Controlling Rheology via Boundary Conditions in Dense Granular Flows

Farnaz Fazelpour, Karen E. Daniels
North Carolina State University

IFPRI
International Fine Particle Research Institute

NSF

NC STATE UNIVERSITY

Granular rheology

- **Questions:** for a given set of particles, can we
    - make flow measurements in one geometry
    - determine the constitutive parameters
    - predict flows in other geometries
- **Status:** nonlocal rheology successfully models granular flows across different packing densities, particle sizes and shapes, and shear rates
    - but also need to know wall velocities a priori
- **Aim:** separate which properties are set by the particle properties, and which by the walls
    - develop predictive model for wall slip

Local rheology has some problems

- stress ratio, ratio between:

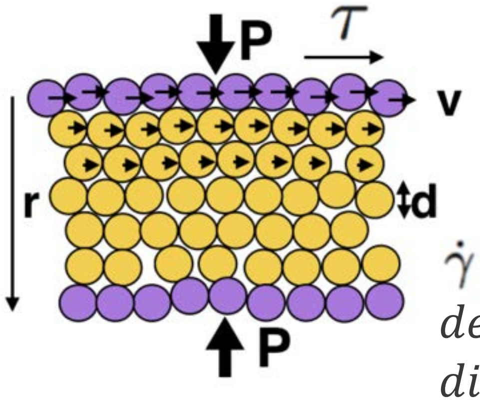
$$\mu = \frac{\tau(r)}{P(r)}$$

shear stress: τ

normal pressure: P

- inertial number, ratio between:
  - Microscopic timescale T to squeeze a particle into a hole: $T = \frac{d}{\sqrt{P/\rho}}$
  - Macroscopic timescale of deformation: $\frac{1}{\dot{\gamma}}$

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}$$



$\dot{\gamma} = dv/dr$

density ρ

diameter d

- Flow can exist below critical stress ratio: $\mu < \mu_s$
- Fails to explain phase transition from inertial to quasistatic regimes
- Fails to describe how shear/vibration in one region can fluidize distant region
- Fails to explain how shear band width depends on geometry

(Forterre et al. *Annu. Rev. Fluid Mech.*, 2008)
(Cheng et al. Phys. Rev. Lett, 2006)
(Nichol et al. Phys. Rev. Lett, 2010)
(Koval et al. Phys. Rev. Lett, 2009)
(Reddy et al. Phys. Rev. Lett, 2011)
(MiDi *Eur. Phys. J. E: Soft Matter Biol. Phys.*, 2004)

Nonlocal rheology (Cooperative model)

Measured values

Model parameters

granular fluidity: $g \equiv \frac{\dot{\gamma}}{\mu}$

- Locally, model the granular material as a Bingham fluid:

$$g_{loc} = \begin{cases} \sqrt{\frac{P}{\rho d^2}} \frac{\mu - \mu_s}{b\mu} & \mu > \mu_s \\ 0 & \mu < \mu_s \end{cases}$$

- The cooperative length scale ξ is set by:

$$\frac{\xi}{d} = A\sqrt{\frac{1}{|(\mu - \mu_s)|}}$$

local parameter + grain parameters + nonlocal parameter

b, μ_s ρ, d A

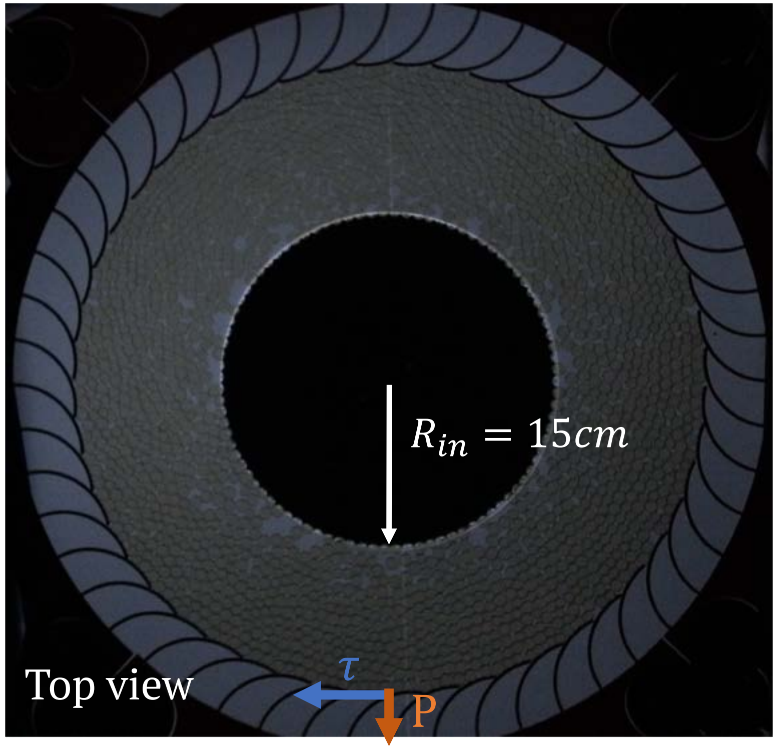
fluidity is determined by both local fluidity and nonlocal cooperative effect:

$$\nabla^2 g = \frac{1}{\xi^2}(g - g_{loc})$$

(Kamrin and Koval, Phys. Rev. Lett, 2012)

(Hennan et al. PNAS,2013)

