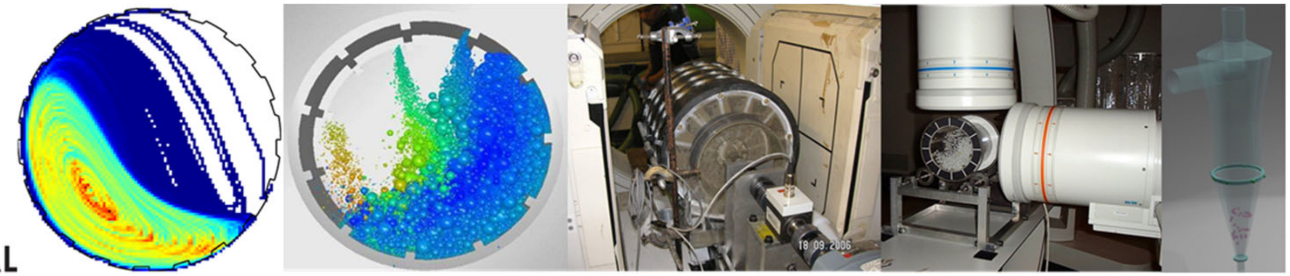

SCALE-UP RULES FOR MIXING MECHANISMS IN ROTATING DRUM FLOWS

By

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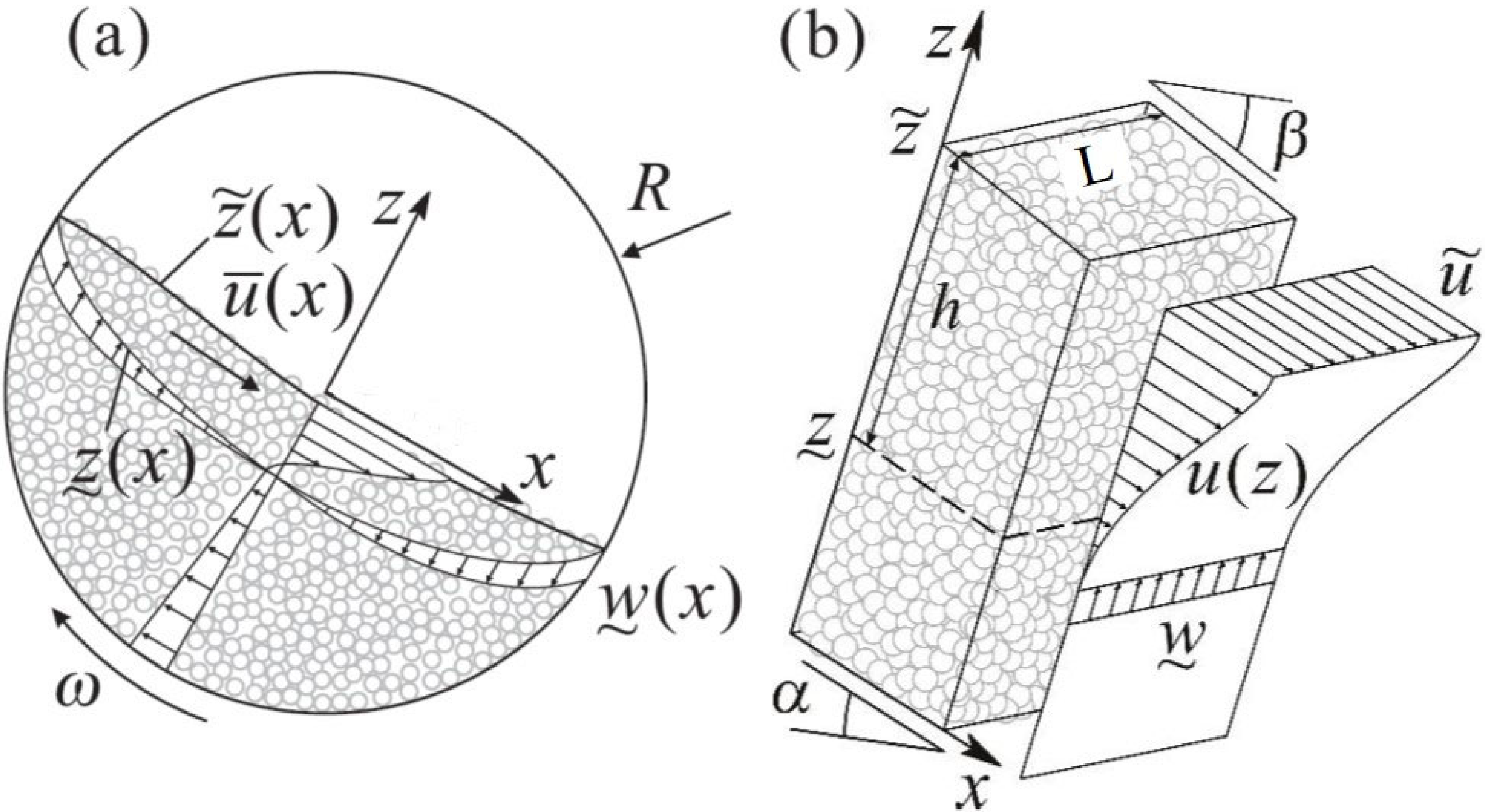
Aim

To identify a suitable dimensionless number from a fundamentally-derived granular flow model of rotating drum flows that facilitates scale-up of the dominant mixing mechanisms spanning rolling-to-cascading flow regimes

1) Model formulation
2) Validation of the model
3) Identification of an energy proxy for dominant mixing mechanisms
4) Dimensionless number that facilitates scale-up

