New perspectives on the mechanism of shear thickening

Helen J. Wilson

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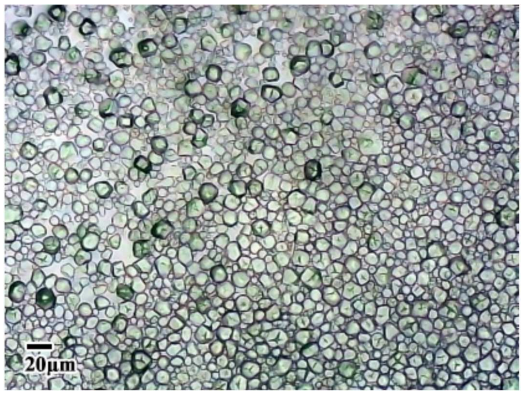
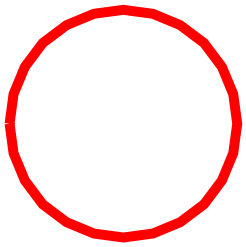


Model Suspensions in Shear: Definitions

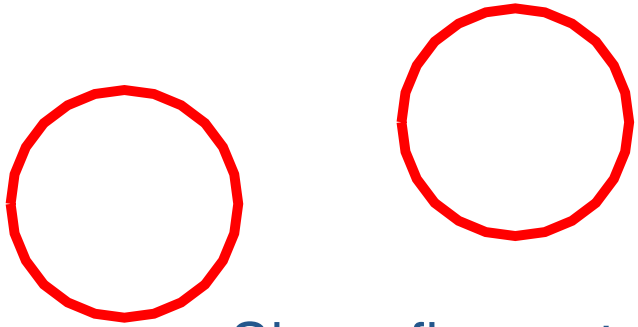
Solid particles, radius a



Viscous fluid

Viscosity μ

Density ρ



Shear flow, rate $\dot{\gamma}$

Particles are identical, spherical, non-colloidal: large enough to neglect Brownian Motion.

Reynolds number $Re = \rho\dot{\gamma}/\mu \ll 1$

Solid volume, area fraction $\phi, \quad c$

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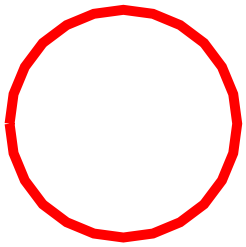

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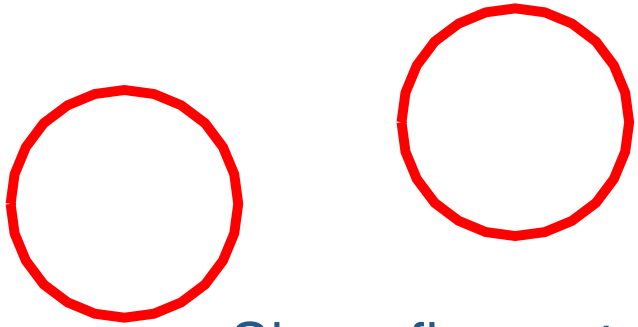

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Key Questions:

- What is the macroscopic viscosity?
- What are the normal stress differences?

Background: Stokes Flow

Governing equations for Newtonian fluid mechanics are the Navier-Stokes equations. When $Re \ll 1$ and with no body forces they reduce to the Stokes Equations:

$$\sigma = -p\mathbf{I} + \mu\left(\nabla\,\mathbf{u} + \nabla\,\mathbf{u}^{\top}\right) \qquad \nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \sigma = \mathbf{0}$$

$p =$ pressure $\qquad \mathbf{u} =$ velocity $\qquad \sigma =$ stress

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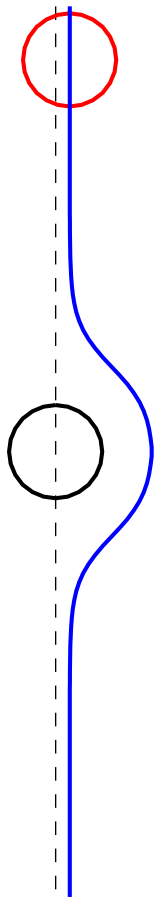
- Linear equations
  - Flow vector $\mathbf{u}$ depends linearly on boundary data
- Quasi-static (no $\partial/\partial t$ term)
  - $\mathbf{u}$ responds instantly to any changes; history irrelevant
- Reversible
  - Reverse boundary data/forcing, $\mathbf{u} \to -\mathbf{u}$ and all trajectories backtrack

Background: Experimental Evidence of Contact



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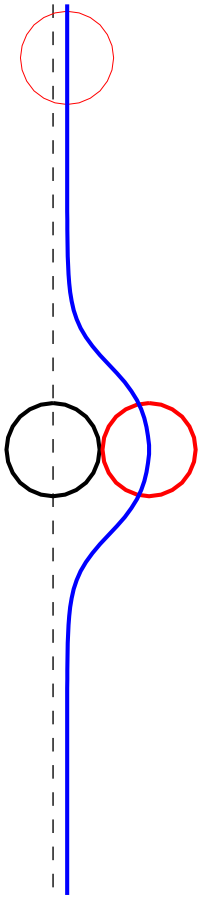

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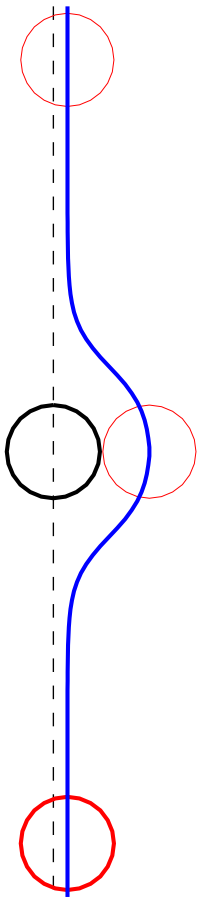