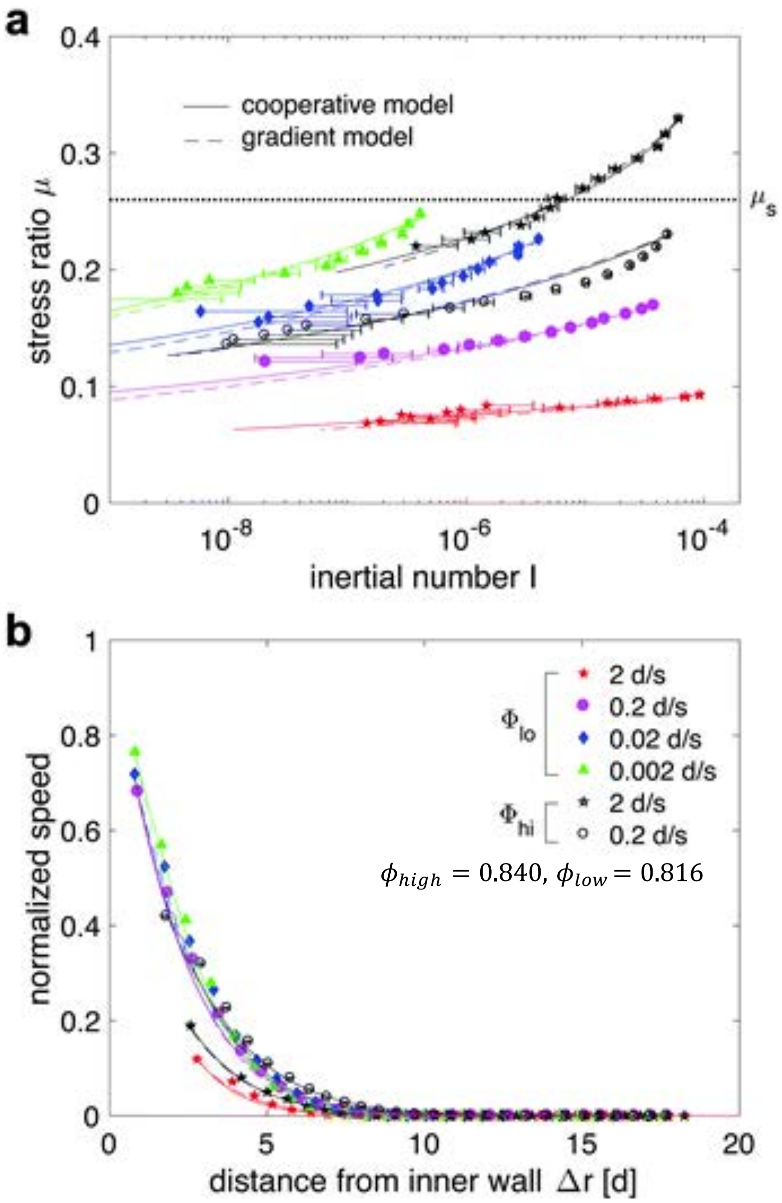
Success of nonlocal rheology



★ The model is able to capture:

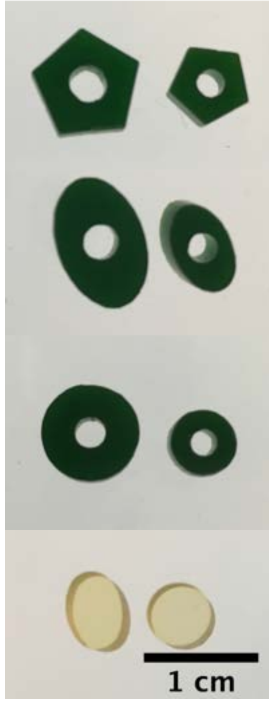
1) the measured $\mu(I)$ and $v(r)$ data
2) using a single set of parameters
3) across all packing fraction and shearing conditions

$A(nonlocal)$	$b(local)$	μ_s
0.402 ± 0.003	1.1 ± 0.5	0.26 ± 0.01

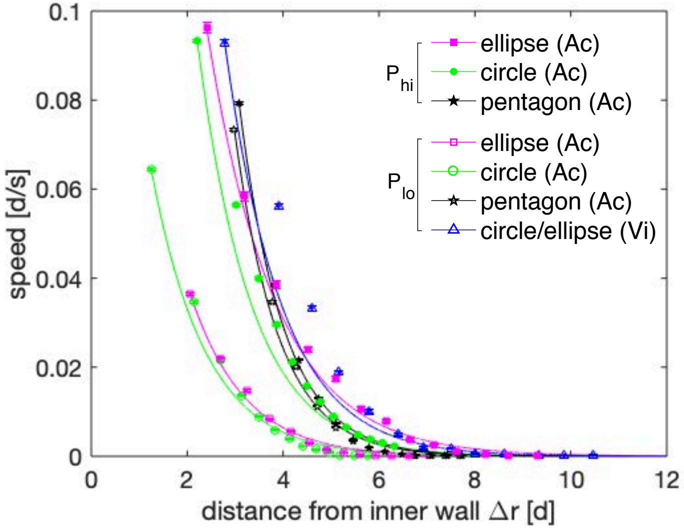


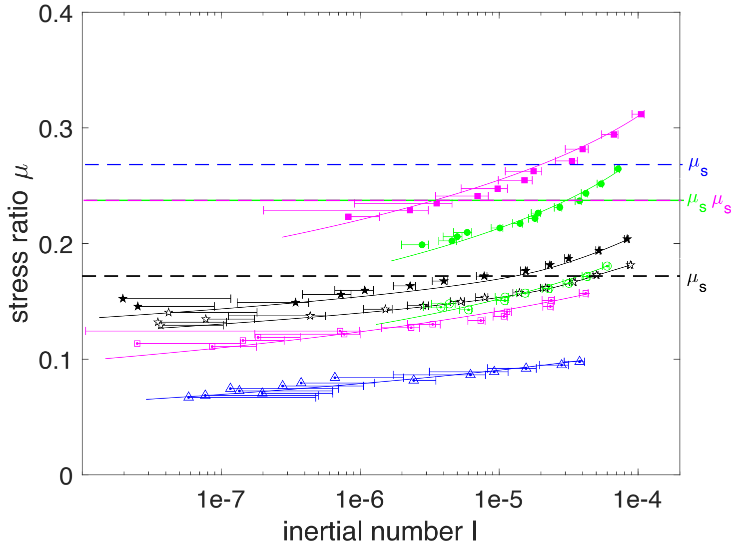
Tang, Brzinski, Shearer and Daniels and , Soft Matter, 2018