1. Continuum Model: Preamble



Capart et al. (2015) and Hung et al. (2016)

1. Continuum Model: Assumptions

a) Based on linearized viscoplastic dense granular flow rheology by Jop et al. (2006)

$$\tau_{xz} = \mu_0 \sigma + \chi D \dot{\gamma}_{xz} \sqrt{\rho \sigma}$$

Normal Stress: σ

Basal Friction Coefficient: μ_0

Rheological Coefficient: χ

Particle Diameter: D

Shear Rate: $\dot{\gamma}_{xz}$

Particle Density: ρ

b) Shallow flow approx. (bed depth << flow length)

c) Incompressible flow (Tested with full DEM tensorial analysis across all flow regimes)

d) Uniform axial flow (We are ignoring axial flow for now)

e) Coulomb-like friction along side walls

f) Non-equilibrium velocity profile remains similar to equilibrium velocity profile

g) The following boundary conditions apply:

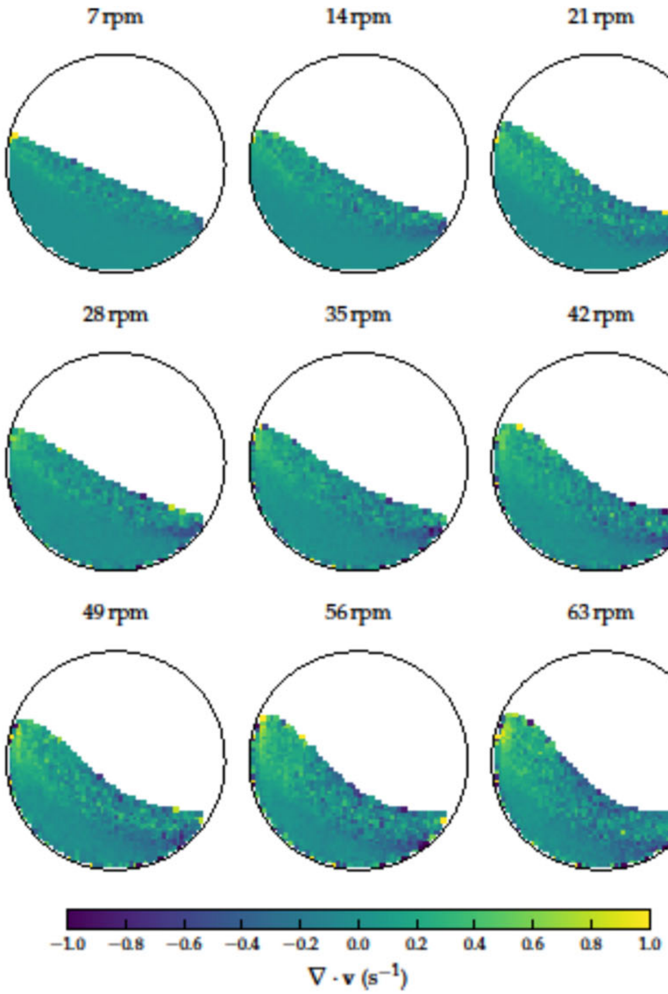
<u>Free Surface</u>

$$\begin{aligned} \tilde{w} &= \frac{\partial \tilde{z}}{\partial t} + \tilde{u}\frac{\partial \tilde{z}}{\partial x} \\ \tilde{\sigma} &= 0, \\ \tilde{\tau} &= 0. \end{aligned}$$

<u>Basal Interface</u>

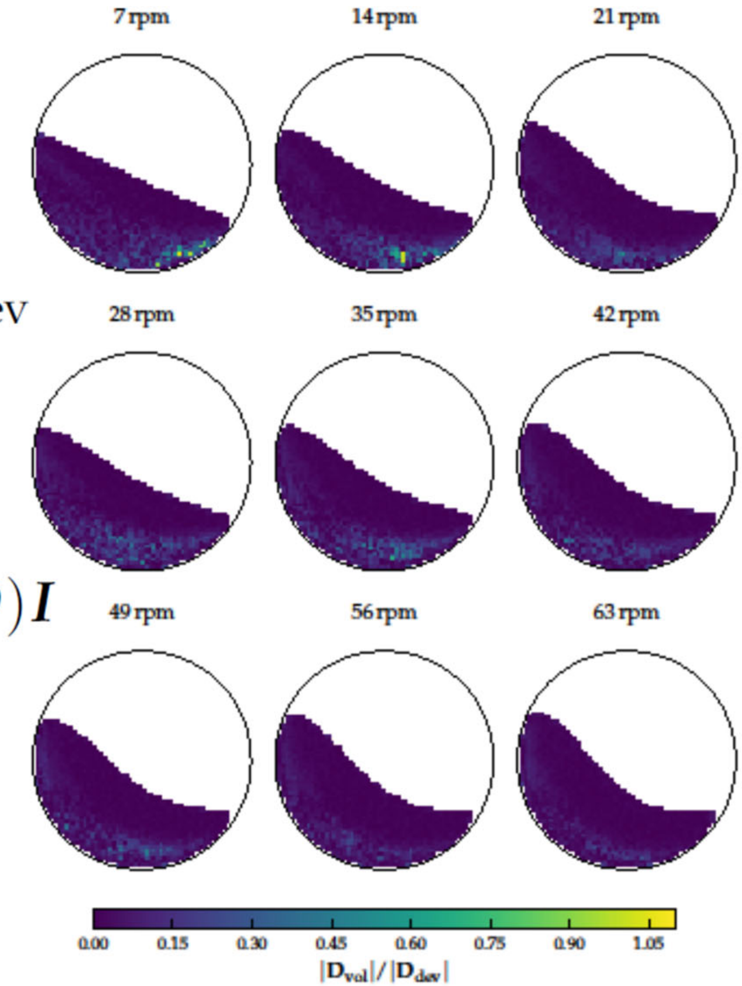
$$\begin{aligned} \underset{\sim}{u} &= 0, \\ \underset{\sim}{w} &= -\omega x \\ \underset{\sim}{\tau} \equiv \tau_0 &= \mu_0 \underset{\sim}{\sigma}. \end{aligned}$$