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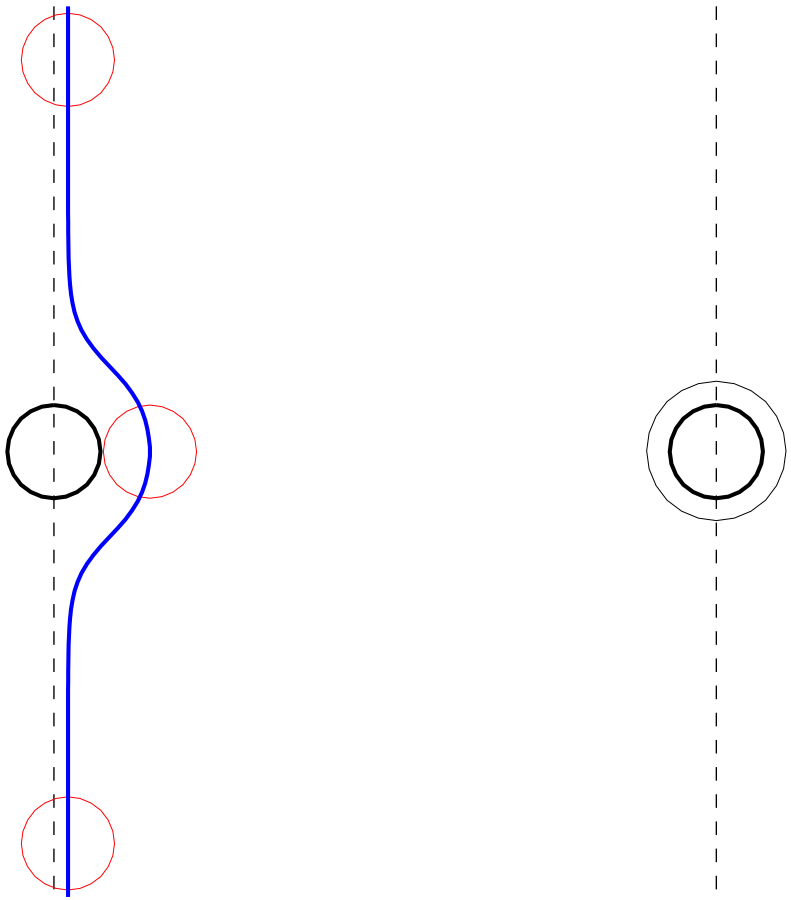

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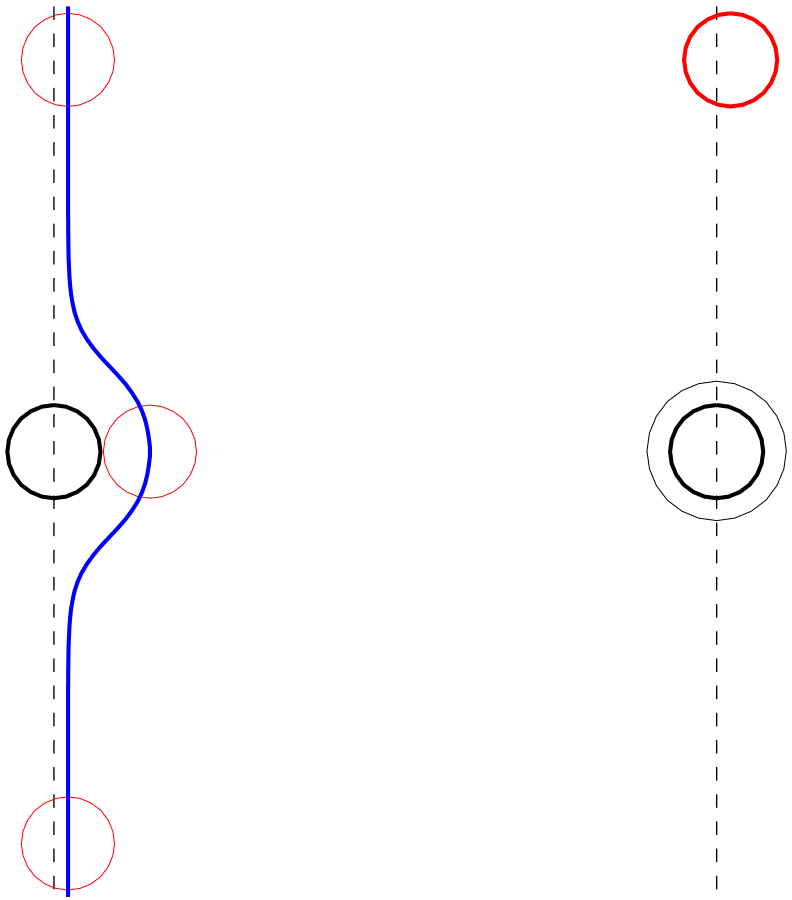

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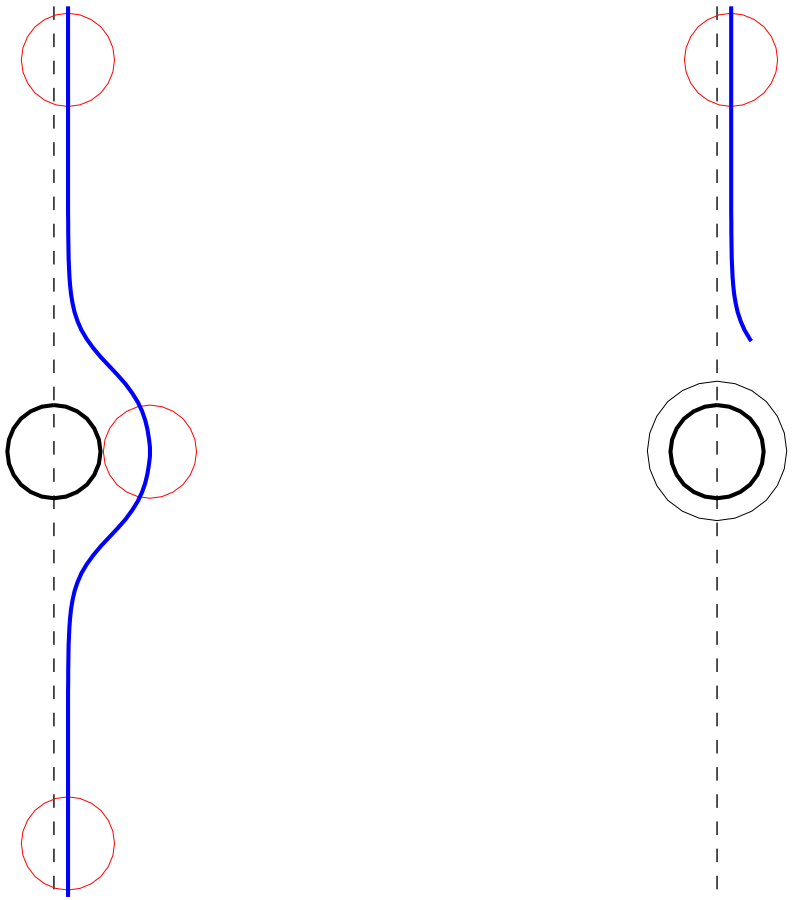

