For steady flow the depth-integrated equations are :

$$\frac{\mathrm{d}}{\mathrm{d}x}(h\overline{u}) = w,$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{77}{48}h\overline{u}^2\right) = -g_\perp h\frac{\mathrm{d}\tilde{z}}{\mathrm{d}x} - \frac{\mu_W}{W}g_\perp h^2,$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\kappa h\overline{u}^3) = -g_\perp h\overline{u}\frac{\mathrm{d}\tilde{z}}{\mathrm{d}x} - \frac{5}{9}\frac{\mu_W}{W}g_\perp h^2\overline{u} - \frac{35}{9}\chi D g_\perp^{1/2}\frac{\overline{u}^2}{h^{1/2}}.$$

The non-dimensionlised form is :

$$\frac{d}{d\hat{x}}(\hat{q}) = -\hat{\omega}\,\hat{x} \longrightarrow (i)$$

$$\frac{d}{d\hat{x}}\left(\chi\frac{\hat{q}^2}{\hat{h}}\right) = -\hat{h}\frac{d\hat{\tilde{z}}}{d\hat{x}} - \hat{h}^2 \longrightarrow (ii)$$

$$\frac{d}{d\hat{x}}\left(\kappa\frac{\hat{q}^3}{\hat{h}^2}\right) = -\hat{q}\frac{d\hat{\tilde{z}}}{d\hat{x}} - \frac{35}{9}\frac{\hat{q}^2}{\hat{h}^{5/2}} - \frac{5}{9}\hat{q}\,\hat{h} \longrightarrow (iii)$$

$$\frac{d}{d\hat{x}}\left(\chi\,\frac{\hat{q}^2}{\hat{h}}\right) = -\hat{h}\,\frac{d\hat{\tilde{z}}}{d\hat{x}} - \hat{h}^2 \rightarrow \text{ from (ii), where } \chi = \frac{77}{48} \;\&\; \hat{q} = \hat{h}\,\overline{\hat{u}}$$

$$\Rightarrow \chi\left[\frac{2\hat{q}}{\hat{h}}\left(\frac{d\hat{q}}{d\hat{x}}\right) + \hat{q}^2\left\{-\frac{1}{\hat{h}^2}\left(\frac{d\hat{h}}{d\hat{x}}\right)\right\}\right] = -\hat{h}\,\frac{d\hat{\tilde{z}}}{d\hat{x}} - \hat{h}^2$$

$$\Rightarrow \frac{2\hat{q}\chi}{\hat{h}}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{\chi\hat{q}^2}{\hat{h}^2}\left(\frac{d\hat{h}}{d\hat{x}}\right) + \hat{h}^2 = -\hat{h}\,\frac{d\hat{\tilde{z}}}{d\hat{x}}$$

$$\Rightarrow \boxed{\frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{\chi\hat{q}^2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right) - \frac{2\chi\hat{q}}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \hat{h}} \rightarrow \boxed{1}$$

Similarly from eq. (iii):

$$\frac{d}{dx}\left(K\frac{\hat{q}^3}{\hat{h}^2}\right) = -\hat{q}\frac{d\hat{\tilde{z}}}{d\hat{x}} - \frac{35}{9}\frac{\hat{q}^2}{\hat{h}^{5/2}} - \frac{5}{9}\hat{q}\hat{h}$$

$$\Rightarrow K\left[\frac{1}{\hat{h}^2}\frac{3\hat{q}^2}{1}\left(\frac{d\hat{q}}{d\hat{x}}\right) + \hat{q}^3\left\{-\frac{2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right)\right\}\right] = -\hat{q}\frac{d\hat{\tilde{z}}}{d\hat{x}} - \frac{35}{9}\frac{\hat{q}^2}{\hat{h}^{5/2}} - \frac{5}{9}\hat{q}\hat{h}$$

$$\Rightarrow \frac{3K\hat{q}^2}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{2K\hat{q}^2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right) + \frac{35}{9}\frac{\hat{q}^2}{\hat{h}^{5/2}} + \frac{5}{9}\hat{q}\hat{h} = -q\frac{d\hat{\tilde{z}}}{d\hat{x}}$$

$$\Rightarrow \boxed{\frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{2K\hat{q}^2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right) - \frac{3K\hat{q}}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{35}{9}\frac{\hat{q}}{\hat{h}^{5/2}} - \frac{5}{9}\hat{h}} \Rightarrow \boxed{2}$$