1. Model Assumption (c): Incompressibility



$$\boldsymbol{D}_{\text{vol}} = \boldsymbol{D} - \boldsymbol{D}_{\text{dev}}$$

$$\boldsymbol{D}_{\text{dev}} = \boldsymbol{D} - \frac{1}{3}\text{tr}(\boldsymbol{D})\boldsymbol{I}$$



$\boldsymbol{D}$ = symmetric velocity gradient
$\boldsymbol{D}_{\text{dev}}$ = deviatoric component of $\boldsymbol{D}$
$\boldsymbol{D}_{\text{vol}}$ = volumetric component of $\boldsymbol{D}$

- The flowing layer is very slightly incompressible
- The rising layer (below basal interface) is incompressible
- Incompressibility valid over wide Froude range

1. Steady-State Governing Equations

Free Surface Profile

$$\frac{d\hat{\tilde{z}}}{d\hat{x}} = \frac{2695\sqrt{\hat{h}}\hat{q} - \hat{h}^4\left(864\kappa - 385\right) - 693\kappa\hat{q}\hat{h}\left(\frac{d\hat{q}}{d\hat{x}}\right)}{9\hat{h}^3\left(96\kappa - 77\right)}$$

Flowing Layer Depth Profile

$$\frac{d\hat{h}}{d\hat{x}} = \frac{6\left(72\kappa - 77\right)\hat{h}\hat{q}\left(\frac{d\hat{q}}{d\hat{x}}\right) + 560\sqrt{\hat{h}}\hat{q} - 64\hat{h}^4}{3\hat{q}^2\left(96\kappa - 77\right)}$$

Velocity field

$$u(x,z) = \overline{u}(x) f(\hat{\eta}(x,z)) + \omega z \qquad \text{... } x\text{-velocity profile}$$

$$w(x,z) = -\frac{\partial \tilde{z}}{\partial x}\overline{u}(x) f(\hat{\eta}(x,z)) - \omega x, \quad \text{with} \qquad \text{... } z\text{-velocity profile}$$

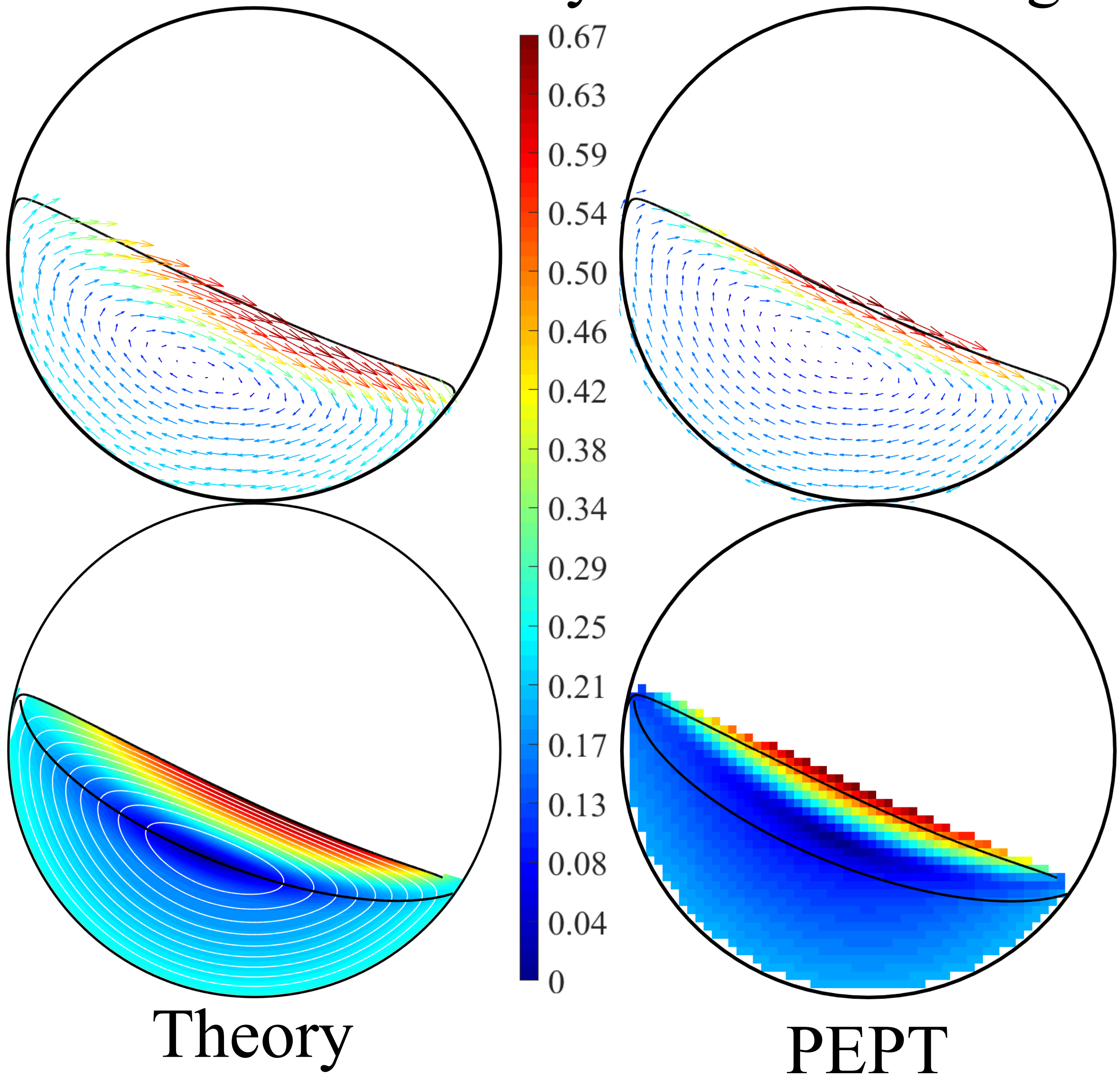
$$f(\hat{\eta}(x,z)) = \begin{cases} \frac{7}{3} - \frac{35}{6}\hat{\eta}^{\frac{3}{2}} + \frac{7}{2}\hat{\eta}^{\frac{5}{2}}, & \text{if } 0 \leq \hat{\eta} \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

