The Vagaries of Particulate Flow

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A New Multi-Scale Paradigm for Particulate Flows

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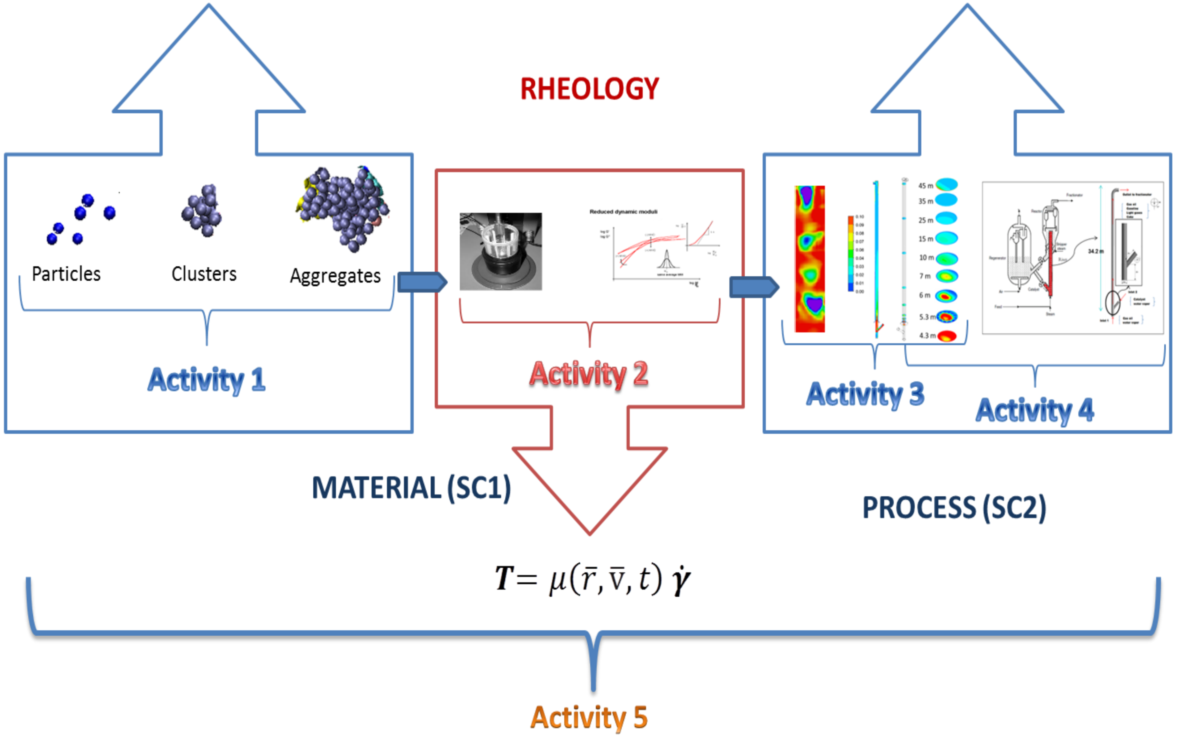
$$\frac{\partial}{\partial t}(\varepsilon_g \rho_g \mathbf{u}_g) + \nabla.(\varepsilon_g \rho_g \mathbf{u}_g \mathbf{u}_g) =$$

$$\nabla.\left[\varepsilon_g \mu_g \left(\nabla \mathbf{u}_g + (\nabla \mathbf{u}_g)^T\right)\right] + \varepsilon_g \rho_g \mathbf{g} - \varepsilon_g \nabla p$$

$$\frac{\partial f}{\partial t} = \bar{v}\cdot\nabla f + \frac{\partial}{\partial \bar{v}}(\bar{L}f) + \bar{g}\cdot\frac{\partial f}{\partial \bar{v}} = \boldsymbol{T}(f,f) + \bar{C}\cdot\nabla\bar{v}$$



To develop a *user-inspired* theory that help master the hydrodynamics of particulate media and improve the reliability of their industrial processing