$\boxed{1} = \boxed{2}$

$$\Rightarrow \frac{\gamma \hat{q}^2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right) - \frac{2\gamma \hat{q}}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \hat{h} = \frac{2K\hat{q}^2}{\hat{h}^3}\left(\frac{d\hat{h}}{d\hat{x}}\right) - \frac{3Kq}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{35}{9}\frac{\hat{q}}{\hat{h}^{5/2}} - \frac{5}{9}\hat{h}$$

$$\Rightarrow \left(\frac{d\hat{h}}{d\hat{x}}\right)\left[\frac{\hat{q}^2}{\hat{h}^3}\right](\gamma - 2K) = \left(\frac{d\hat{q}}{d\hat{x}}\right)\left[\frac{\hat{q}}{\hat{h}^2}\right](2\gamma - 3K) - \frac{35}{9}\frac{\hat{q}}{\hat{h}^{5/2}} + \frac{4}{9}\hat{h}$$

mult. by $\dfrac{\hat{h}^3}{\hat{q}^2(\gamma - 2K)}$

$$\Rightarrow \frac{d\hat{h}}{d\hat{x}} = \frac{\left(\frac{d\hat{q}}{d\hat{x}}\right)(2\gamma - 3K)\hat{h}}{\hat{q}(\gamma - 2K)} - \frac{\frac{35}{9}\hat{h}^{\frac{1}{2}}}{\hat{q}(\gamma - 2K)} + \frac{\frac{4}{9}\hat{h}^4}{\hat{q}^2(\gamma - 2K)}$$

$$\Rightarrow \frac{d\hat{h}}{d\hat{x}} = \frac{(2\gamma - 3K)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{35}{9}\hat{h}^{\frac{1}{2}}\hat{q} + \frac{4}{9}\hat{h}^4}{\hat{q}^2(\gamma - 2K)}$$

now mult. R.H.S. by $\left(\frac{-1}{-1}\right)$

$$\therefore \boxed{\frac{d\hat{h}}{d\hat{x}} = \frac{(3K-2\chi)\hat{h}\,\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + \frac{35}{9}\hat{h}^{\frac{1}{2}}\hat{q} - \frac{4}{9}\hat{h}^{4}}{\hat{q}^{2}(2K-\chi)}} \longrightarrow \boxed{3}$$

But $\chi = \frac{77}{48}$

$$\Rightarrow \frac{d\hat{h}}{d\hat{x}} = \frac{\left(\frac{144K-154}{48}\right)\hat{h}\,\hat{q}\,\frac{d\hat{q}}{d\hat{x}} + \frac{35}{9}\hat{h}^{\frac{1}{2}}\hat{q} - \frac{4}{9}\hat{h}^{4}}{\hat{q}^{2}\left[\frac{96K-77}{48}\right]}$$

where $\frac{3K}{1} - \frac{2(77)}{48} = \frac{144K-154}{48}$ and $\frac{2K}{1} - \frac{77}{48} = \frac{96K-77}{48}$

$\therefore$ $$\frac{d\hat{h}}{d\hat{x}} = \frac{(144K-154)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + \frac{560}{3}\hat{h}^{1/2}\hat{q} - \frac{64}{3}\hat{h}^4}{\hat{q}^2(96K-77)}$$

now mult. R.H.S. by $\left(\frac{3}{3}\right)$

$$\Rightarrow \frac{d\hat{h}}{d\hat{x}} = \frac{3(144K-154)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + 560\hat{h}^{\frac{1}{2}}\hat{q} - 64\hat{h}^4}{3\hat{q}^2(96K-77)}$$