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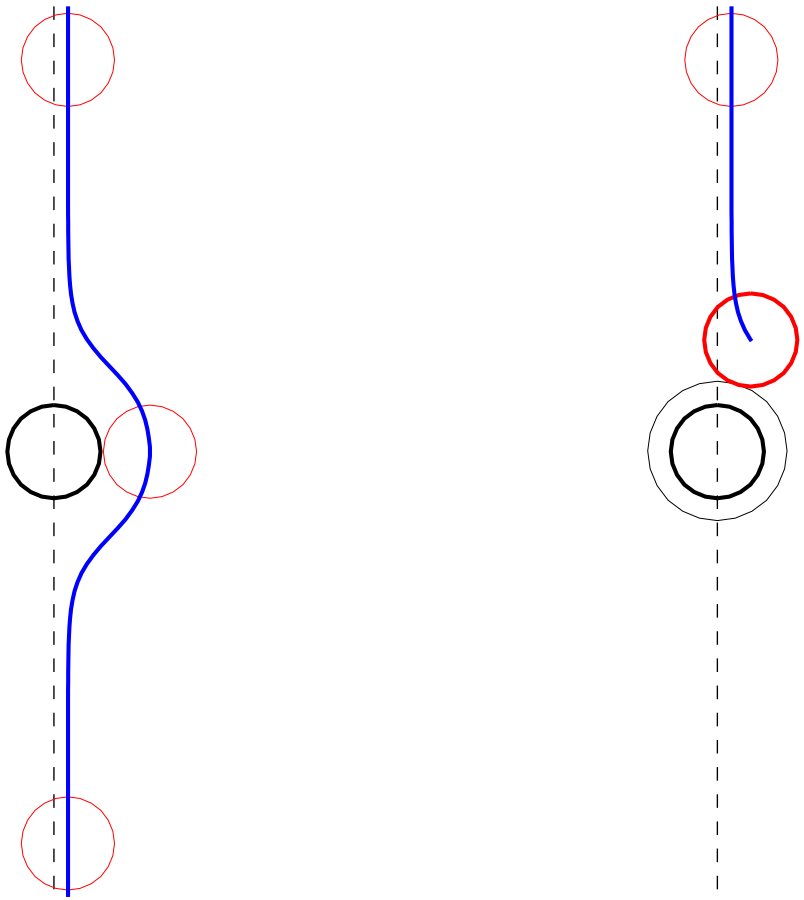

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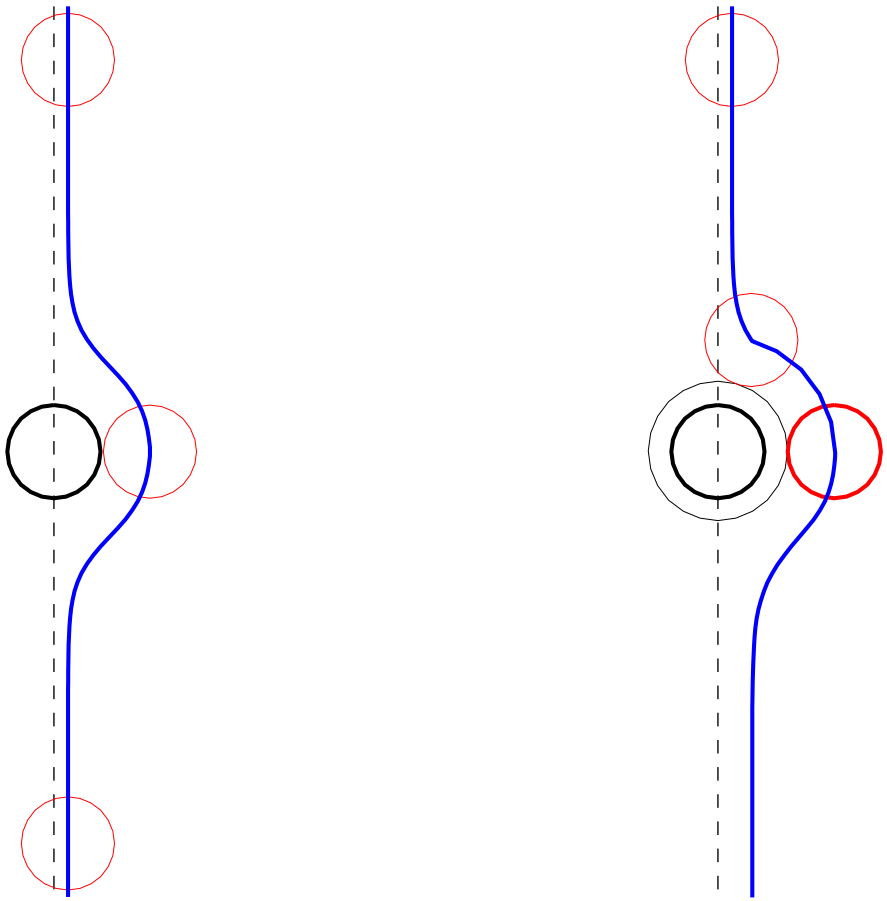

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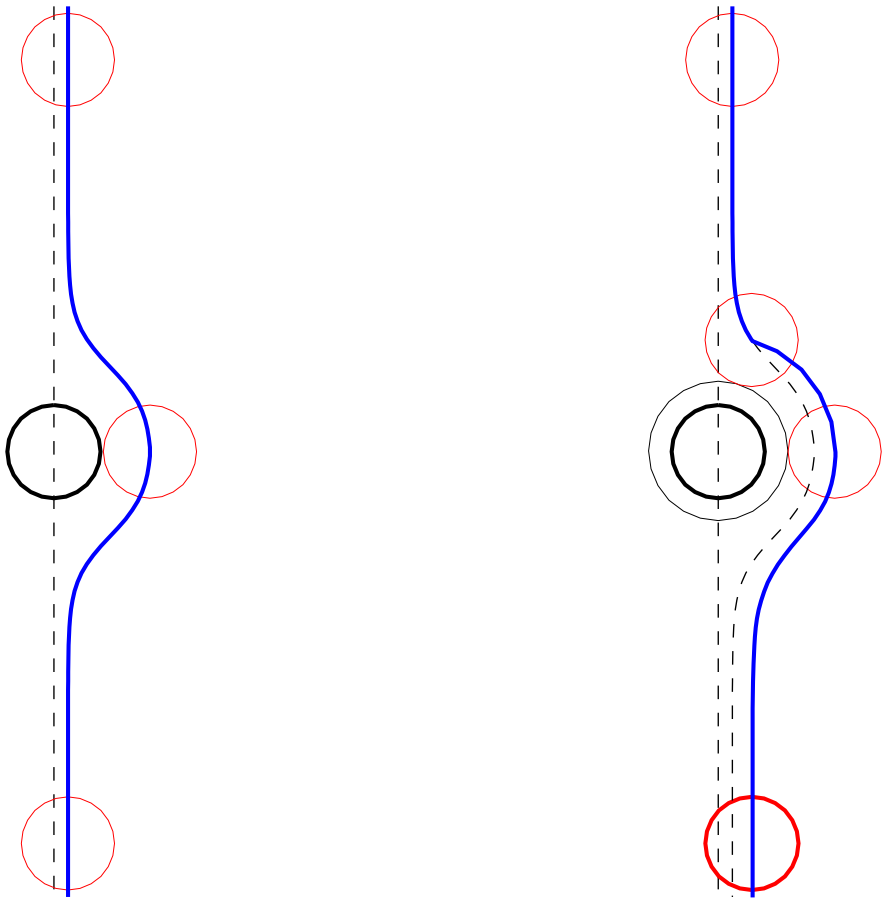
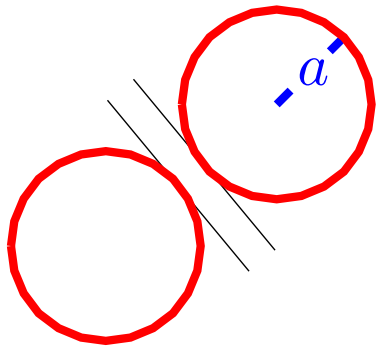