Success of nonlocal rheology for particles of different shapes



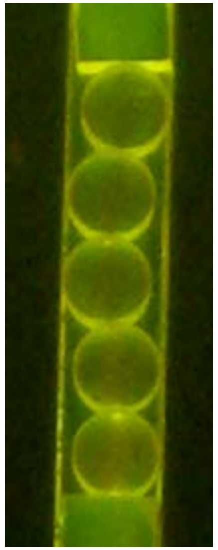
μ_s	b(local)	A(nonlocal)
0.17 ± 0.01	1.1 ± 0.6	0.10 ± 0.001
0.24 ± 0.02	1.1 ± 0.5	0.23 ± 0.003
0.24 ± 0.02	1.1 ± 0.6	0.28 ± 0.003
0.26 ± 0.02	1.1 ± 0.3	0.40 ± 0.003



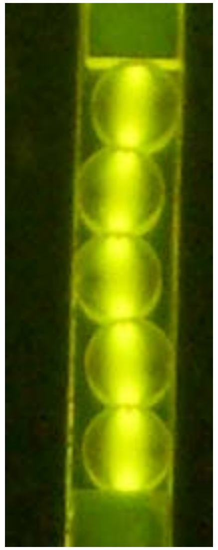


Changing boundary roughness

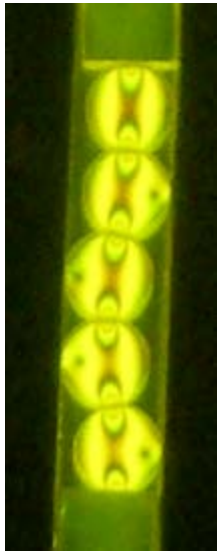
- Laser-cut boundaries with controlled roughness
- 9 & 11 mm diameter photoelastic particles
- Velocity profile from particle tracking
- Shear and normal stresses from photoelasticity