... velocity shape function with

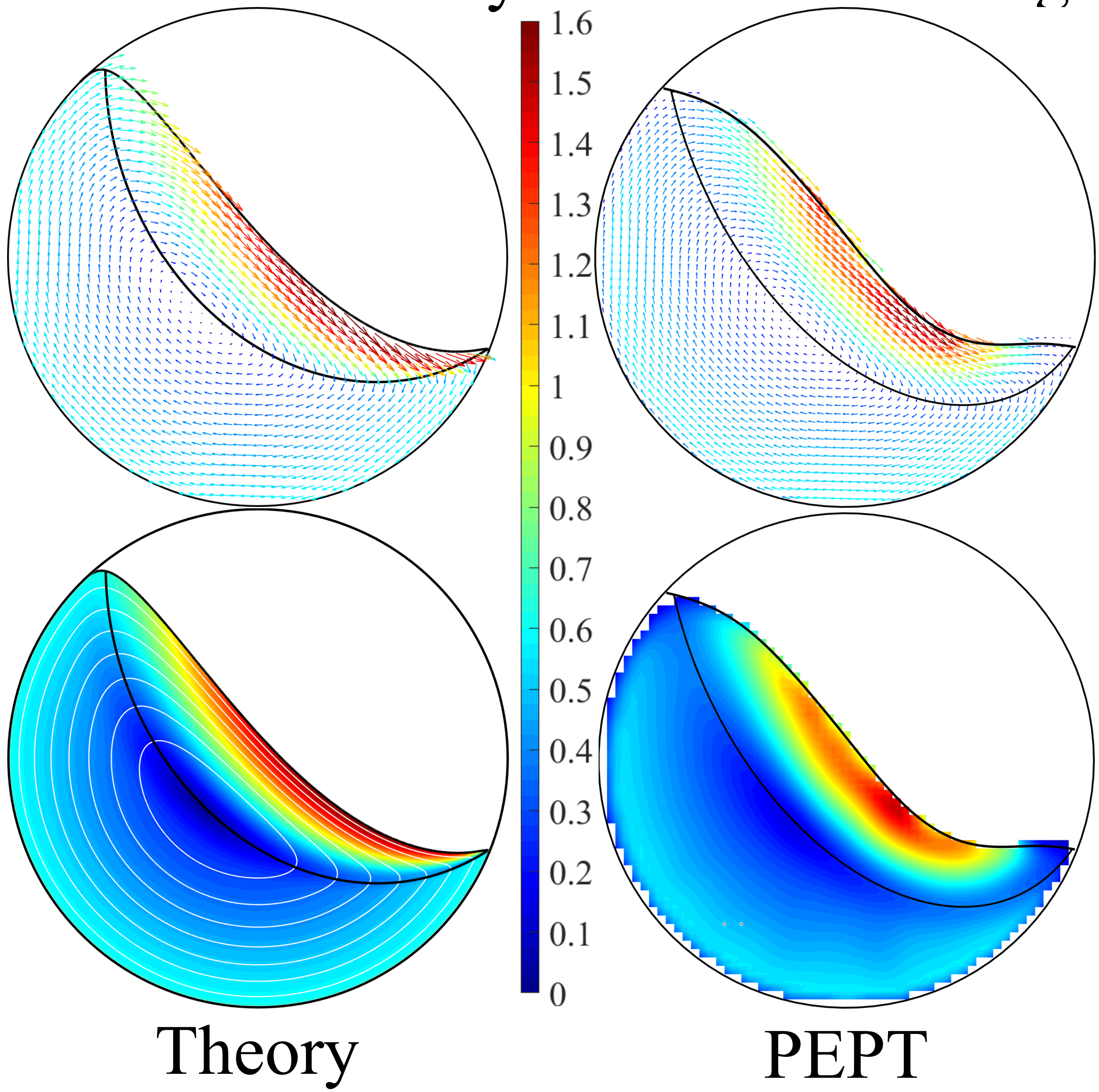
$$\eta = \tilde{z} - z$$

$$\hat{\eta} = \frac{\eta}{h}$$

2. Validation: Velocity Field in Rolling Regime



2. Validation: Velocity Field in Cascading Regime



3 Physical Significance of Energy Balance

$$u\rho\frac{\partial u}{\partial t} + \rho\left(u^2\frac{\partial u}{\partial x} + uw\frac{\partial u}{\partial z}\right) = u\rho g\sin(\beta) - u\frac{\partial \sigma}{\partial x} - \frac{2u\tau_W}{L} + u\frac{\partial \tau}{\partial z}$$

1$^{\text{st}}$ term: Rate of change of kinetic energy.