The Fellowship –linking the micro to the macro through the meso

EPSRC
Engineering and Physical Sciences Research Council

Deliverables

- Constitutive Models –Unified, *user inspired* Theory (Activity 1)
- Reliable Measurements (Activity 2)
- Improvement of CFD Codes (Activity 1, 2 & 3)
- Industrial Translation (Activity 4)

- Advocacy and Leadership
- Transferable Skills and Training
- Public Engagement

} (Activity 5)

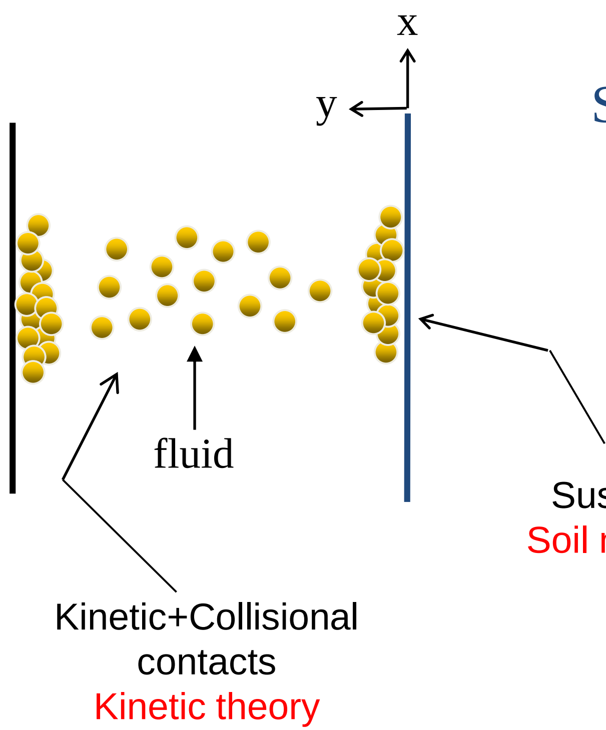
Continuum Modelling (Back to Last Century….)

- Anderson & Jackson (1967)
    - Starting from the equation of motion of a **single** solid particle and the Navier-Stokes equation for the fluid motion, a set of equation was developed for a system of fluidised particles
    - Replacement of point mechanical variables with averages quantities, i.e. averages introduced to jump from one small scale (the particle) to a larger scale (the collection of particles)
    - The (only) assumption made: *the "macroscopic" variation was much larger than the particle spacing*
    - "**Undetermined terms in the equations of motion**" were needed: particle-particle and particle-fluid interactions. Those were determined by "averaging" physics happening at the "small scale"

- Lun et al (1984)
    - Solid particles to be treated in a similar fashion of gas molecules (**kinetic theory**) –with all the assumptions that the k-t implies!
    - Constitutive equations obtained in a "rigorous" way, i.e. trough averages linking the particle scale to the bulk properties

Kinetic-collisional + Frictional models

[Johnson and Jackson, 1987]

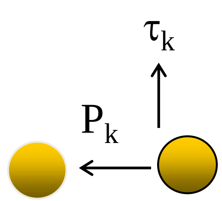


Simple addition of the two solid stress components:

$$P_s = P_k + P_f$$

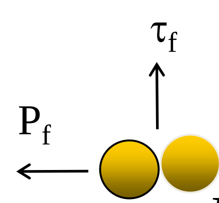
$$\tau_s = \tau_k + \tau_f$$

Assuming negligible frictional effects at $\varepsilon_s < \varepsilon_{s,critical}$



$$P_k = \rho_s \varepsilon_s T + 2 g_o \rho_s \varepsilon_s^2 T (1 + e)$$

$$\tau_k = 2 \mu \frac{dv}{dy}$$



$$P_f = C \frac{(\varepsilon_s - \varepsilon_{s,\,critical})^n}{(\varepsilon_{s,\max} - \varepsilon_{s,\,critical})^p}$$

$$\tau_f = P_f \sin \phi$$

Adding friction was not sufficient…

- …when particles "deviate" from being "well behaved"
    - Agglomerates
    - Clusters
    - …..