Simple Models of Contact

Dimensionless roughness height ξ

- Hydrodynamics unaffected
- Approach closer than ξa prevented by normal contact force F_n
- Particles free to separate



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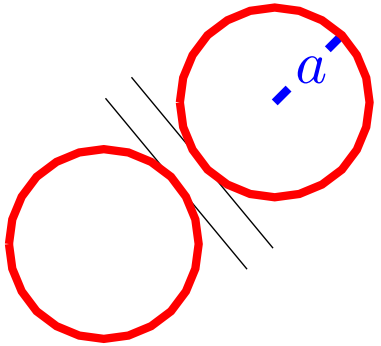

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Coefficient of friction ν

- Applies tangential friction force F_t
- Rolling contact if F_n large enough: $|F_t| \leq \nu |F_n|$ and no relative motion at contact point
- Slipping contact otherwise: $|F_t| = \nu |F_n|$ and particles roll and slide past one another
- Extra: critical load F_c: friction applied only if $|F_n| > F_c$

History: Dilute Suspensions

'Expand' about the state of no particles at all:

- $\phi = 0$, Newtonian fluid with viscosity $\eta = \mu$.
- Einstein (1906 & 1915) calculated stress to order ϕ
  Single free solid sphere at the origin of a flow field
  $\mathbf{U} = \mathbf{U}^{\infty} + \boldsymbol{\Omega} \times \mathbf{x} + \mathbf{E} \cdot \mathbf{x}$ causes extra dissipation (stresslet)

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The suspension is a Newtonian fluid with this viscosity.

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Batchelor & Green (1972) calculated stress to order ϕ^2 for smooth spheres

- A pair of particles at positions $\mathbf{x}$ and $\mathbf{x}+\mathbf{r}$ in the linear flow field cause the stresslets they would each cause alone plus an extra interaction stresslet

$$\mathbf{S}(\mathbf{r})$$

- The probability of finding a pair in this arrangement can be calculated using trajectory analysis; fine as long as all trajectories come from infinity
- The total extra stress at order ϕ^2 is calculated from

$$\int \mathbf{S}(\mathbf{r})\mathbf{P}(\mathbf{r})\,\mathrm{d}\mathbf{r}.$$

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Batchelor & Green's calculation (1972) to order ϕ^2

- In shear flow $\mathbf{U} = (\dot{\gamma} y, 0, 0)$, not all particle trajectories come from infinity: there are closed orbits where P cannot be found.

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- In the absence of any forces except hydrodynamics, P is spherically symmetric
- By symmetry, N_1 and N_2 are zero so stress is Newtonian to $O(\phi^2)$
- Viscosity η cannot be calculated: unknown quantity of particles P on bound trajectories

Effect of Contact: Dilute Suspensions

Batchelor theory for smooth spheres

- Calculate stress from

$$\int \mathbf{S}(\mathbf{x}_0, \mathbf{x}_0 + \mathbf{r}) P(\mathbf{r}) d^3\mathbf{r}$$

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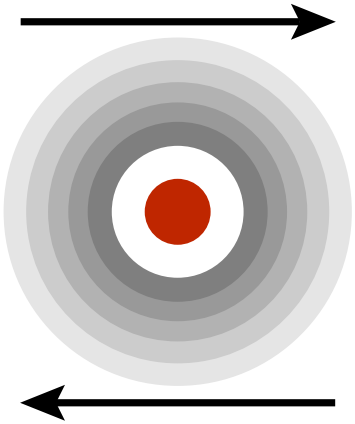
Perturbation by contact

- Stresslet $\mathbf{S}$ hardly changes
- Pair distribution P of spheres strongly affected

