINO, CONOR SCHLICK, JÖRG THEUERKAUF, PAUL B. UMBANHOWAR,
VIDYA VIDYAPATI, DHAIRYA VYAS, HONGYI XIAO, MENGQI YU, ZAFIR ZAMAN, QIONG ZHANG, YONGZHI ZHAO

Funded by The Dow Chemical Company, Procter & Gamble Company, and
NSF CMMI-1435065, CBET-1511450, CBET-1929265, CBET-2203703



Northwestern | McCORMICK SCHOOL OF
ENGINEERING