…then:

- "Bolted-on" Approach
    - Tweak the original theory by "bolting on" new physics at the meso scale (e.g., Gidaspow and Huilin, 1988; Ocone et al. 2000; Ye et al., 2005; van Wachem and Sasic, 2008).

Proposed inter-particle "cohesive" model

[Ocone et al, 2000]

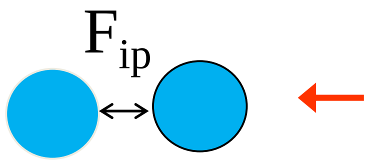
The radial component of the cohesion force is derived:

$$P_c = C_o \frac{6\sqrt{2}F_{ip}\sqrt{T}}{u_t d} \left|\nabla \varepsilon_s\right|$$

Based on experimental data on Group A/B particles, we are taking an average value of F_{ip}=0.2X10^{-8} N

The tangential cohesion force is given by a modified formula of Molerus (1982):

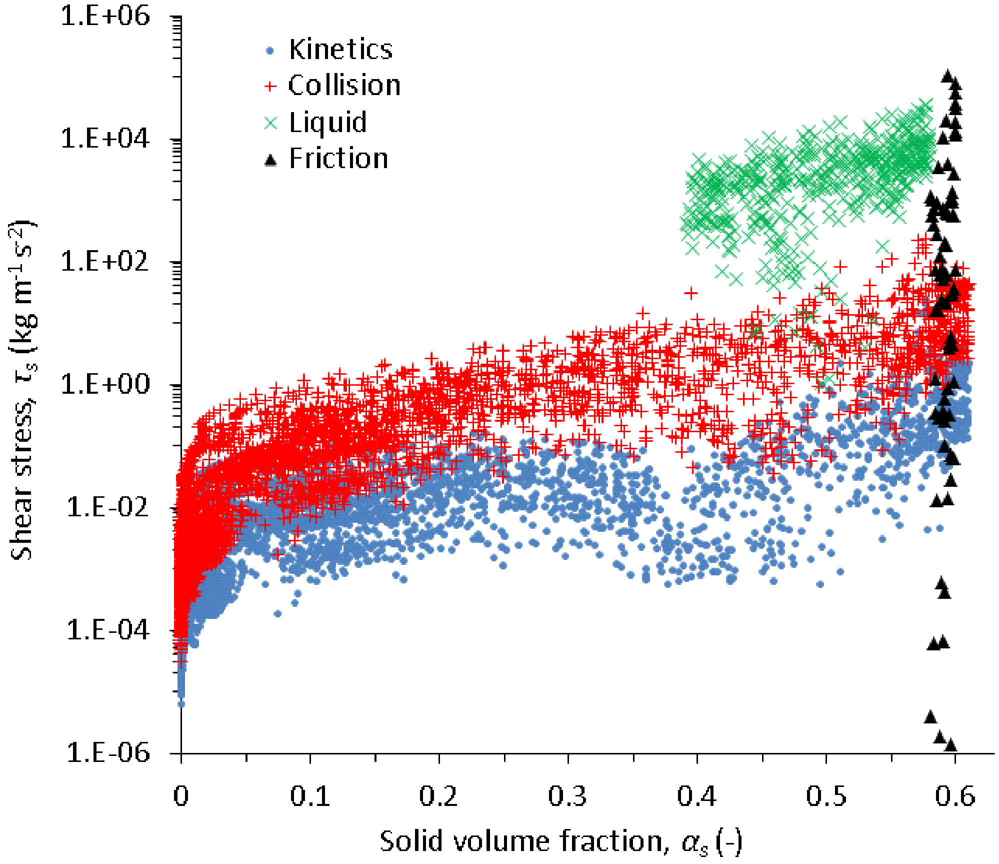
$$\tau_c = P_c \frac{\pi}{6(1-\varepsilon_s)}$$



Two isolated particles in contact

where C_o is a factor introduced due to uncertainty about the exact value of F_{ip} and F_{ip} is the cohesive force

Additive Effects



Predicted solid shear stress in a slightly wet fluidized bed of 15 cm diameter at the gas velocity of 0.8 m/s and liquid presence of $\delta = 0.1 \times 10^{-2}$.