Effect of Contact: Dilute Suspensions



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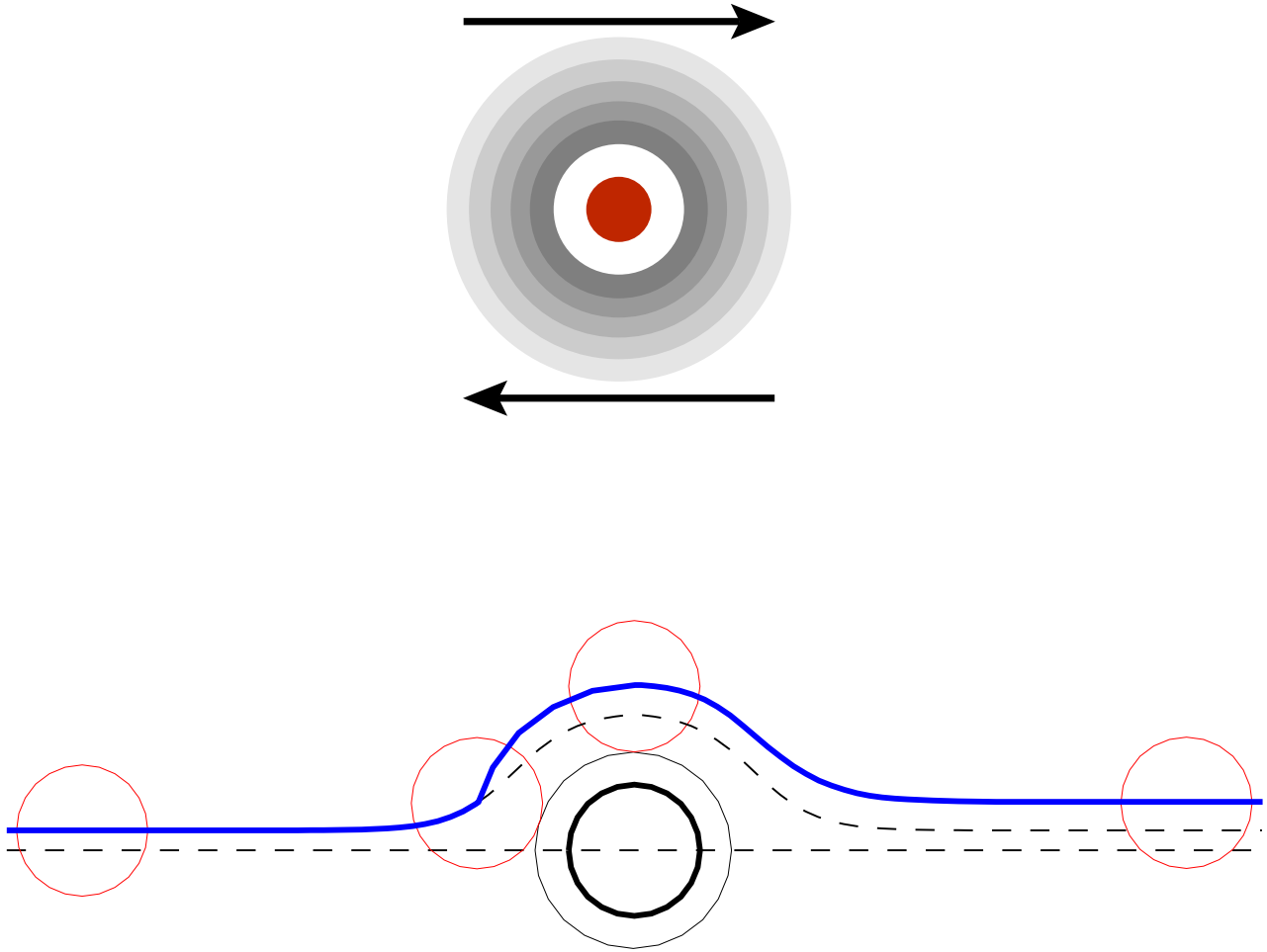

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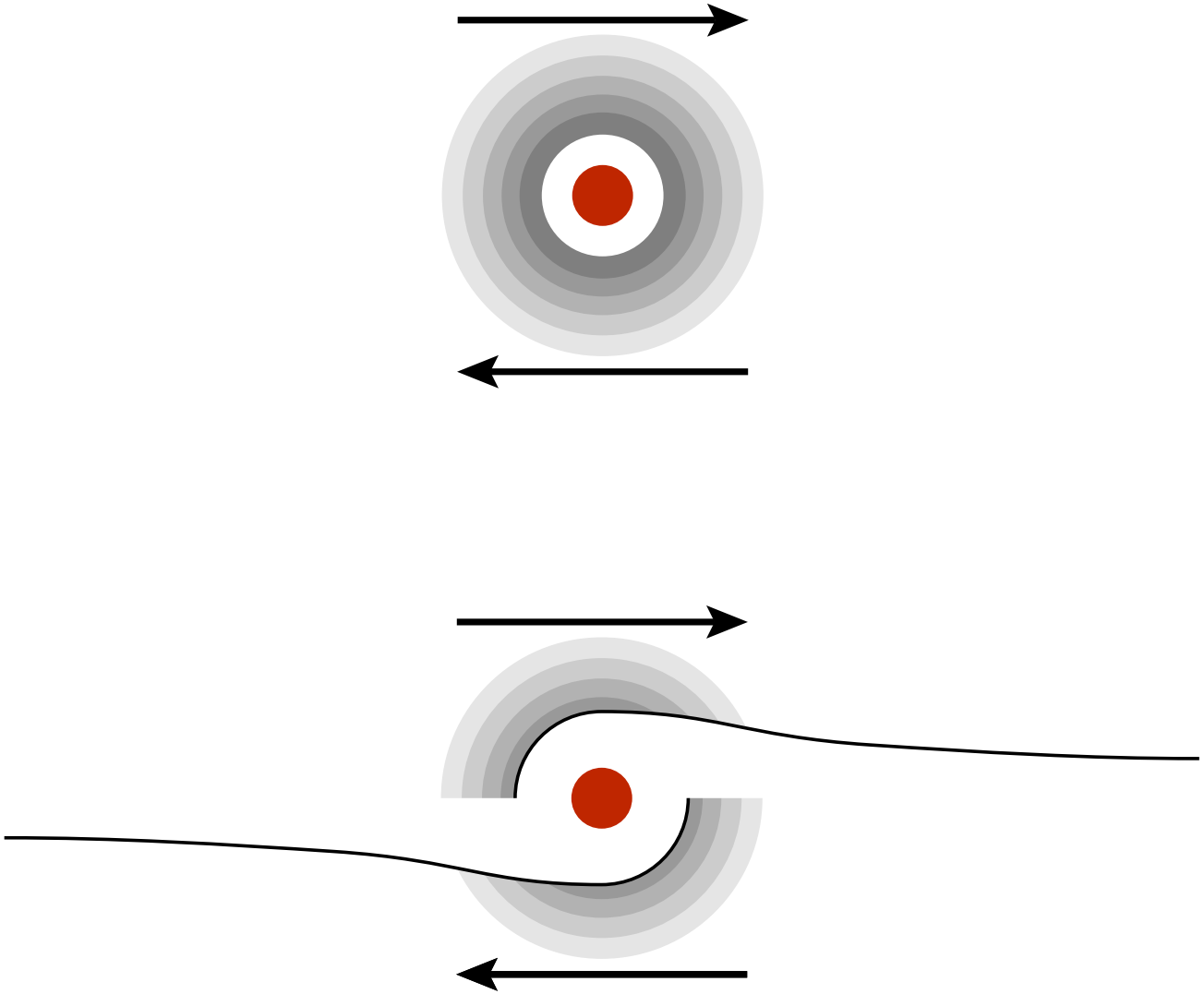
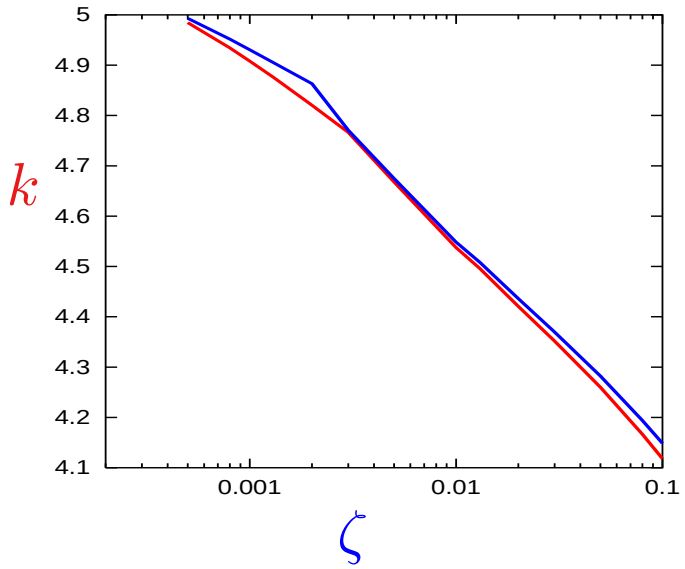