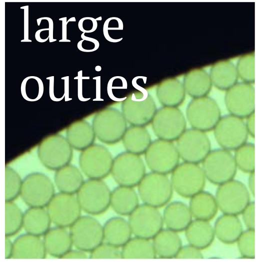
low force

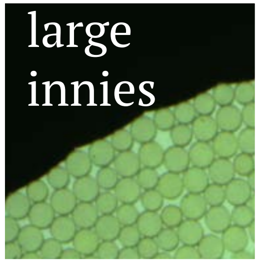


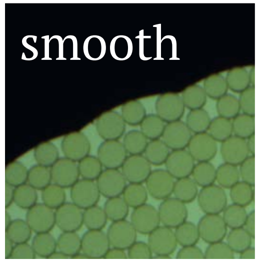
medium force

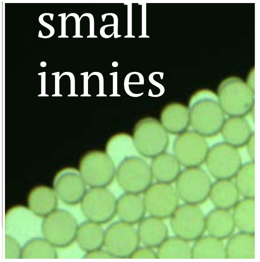


high force



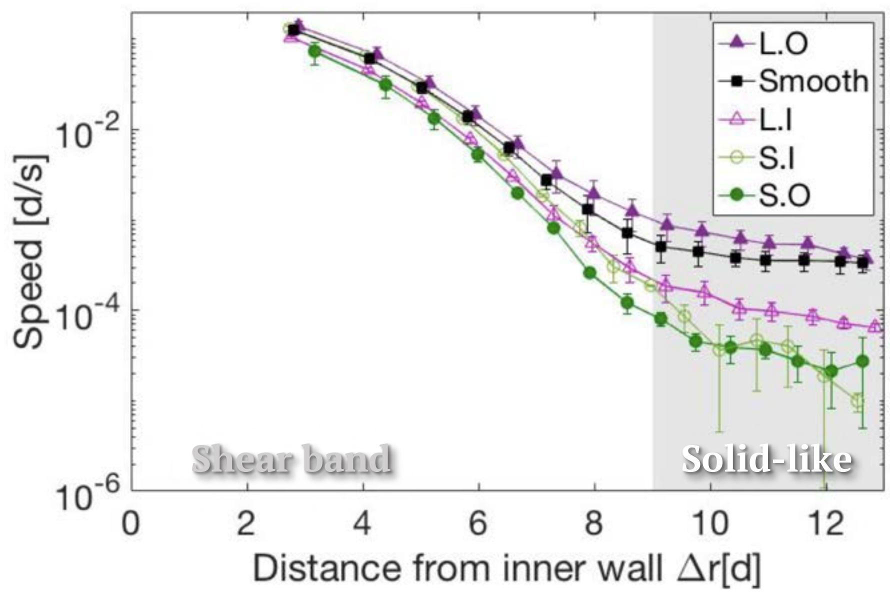






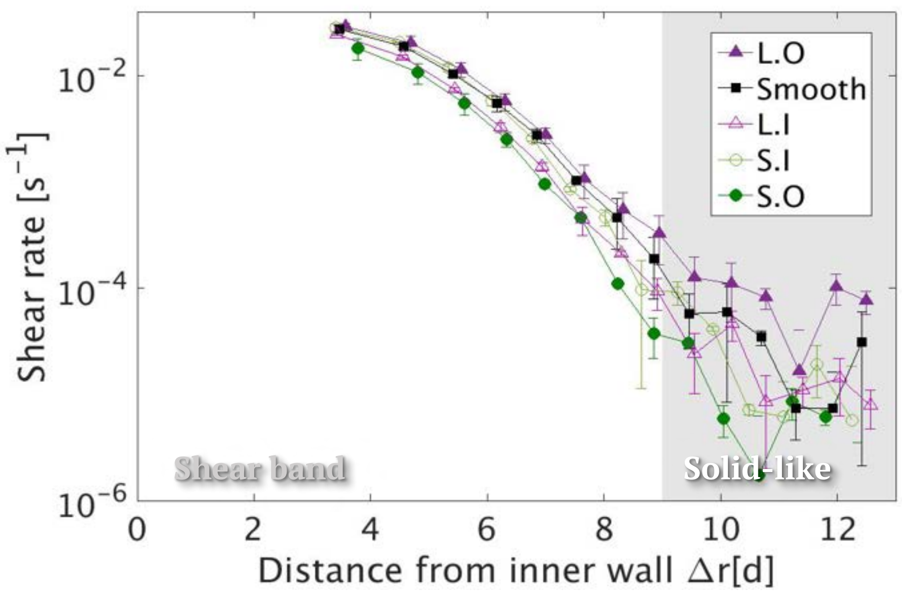
Classic technique: e.g. Frocht 1941

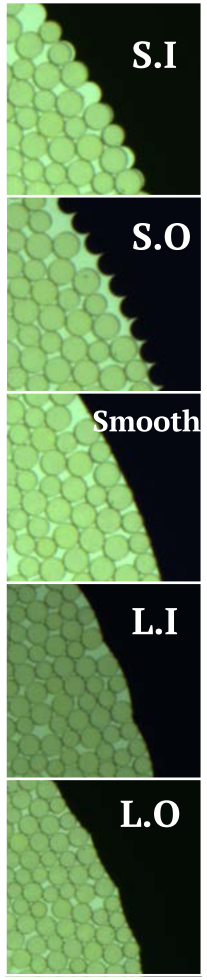
The effect of boundary on flow



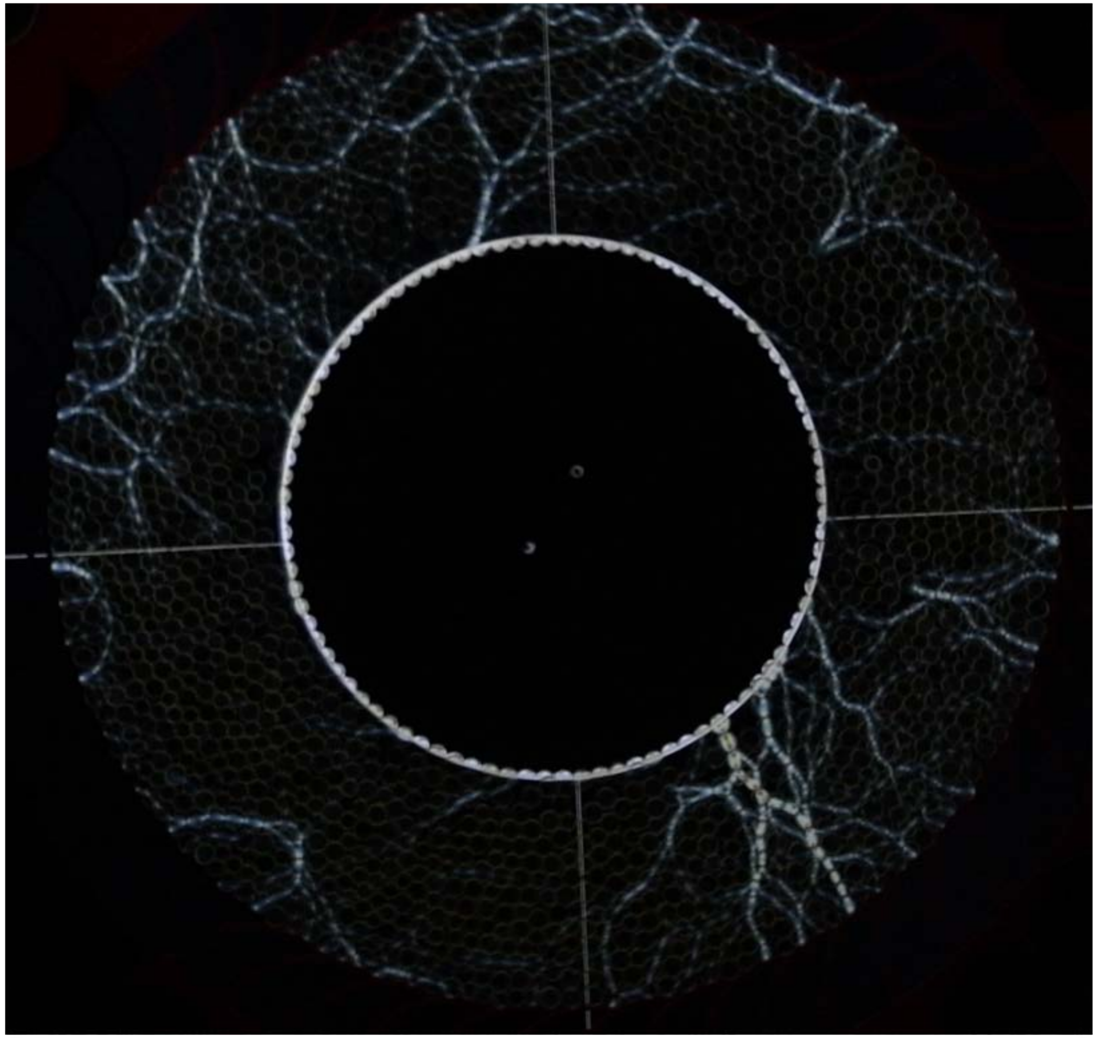
Smoother

Rougher

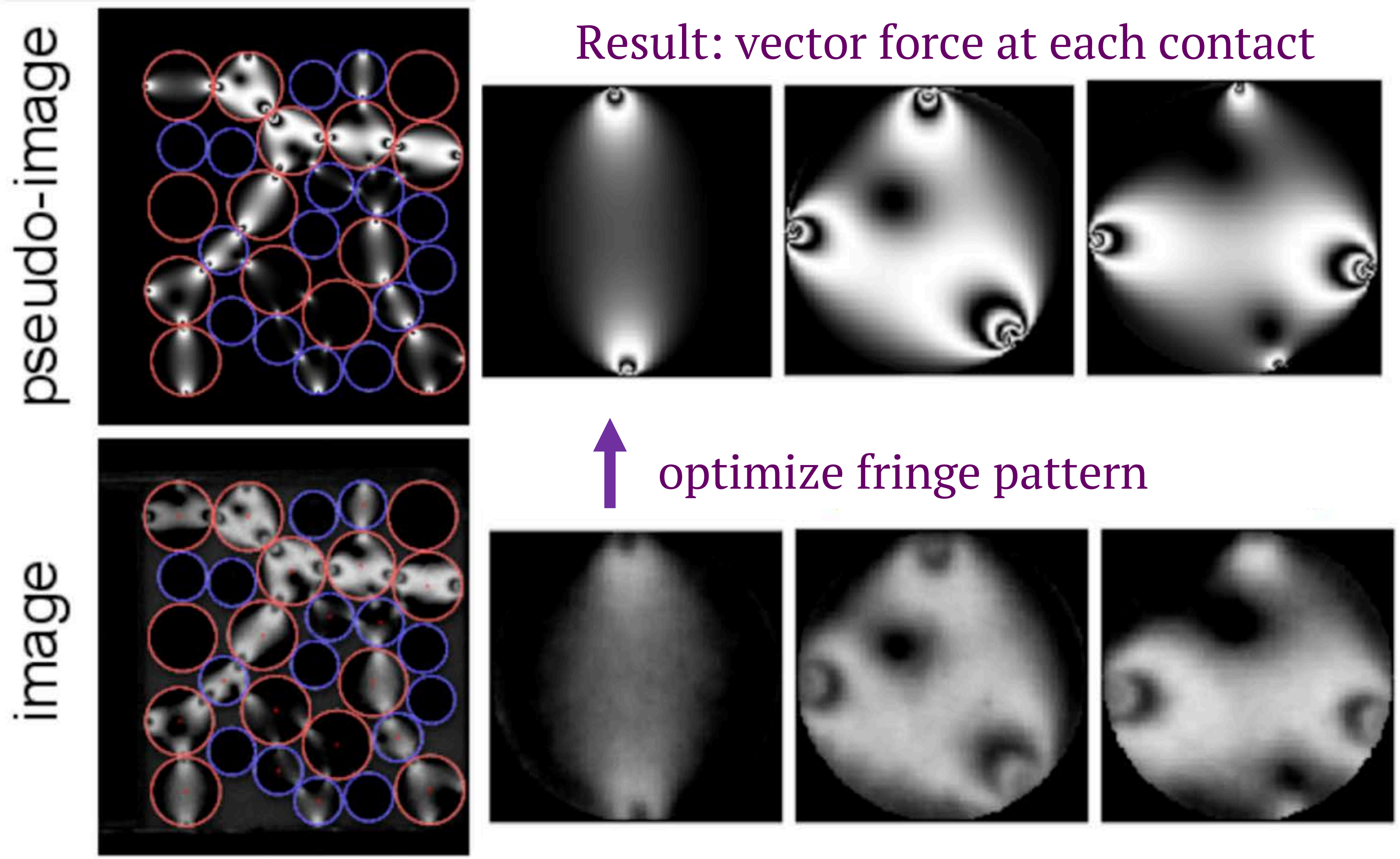