[X.Yu, S. Generalis, R. Ocone, Y. Makkawi, 2014]

Kinetic Theory (Inconsistencies)

- Particles lose their granular temperature (cool down) without any mechanism to replace it
- Clusters cannot be predicted by the kinetic theory:
  - Particles must forget about their past quickly (so that they can regain their Maxwell distribution after the impact – molecular chaos assumption)
- Kinetic theory forbid variation on a scale less than the mean free path between collisions

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MATHEMATICAL PHYSICS

Famous Fluid Equations Are Incomplete

July 21, 2015

A 115-year effort to bridge the particle and fluid descriptions of nature has led mathematicians to an unexpected answer.

Ruslan Khasanov

… because the Navier-Stokes equations — despite being exceptionally useful for modelling the weather, ocean currents, pipes, cars, airplane wings and other hydrodynamic systems, and despite the million-dollar prize offered for their exact solutions — are incomplete.

Challenge:

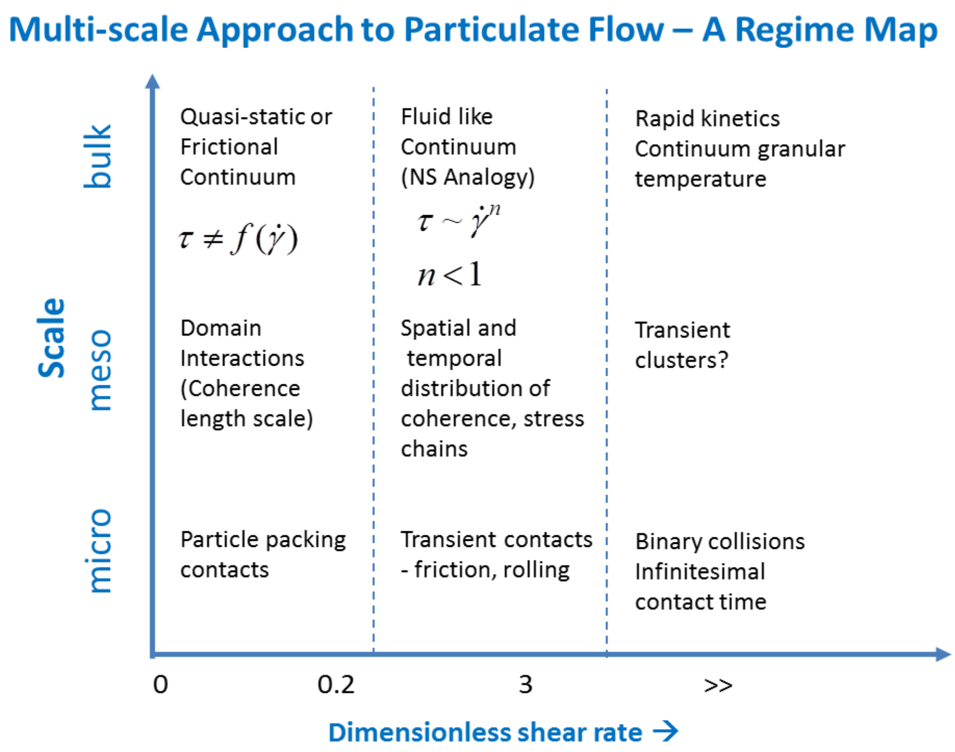
- Do the microscopic and macroscopic pictures rest on equivalent axiomatic foundations, and are thus different manifestations of a single theory?

    - Attempt to mathematically translate the Boltzmann equation, into the Navier-Stokes equations, which describe the gas on larger scales as a continuous, flowing entity

    - **Could the particle and fluid pictures be rigorously linked?**

Let us not be afraid to abandon the kinetic theory!