Viscosity of dilute rough suspensions: 2D

Take the effective volume of the system to be a layer of depth $2a$. Viscosity (for roughness height $\zeta > 2.1 \times 10^{-4}$) and rolling or slipping is $\eta = \mu\left(1 + \frac{5}{3}c + kc^2\right)$.



- Increasing roughness height lowers viscosity
- Changing friction unimportant

Dense Suspensions: Numerical Simulation

Stokesian Dynamics method

- Brady & coworkers 1988 onwards
- Uses linearity of the Stokes flow problem
- Far-field mobility matrix: invert $\rightarrow$ resistance matrix with screening
- Add exact pairwise resistances

$$\begin{pmatrix} \mathbf{F} \\ \mathbf{T} \\ \mathbf{S} \end{pmatrix} = \left[\mathcal{R}_{2B} + (\mathcal{M}^{\infty})^{-1} - (\mathcal{M}^{\infty}_{2B})^{-1} \right] \begin{pmatrix} \mathbf{U} \\ \mathbf{\Omega} \\ \mathbf{E} \end{pmatrix}$$

Implementing Contact

What do others do?

- Add Brownian motion and a step-function repulsive potential
- Implement a continuous repulsive force over a small range of separations
- . . . none of these impose the minimum separation condition

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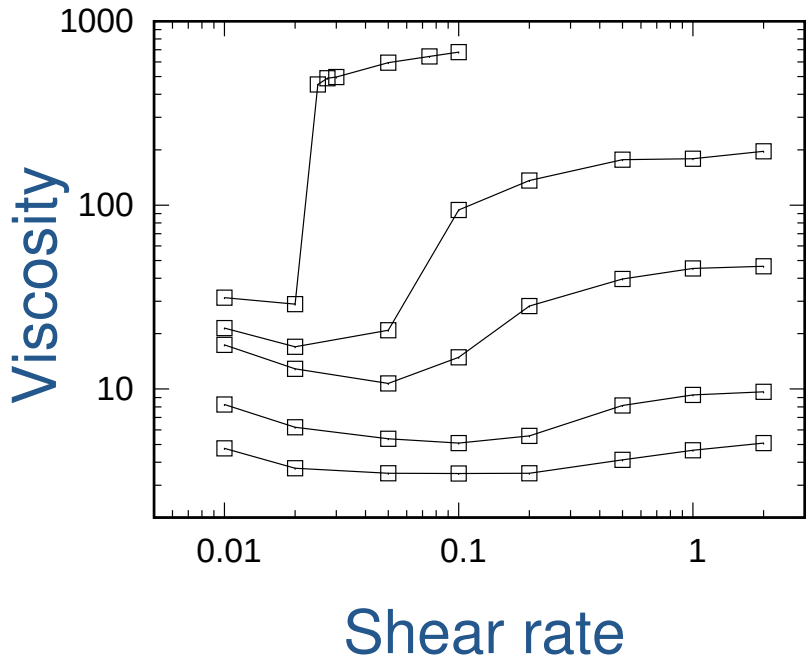
Our approach

- Find normal forces exactly
- Use the linearity of Stokes flow and of SD
- Expand the resistance matrix by one row and column for each contact
- Retain matrix symmetry and positive definiteness
- Approximate tangential force using lubrication theory

Shear-thickening with critical load: Results

Literature results (Seto et al)



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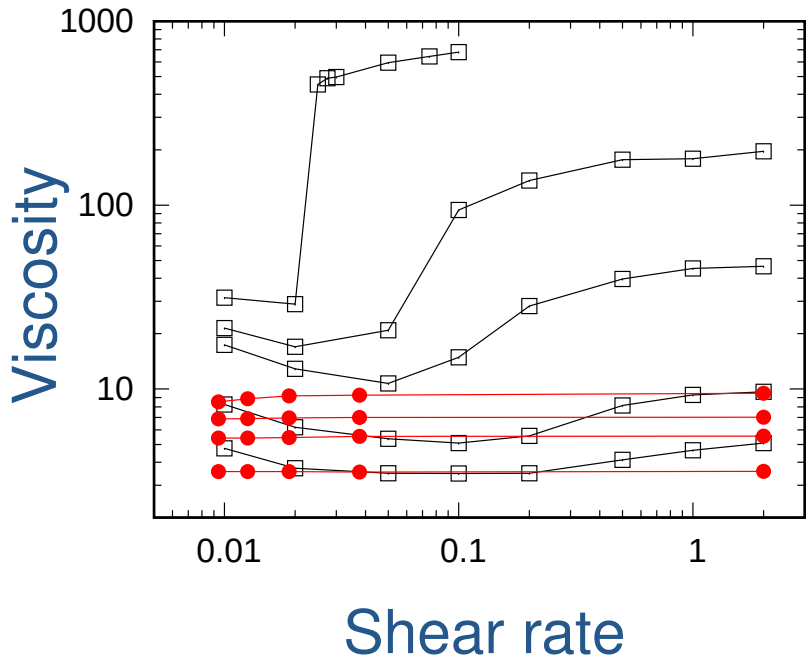