2$^{\text{nd}}$ term: Divergence of the kinetic energy flux.

3$^{\text{rd}}$ term: Gravitational energy flux.

4$^{\text{rd}}$ term: Work rate of the normal stress gradient.

5$^{\text{th}}$ term: Energy dissipation rate due to wall shear stresses.

6$^{\text{th}}$ term: Energy dissipation rate by internal shear stresses.

Term 2 describes advective energy dissipation

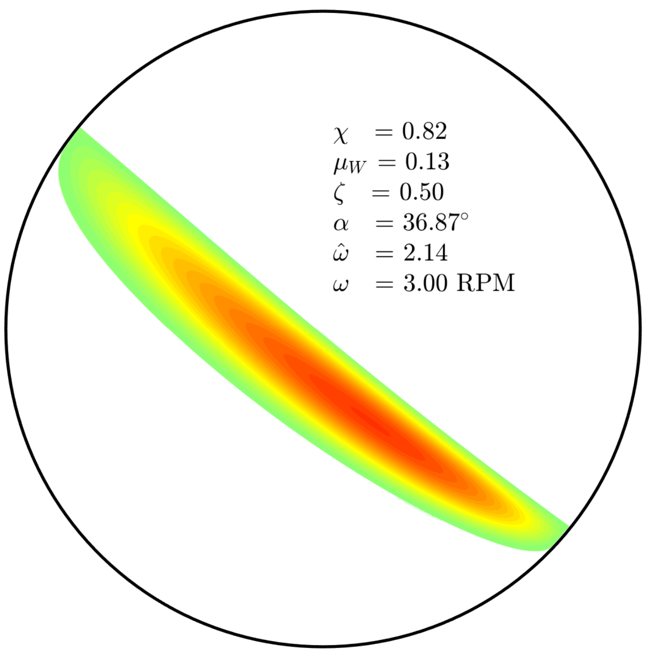
Term 6 describes shear energy dissipation

Typical Energy Distributions (per unit density): [m^2/s^3]

Shear Dissipation

$$\chi D \sqrt{g_\perp h \hat{\eta}} |\dot{\gamma}|^2$$

- Positive definite
- Monopolar

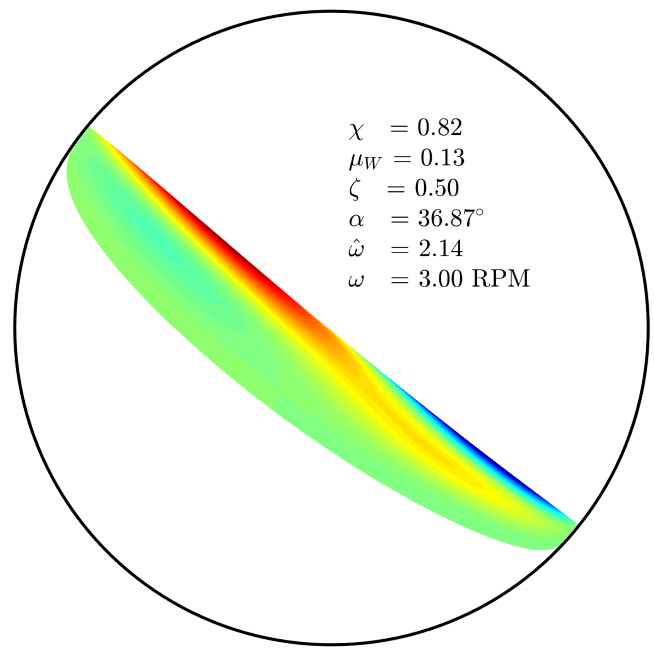


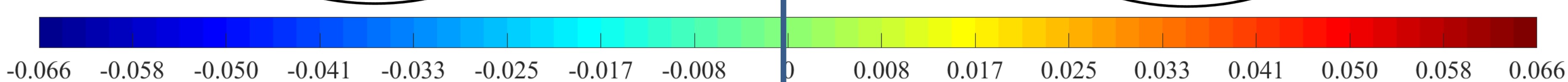
Advective Dissipation

$$\vec{\nabla} \cdot \left(K \rho \vec{V} \right) = \left(\frac{3\rho}{2} \right) u^2 \frac{\partial u}{\partial x} + \rho u w \frac{\partial u}{\partial z}$$