- Smooth boundaries have more than a order of magnitude slip than rough
- Transition to nonlocal regime depends on boundary roughness



Photoelastic inversion



Daniels, Kollmer, Puckett. Rev. Sci. Inst. (2017)
https://github.com/jekollmer/PEGS

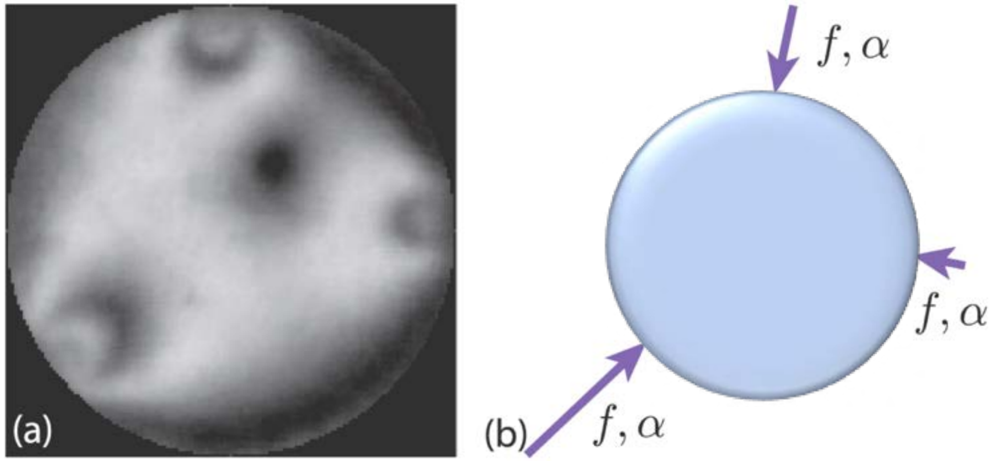
Photoelastic inversion

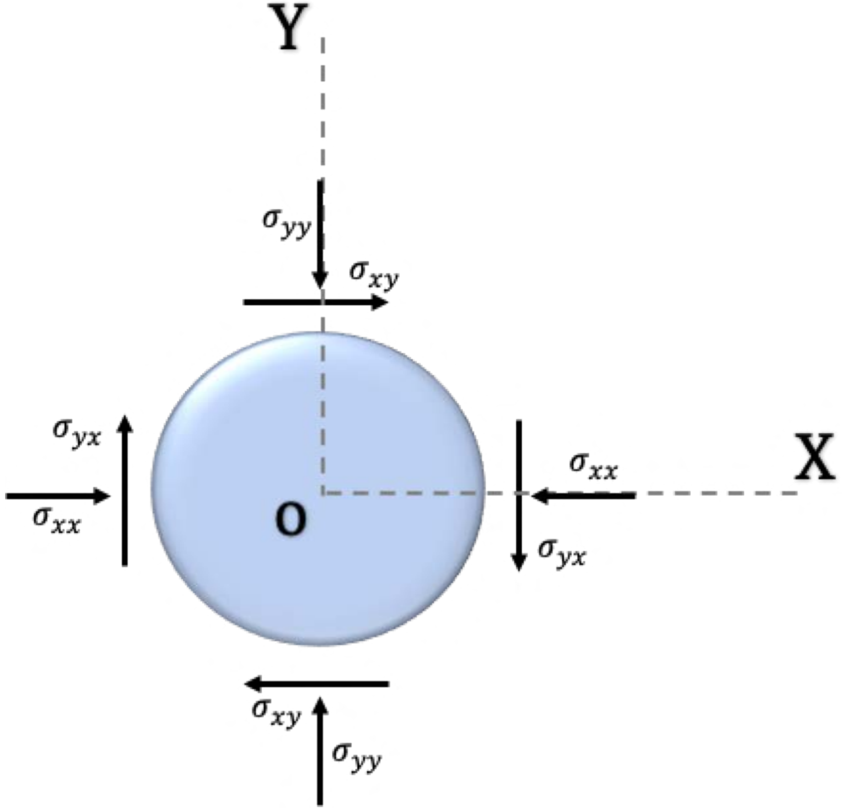
The original image taken on the experiment

Reconstructed pseudo-images, created from fitting the vector contact force at each interparticle contact.