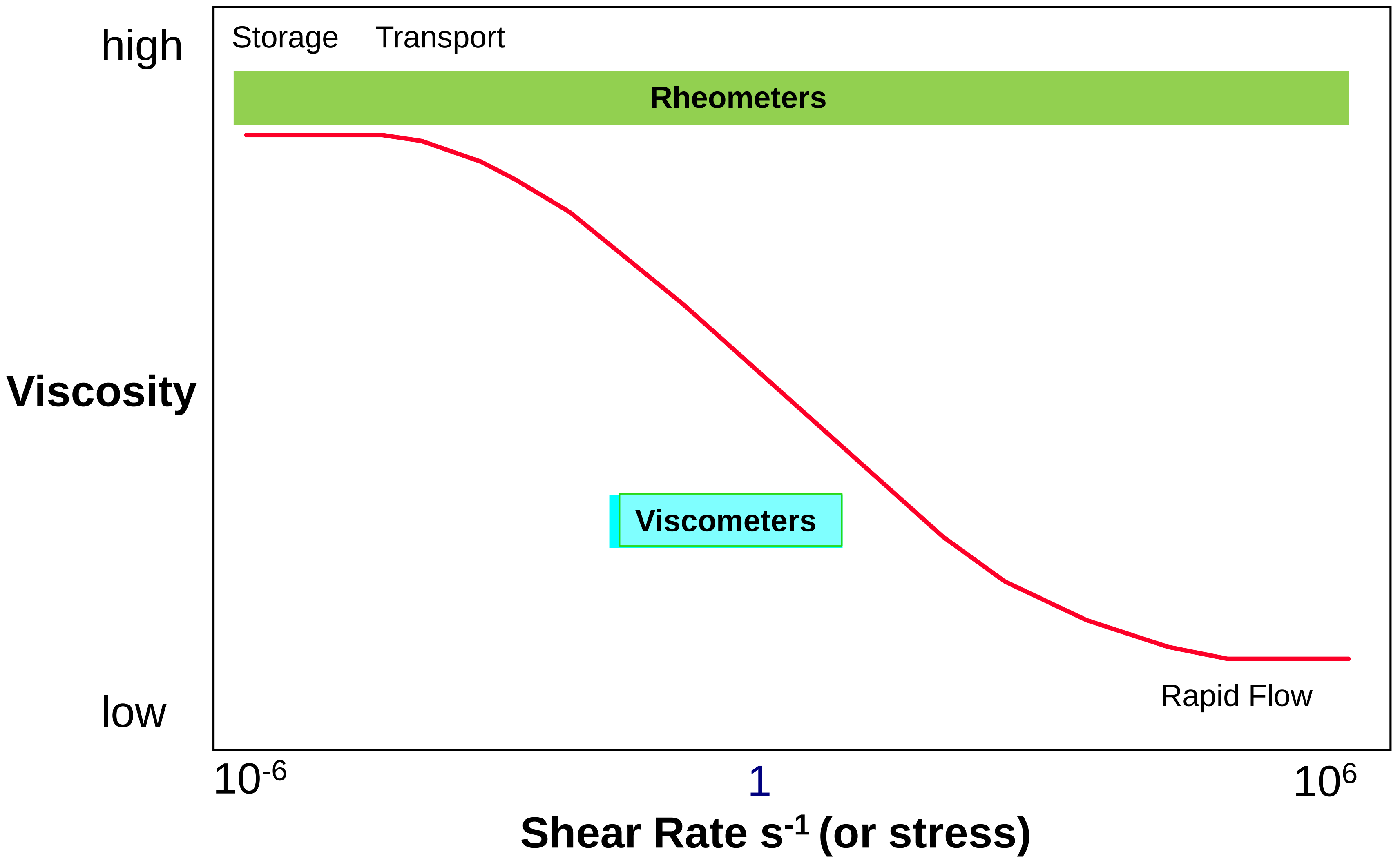
The Classification of Flow Regimes depends on the Applications



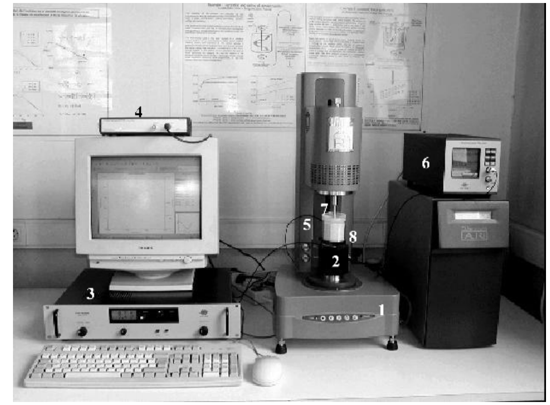
[courtesy of P Mort]