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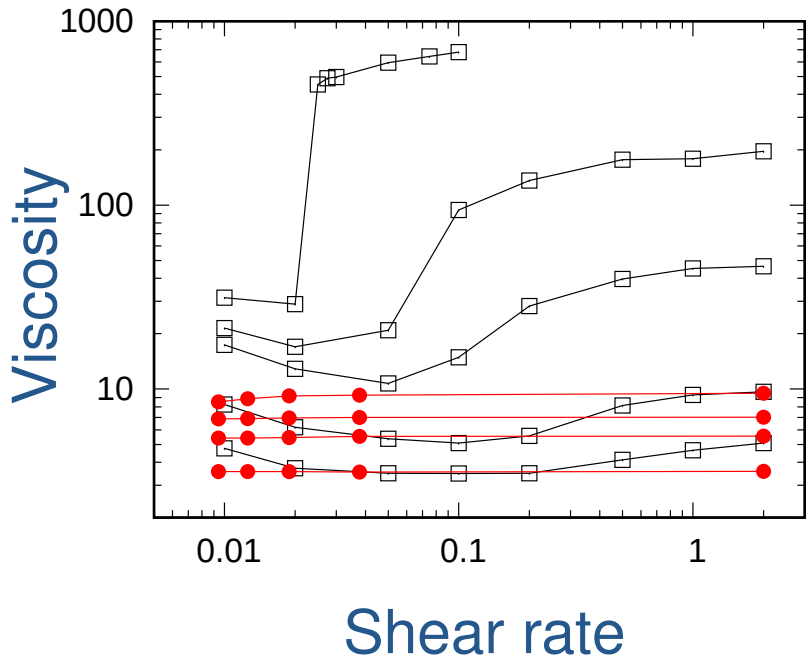

- We see very mild thickening (from $\nu = 0$ to $F_c = 0$)
- Not good enough

Friction and critical load

- Low shear viscosity equivalent to $\nu = 0$
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- Low friction, systems should jam at $Z = 6$ ($Z = 4$ in 2D)
- Rolling contact systems should jam at $Z = 4$ ($Z = 3$ in 2D)
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Need something else?

- Closer particles to harness lubrication dissipation?
- Compressible asperities

Normal stress differences: Dilute Suspensions

- Can do 3D, since closed orbits don't contribute to N_1, N_2
- N_1, N_2 both zero if $\zeta < 0.0002$
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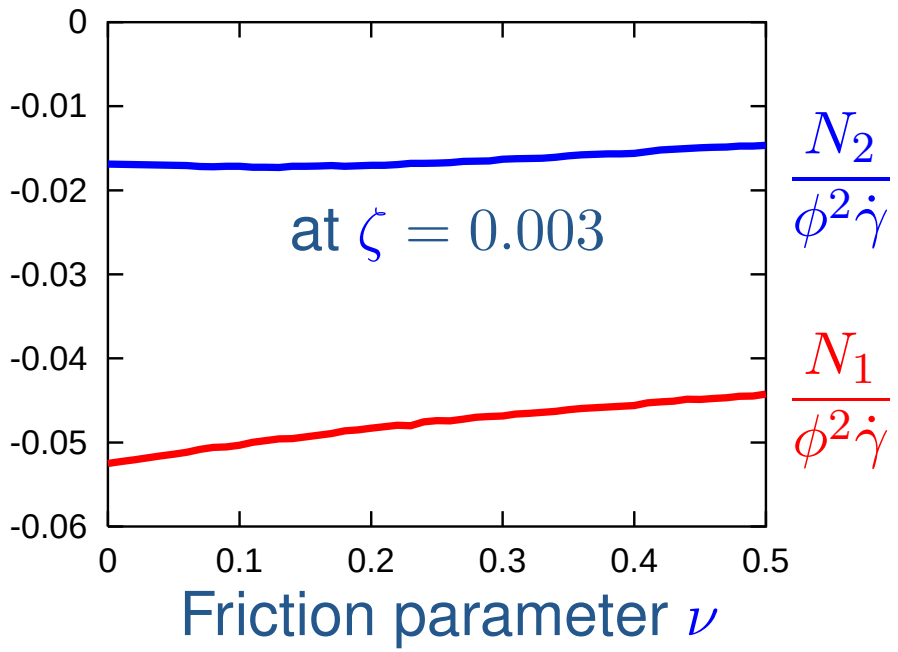
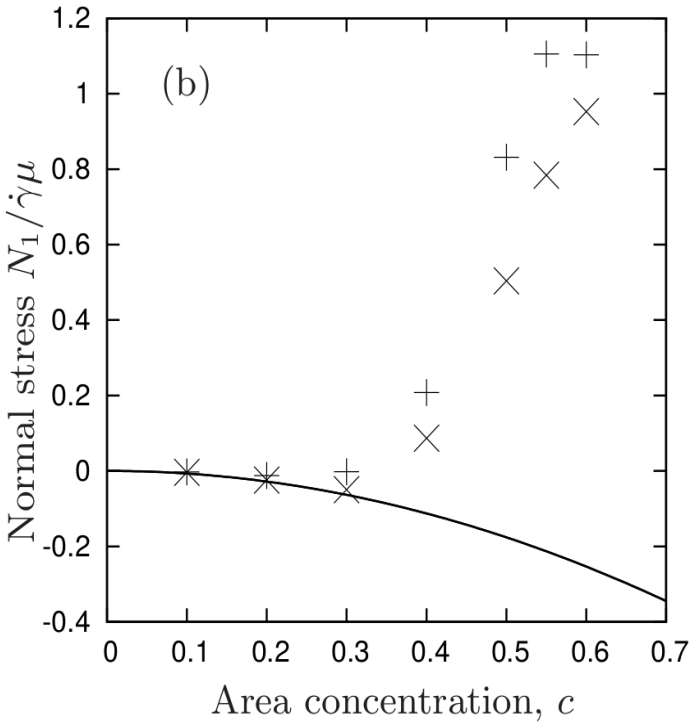
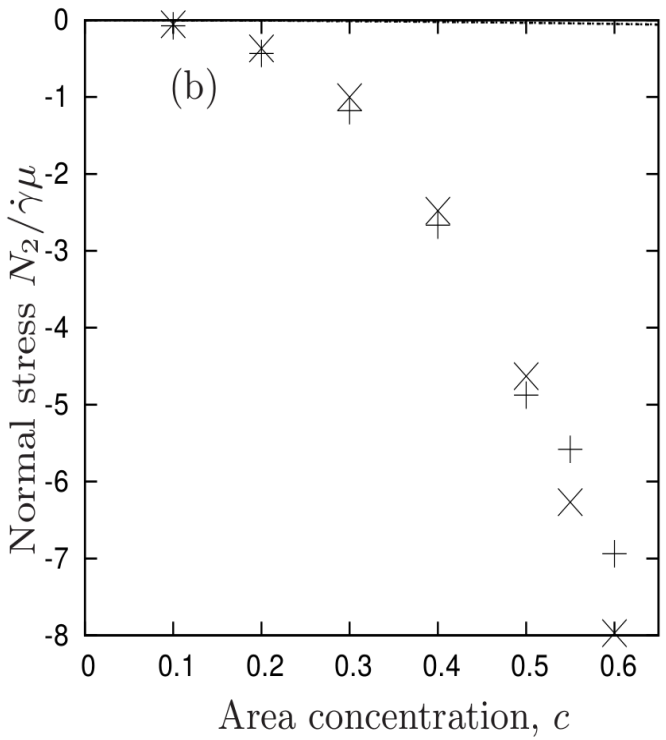


Normal stresses: Concentrated suspensions

Small 2D simulations using SD. Without (+) and with ($\times$) friction.