Stress measurement

contact force vector at each contact ⟷ $dF_i = ds\ \sigma_{ij} n_j$ ⟷ stress at each contact





K. Daniels et al Rev, Sci, Inst, 2017

Coarse graining

Discrete data → **Coarse graining** → **Continuum field**

A macroscopic stress field can be extracted from microscopic stress with a coarse graining function(Lucy function):

$$\mathcal{W}(\mathbf{r}) = \frac{105}{16\pi c^3}\left(-3(r/c)^4 + 8(r/c)^3 - 6(r/c)^2 + 1\right), \text{ if } r := |\mathbf{r}| < c, 0 \text{ else},$$

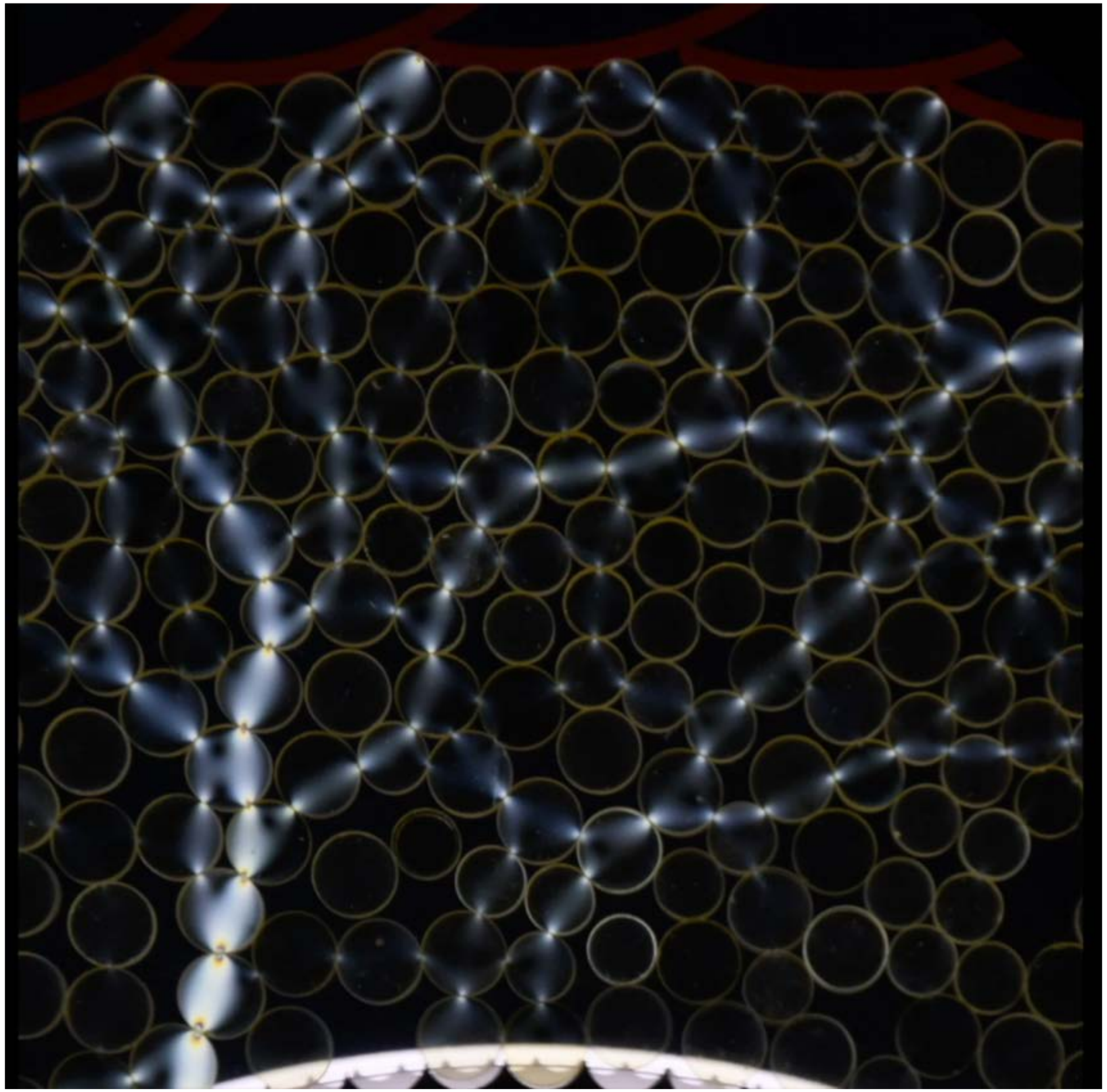
Contact stress is defined:

$$\sigma^c = \sum_{i=1}^{N}\sum_{j=i+1}^{N} f_{ij}\, r_{ij}\int_0^1 \mathcal{W}(r - r_i + s\, r_{ij})ds$$