Bipolar distribution

- Positive divergence ⇒ KE source
- Negative divergence ⇒ KE sink

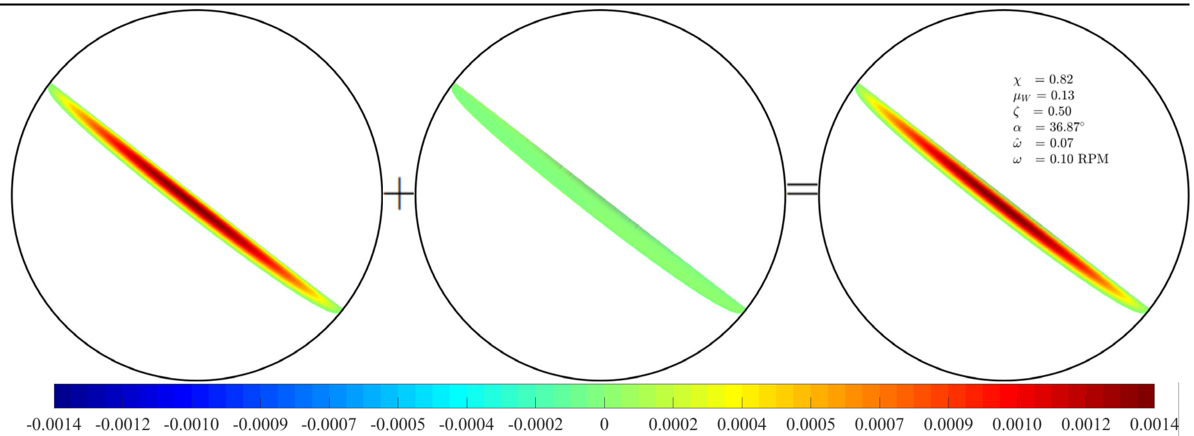




Energy Distributions spanning Rolling-to-Cascading

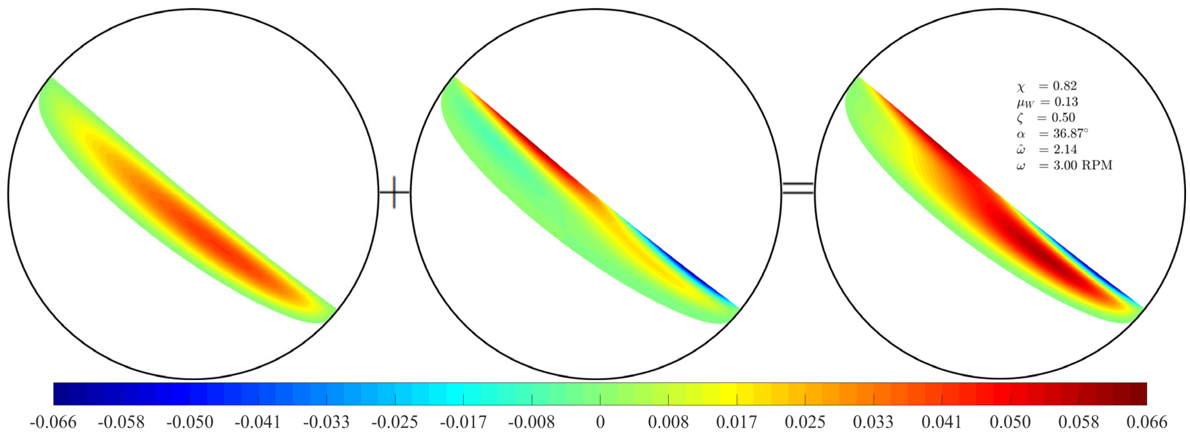
Shear Dominated

ω = 0.10 RPM
R = 239 mm
Fill Frac (ζ) = 0.5



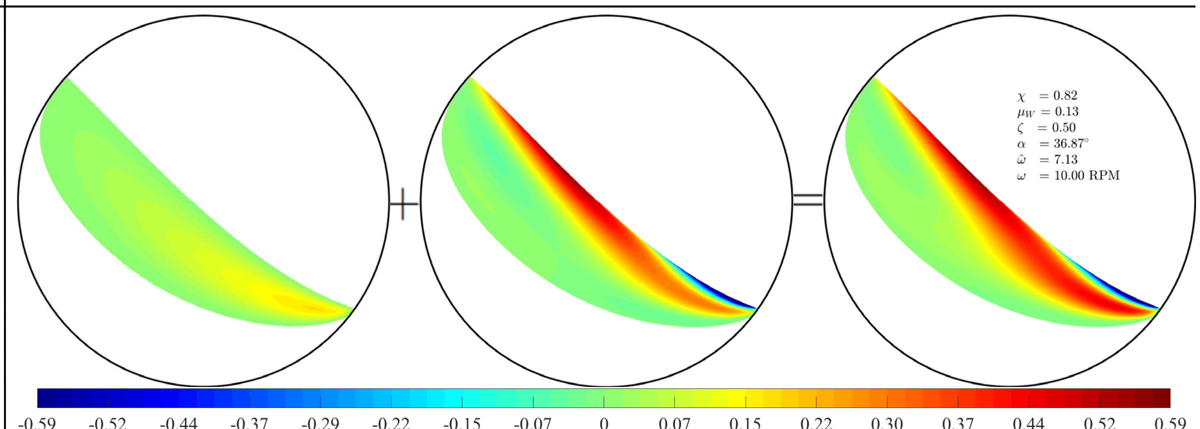
Shear & Advective Dominated

ω = 3.00 RPM
R = 239 mm
Fill Frac (ζ) = 0.5



Advective Dominated

ω = 10 RPM
R = 239 mm
Fill Frac (ζ) = 0.5



3. Scale-Up Rule of Mixing Mechanism

By dimensional analysis we obtain the ratio of forced-to-free Entrainment:

$$\hat{\omega} = \frac{\omega R^{9/8}}{\sqrt{g_{\perp}}\,(\chi D)^{3/4}} \left(\frac{L}{\mu_W}\right)^{1/8}$$