- N_1 changes sign as concentration increases
- N_2 always negative.



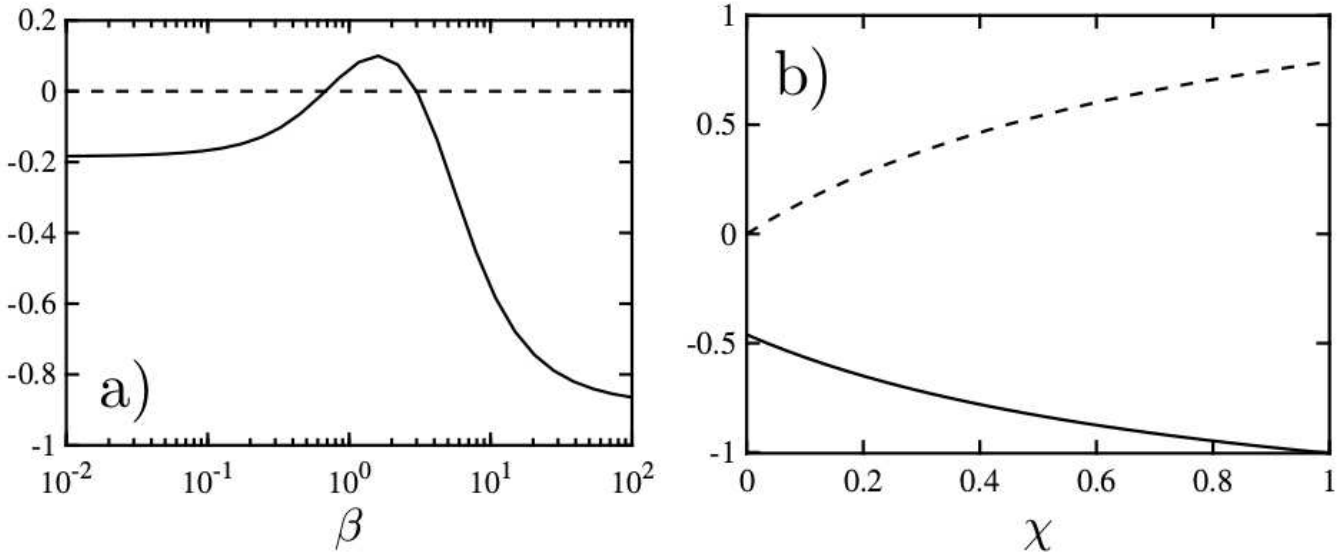


Normal stresses: Concentrated suspensions model

New constitutive model (Gillissen, PRL, just submitted) for near-jammed suspensions (dominated by pairwise contacts)

- Define stress in terms of moments of pair-vector $\mathbf{p}$
- Describe evolution of $\mathbf{p}$ in terms of compressive flux into contact, rotation, and extensional flux out of contact

Predicted N_1 (dashed) and N_2 (solid), scaled by shear stress, without friction (left) and against friction parameter (right):



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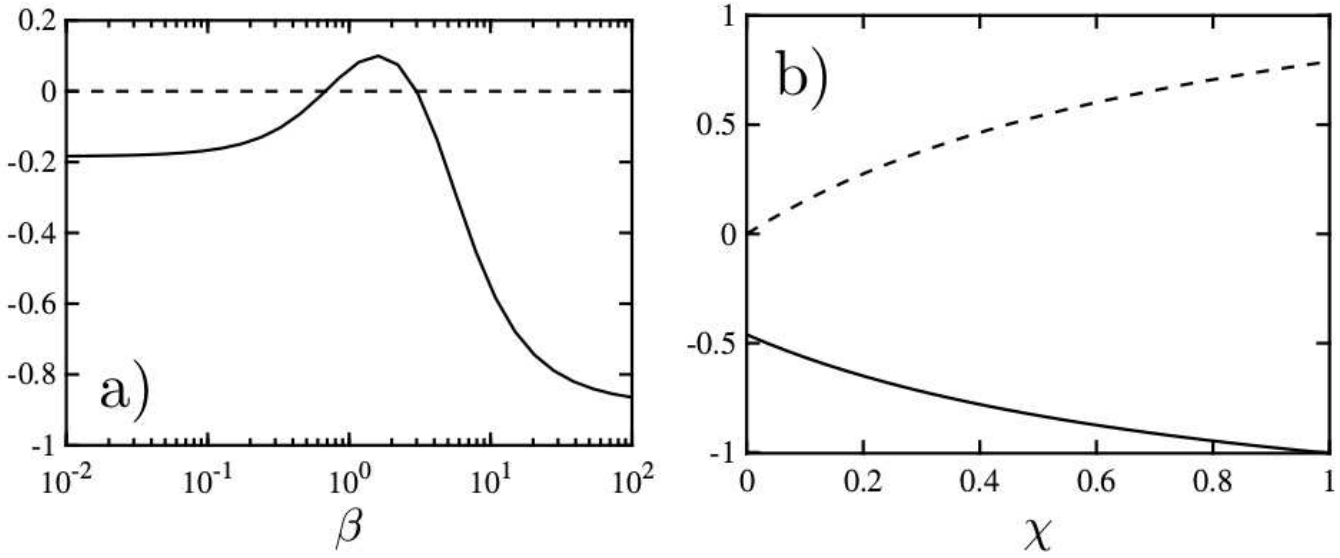

Conclusions

Viscosity

- Dilute systems:
  - Roughness strongly decreases viscosity
  - Friction weakly increases viscosity
- Dense systems (SD simulations):
  - DST still controversial! Soft contacts?

Normal Stress Differences

- Dilute systems:
  - Negative normal stresses for $\zeta > 0.0002$
- Concentrated systems:
  - Simulations: N_1 crossover at higher volume fractions.
  - Jamming: $N_1 > 0$ caused by friction; N_2 mostly negative.

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Postdoc – UCL

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asymptote