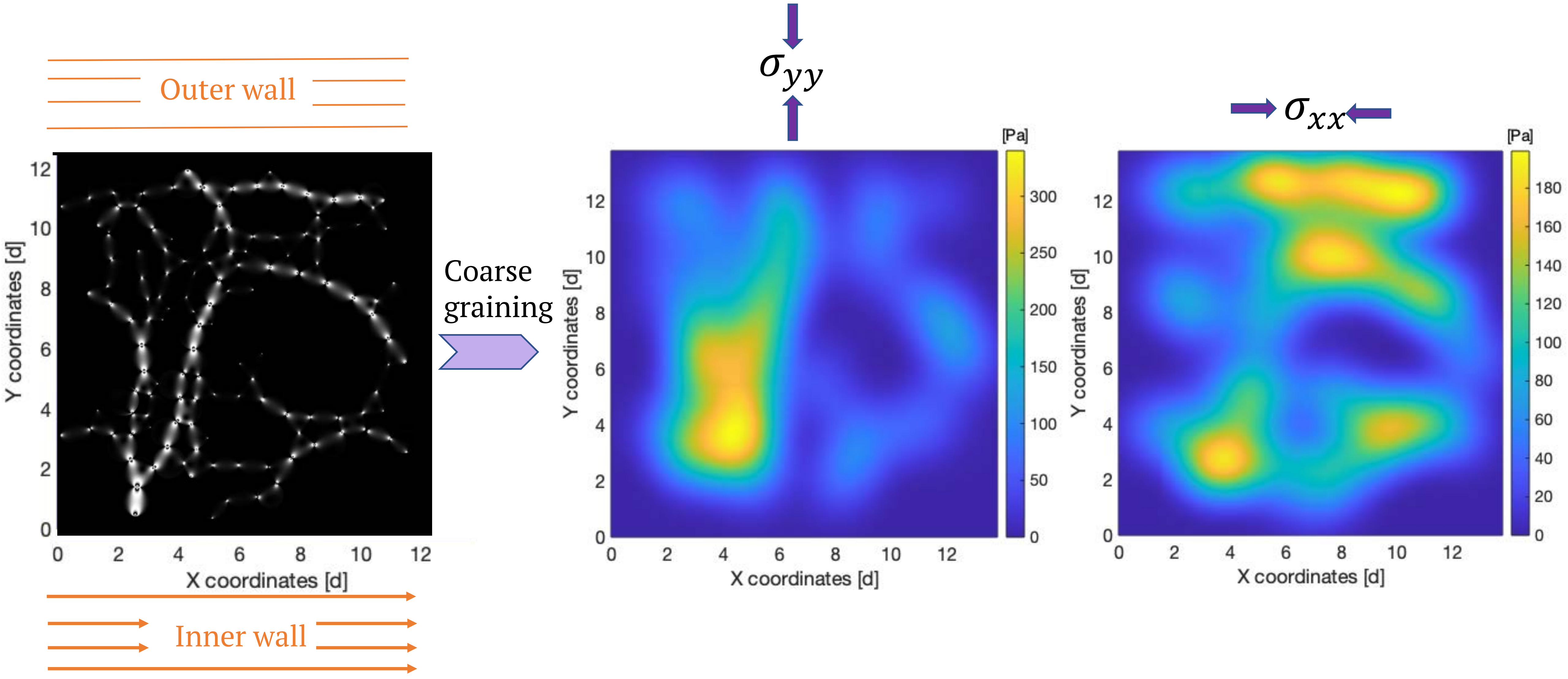
Where r_{ij} is interaction forces and f_{ij} is center to center vector

T Weinhart et al, *Phys. Fluids, 2013*

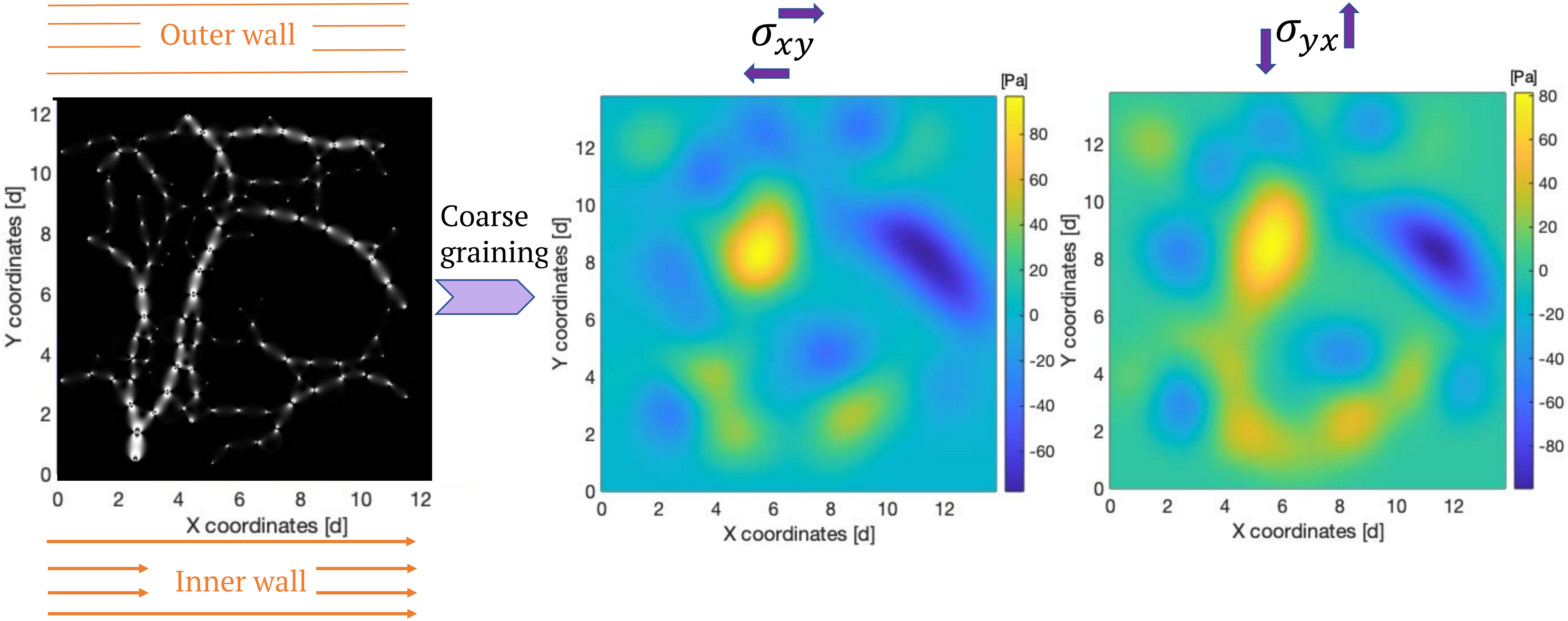
Code-Development Dataset



Coarse graining normal stress (pressure)