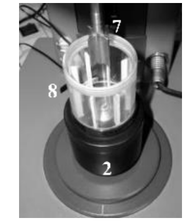
To understand the bulk properties (meso-scale) we need **RHEOLOGICAL** measurements

Rheology vs Viscometry

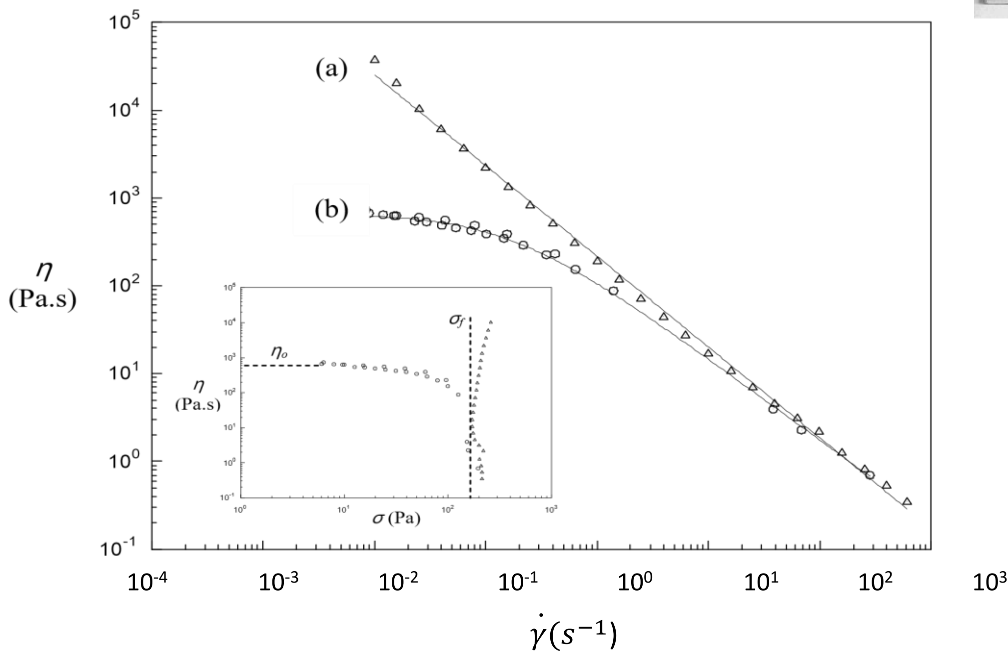


Rheological Measurements





1. Rhéomètre
2. Mini-vibreur
3. Ampli. de puissance
4. Générateur de fonction
5. Accéléromètre
6. Ampli. de mesure
7. Scissomètre
8. Cuve et chicanes



Viscosity evolution without oscillations (a) and with oscillations (b)