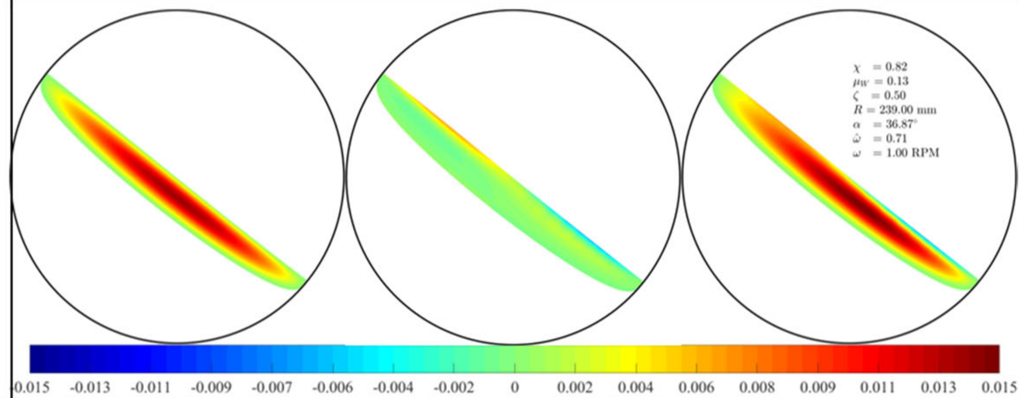
The Crux of the Scale-Up Rule

- Two rotating drum configurations with the same $\hat{\omega}$ will exhibit the same proportions of advective-to-shear energy dissipation.
- So, a bench-scale configuration with a given proportion of advection-to-shear energy dissipation can be scaled (up) to an industrial drum by keeping $\hat{\omega}$ the same in both configurations.
- To illustrate the scale-up rule, we consider the three basic splits of advection-to-shear flow and mixing

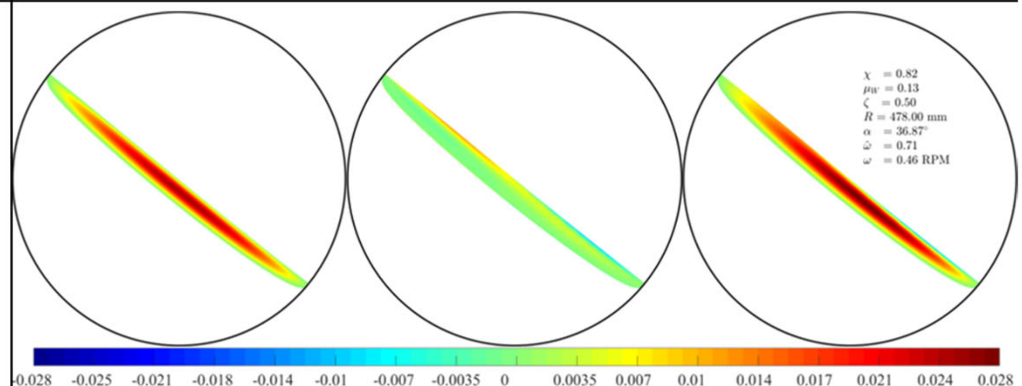
Scale-Up of Shear Dominated Mixing

Scaling Rule	Shear + Advection = Total

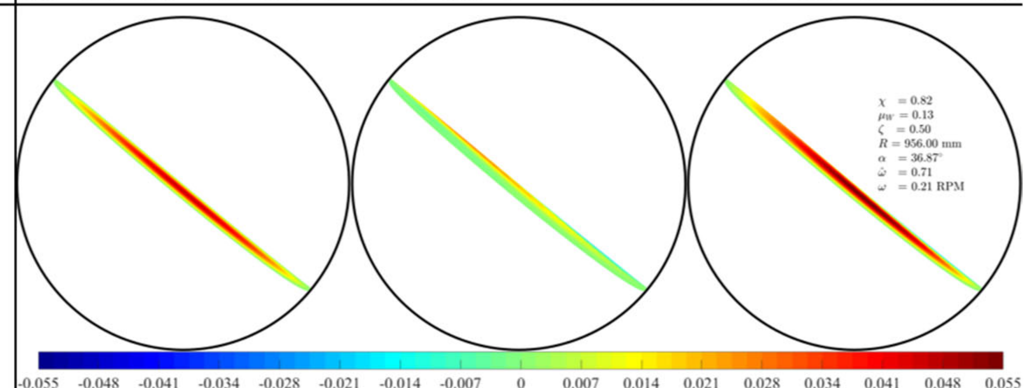
R = 239 mm
ω = 1.00 RPM
Fill Frac (ζ) = 0.5