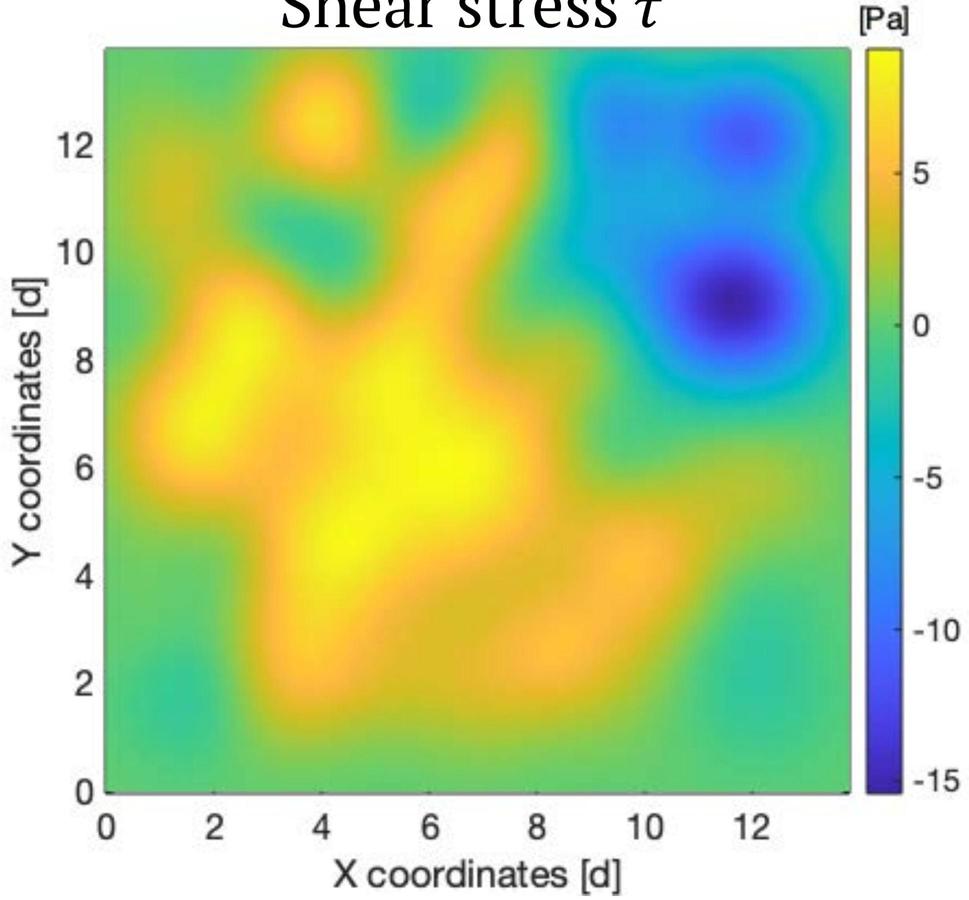


Coarse graining shear stress

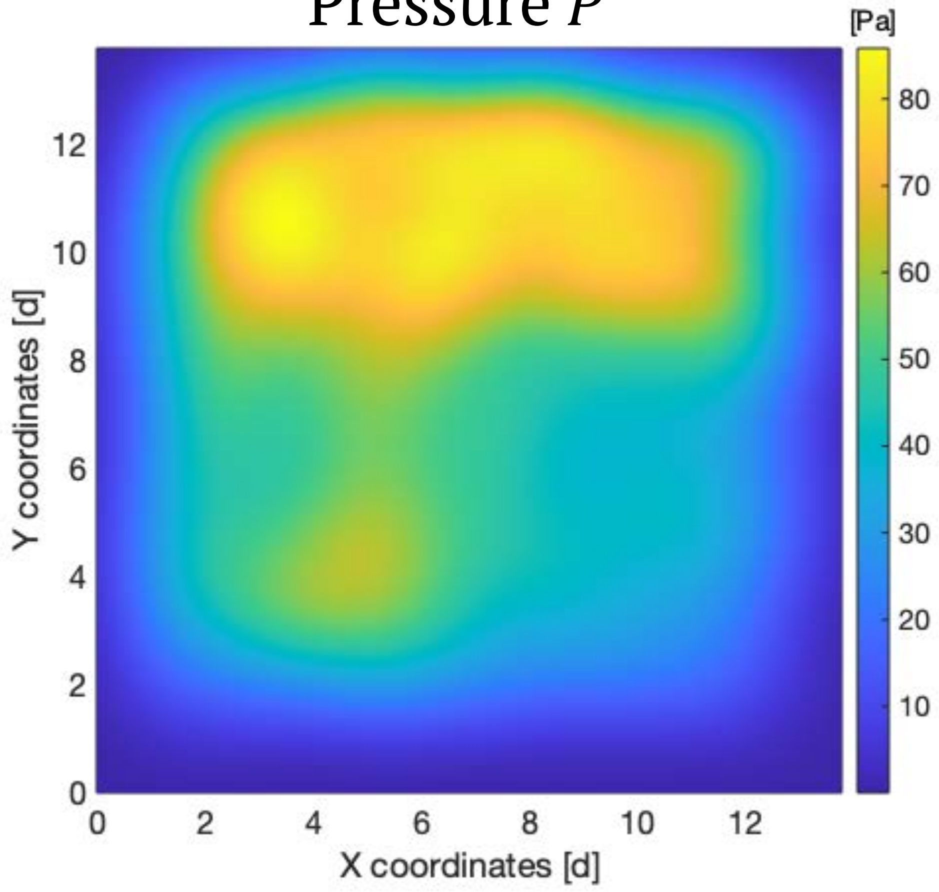


Coarse graining stress, average over 60 frames

Shear stress τ



Pressure P



Conclusions & Next Steps

- we have observed that boundary roughness strongly controls both the flow profile $v(r)$ and shear rate profile $\gamma(r)$.
- Successfully measured stress component at grain size level, using photoelastic methods.
- Developed coarse graining pipeline to obtain stress field.
- When we take more data we will be able to:
  1. obtain stress fields and how boundary properties control stress fields
  2. provides all of the information necessary to calculate the local $\mu(I)$ and fluidity $g(r)$
  3. characterize the dependence of the boundary conditions on both the particle-shape and boundary characteristics.