Most of the questions are regarding variable, derivation, and literature references. In your video you were showing a lot of equations but due to the format of this being a 30min talk it was difficult to explain everything in detail. I think people are interested in learning more on how you got to the equation and relationship that you had. If you provide a bit more context to each equation and variable it would be helpful. Below are the compiled questions from the members

There are a lot of equations and variables, maybe having a nomenclature might be helpful so that the audience can print it out and follow along
The attached derivations will make plain the meaning of the symbols.
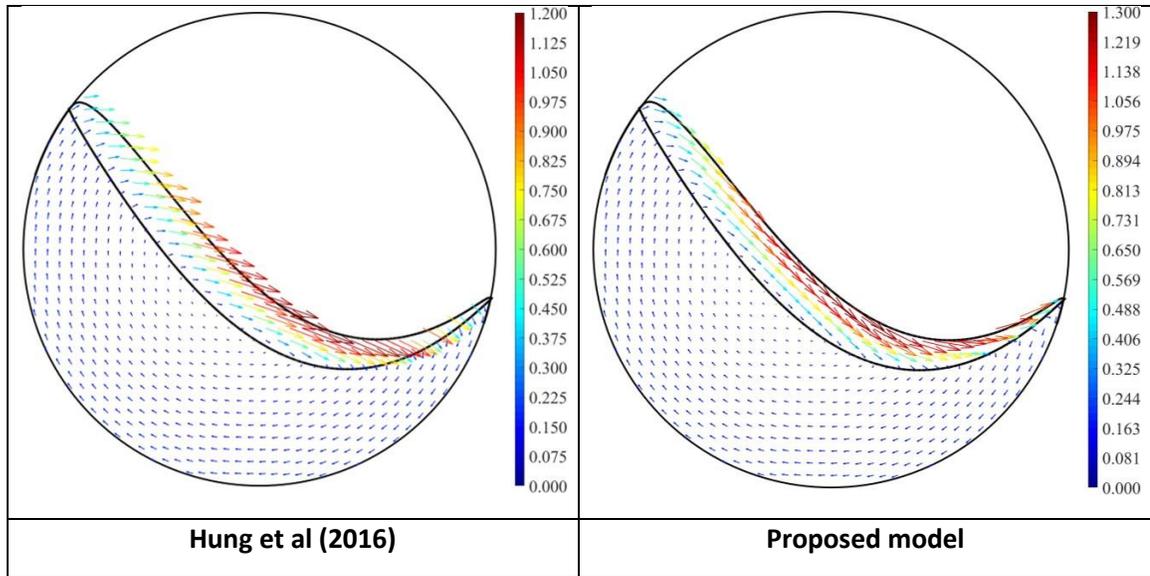
Slide 6
- Can you provide the derivation of the free surface profile and the flowing layer depth profile?
  I have attached my handwritten derivations.
  The derivation of free surface and basal interface combines the depth-integrated governing equations of mass momentum and energy. The derivation of the momentum and energy equations are not difficult; however, they are quite long, and involve several mathematical techniques like Leibniz integration rule, etc …

Slide 7
- Can you describe how the stream function was derive from the velocity profile?
  I have attached my handwritten derivation for the stream function
- **Basically, for an incompressible flow assumption, the gradient of the stream function equals the velocity. Since we know the velocity field, we can integrate out the stream function.**
- You mentioned that the 'current literature model fail to achieved this level of agreement'. Can you provide more context to this statement (i.e. which model and why it does not capture the direction correctly)
  All literature models essentially apply the shallow flow approximation due to lack of information (or accurate measurements) about the z-component of velocity. Consequently, none of them account for the z-component of velocity, leading to an incorrect velocity field. The tolerance for this approximation lies in the fact that the z-component is relatively small and under certain conditions, do not make a big difference to the magnitude:
  The best model in the literature is by Hung et al (2016)—see reference list at the end of this document. Plotted below is the model of Hung against mine:

| Hung et al (2016) | Proposed model |

In my November-2020 report, I will show these differences for all configurations investigated

Slide 9
- The 5th term does not seem to have the same dimension as the other term, can you please explain this
  That's a typo in my slide—it's clearly missing the velocity *u*.
  I have updated this in my slides and attached the updated pdf for your convenience.

Slide 10
- What is K in the equation
  K = ½ V^2 is the kinetic energy per unit mass. The derivation of the energy dynamics is attached.
  Therein you will clearly see what all the terms mean.

Slide 11
- Can you walkthrough how you derive the equation
  I assume you mean the derivation of shear and advective dissipation energy??
  The derivation is attached.

Slide 12
- Can you provide derivation for the scale up rule
  The derivation of entrainment number and all other dimensionless numbers and equations are attached.
  The scale up rules are then simply a matter of changing values of individual terms in the Entrainment number such that the Entrainment number itself does not change.

Can you define and discuss the derivation for:
- Entrainment number
- Shear energy dissipation
- advection energy dissipation
- forced entrainment and free entrainment

The attached documents on derivations answer these questions.

References

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The above references were used to develop the granular flow model of rotating drum flows