$\therefore$ $$\boxed{\frac{d\hat{h}}{d\hat{x}} = \frac{6(72K-77)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + 560\hat{h}^{\frac{1}{2}}\hat{q} - 64\hat{h}^4}{3\hat{q}^2(96K-77)}} \rightarrow \boxed{4}$$

NB: Same as Hung's equ. (11) but without $\left(\hat{\omega}^{7/8}\right)$

To solve for $\left(\frac{d\hat{z}}{d\hat{x}}\right)$, subst ③ into ① $\Longrightarrow$

$$\frac{d\hat{z}}{d\hat{x}} = \frac{-2\gamma\hat{q}}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \hat{h} + \frac{\gamma\hat{q}^2}{\hat{h}^3}\frac{d\hat{h}}{d\hat{x}}$$

$$= \frac{-2\gamma\hat{q}}{\hat{h}^2}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \frac{\hat{h}}{1} + \frac{\gamma\hat{q}^2}{\hat{h}^3}\left[\frac{(3K-2\gamma)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + \left(\frac{35}{9}\right)\hat{h}^{\frac{1}{2}}\hat{q} - \frac{4}{9}\hat{h}^4}{\hat{q}^2(2K-\gamma)}\right]$$

$$= \frac{-\hat{h}(2K-\gamma)2\gamma\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \hat{h}^4(2K-\gamma) + \gamma\hat{q}\hat{h}\left(\frac{d\hat{q}}{d\hat{x}}\right)(3K-2\gamma) + \frac{35}{9}\hat{h}^{\frac{1}{2}}\gamma\hat{q} - \frac{4}{9}\hat{h}^4\gamma}{\hat{h}^3(2K-\gamma)}$$

$$\therefore \frac{d\hat{z}}{d\hat{x}} = \frac{\left(\frac{d\hat{q}}{d\hat{x}}\right)\gamma\hat{q}\hat{h}\left[3K-2\gamma-4K+2\gamma\right] - \hat{h}^4\left[2K-\gamma+\frac{4}{9}\gamma\right] + \frac{35}{9}\hat{h}^{\frac{1}{2}}\gamma\hat{q}}{\hat{h}^3(2K-\gamma)}$$

$$\therefore \frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{-K\gamma\hat{q}\hat{h}\left(\frac{d\hat{q}}{d\hat{x}}\right) - \hat{h}^4\left[\frac{18K-5\gamma}{9}\right] + \frac{35\hat{h}^{\frac{1}{2}}\gamma\hat{q}}{9}}{\hat{h}^3\left[2K-\gamma\right]}$$

$$\Rightarrow \boxed{\frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{35\hat{h}^{\frac{1}{2}}\gamma\hat{q} - \hat{h}^4(18K-5\gamma) - 9K\gamma\hat{q}\hat{h}\left(\frac{d\hat{q}}{d\hat{x}}\right)}{9\hat{h}^3\left[2K-\gamma\right]}} \longrightarrow \boxed{5}$$

Now subst. $\gamma = 77/48$

$$\Rightarrow \frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{\frac{35(77)}{48}\hat{h}^{\frac{1}{2}}\hat{q} - \hat{h}^4\left[\frac{18(48)K-5(77)}{48}\right] - \frac{9(77)}{48}K\hat{q}\hat{h}\frac{d\hat{q}}{d\hat{x}}}{9\hat{h}^3\left[\frac{96K-77}{48}\right]}$$

$$\therefore \boxed{\frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{2695\,\hat{h}^{\frac{1}{2}}\hat{q} - \hat{h}^{4}\left[864K - 385\right] - 693K\hat{q}\hat{h}\left(\frac{d\hat{q}}{d\hat{x}}\right)}{9\hat{h}^{3}\left[96K - 77\right]}} \rightarrow \boxed{6}$$