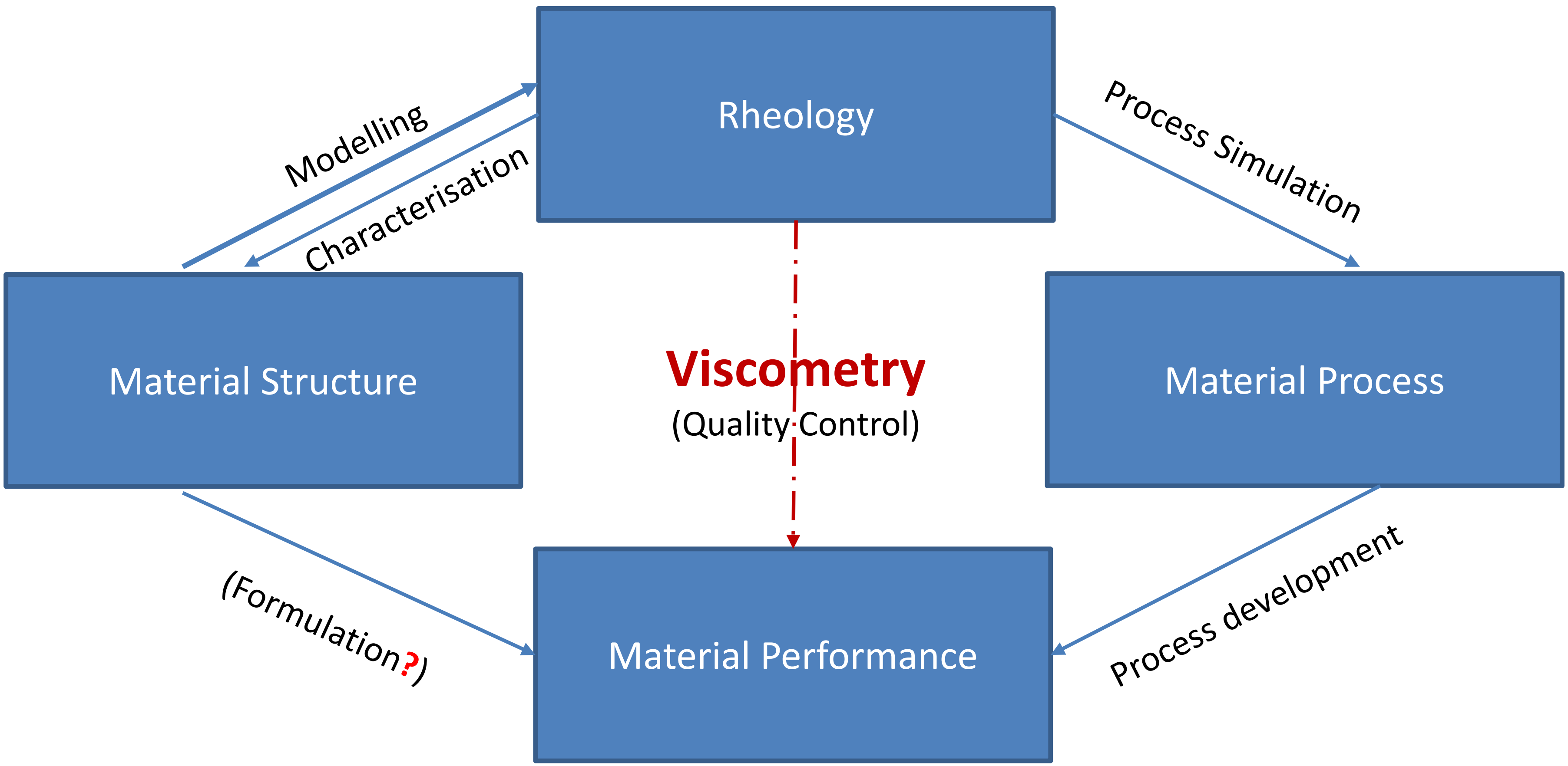
[courtesy of P Marchal, ENSIC)

vibration of de cell
frequency f = 10-100Hz - amplitude A = 100-1000µm

oscillatory rheological tests (frequency sweep, stress sweep, strain sweep tests): frequency w = 0.01-100rad.s-1 - stress sigma = 1-100Pa - strain gamma = 0.01-10 (1%-1000%)

Viscoelastic material with a viscous and an elastic viscosity (**G’** and **G”**)

RELIABLE MEASUREMENTS



Lessons Learnt

- Scale is important (but let us not forget about the time scales)
- The meso-scale needs to be experimentally investigated (and models need to be “linked”)
- Models can give back only what we gave them (and often they are qualitative and not quantitative)
- Experiments need to be complemented with insights on the physics
- Genuine multi-scale approaches are needed
- Understanding of the meso-scale –develop constitutive equations
- Viscometry alone cannot give information on the meso-structure
- Let us not forget about multi-disciplinary interactions

Conclusion

Granular materials present a number of vagaries which need sound mathematics and physics complemented with reliable measurements to be (possibly) solved

"Complex systems…. admit many descriptions, each of which is only partially true…"
-Rosen (1979)