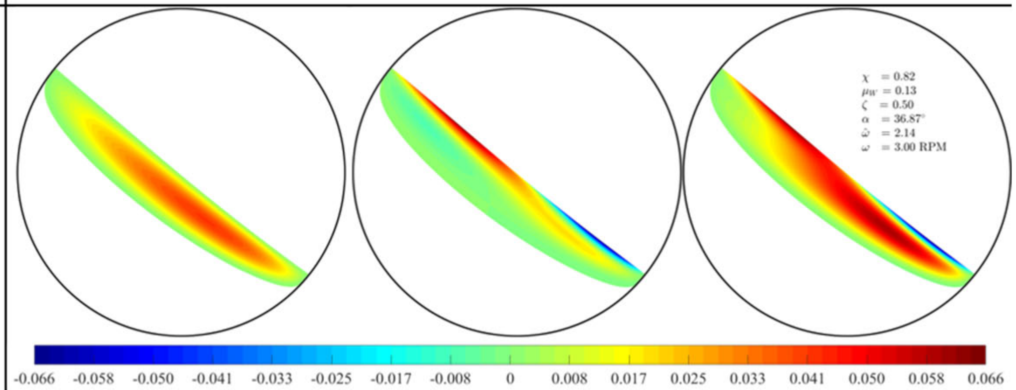
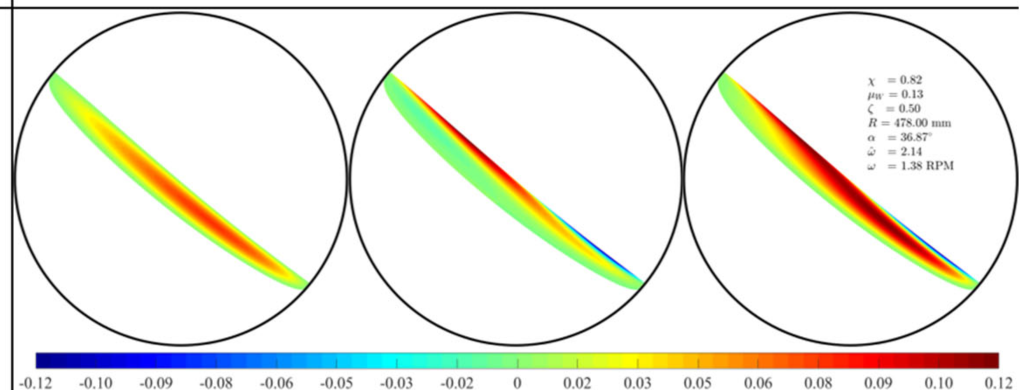
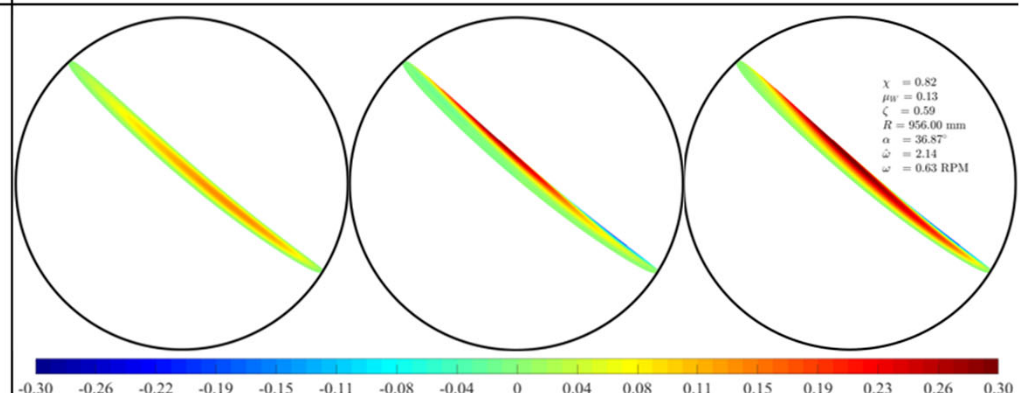
R → 2R
ω → $\omega/(2^{9/8})$
Fill Frac (ζ) = 0.5



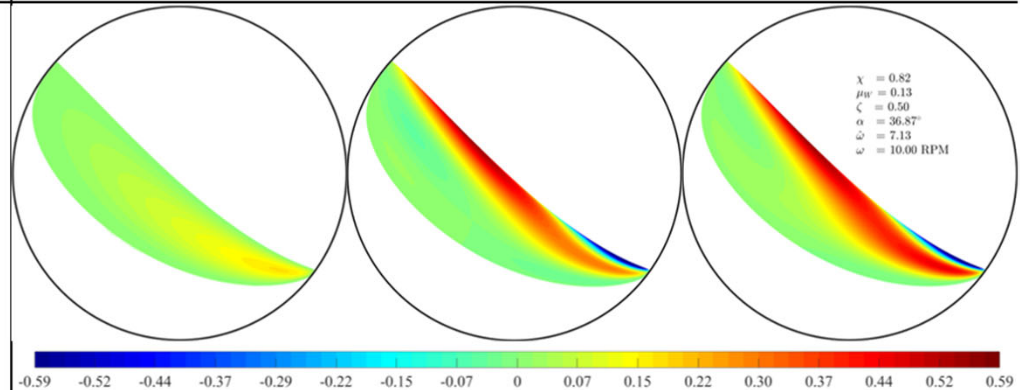
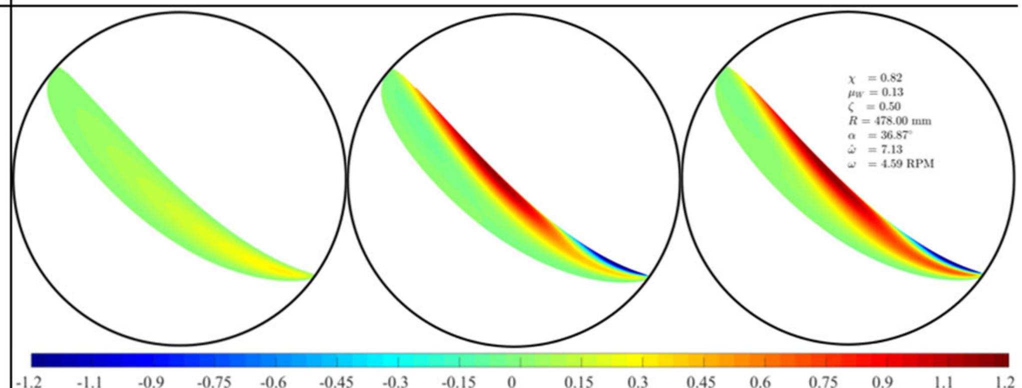
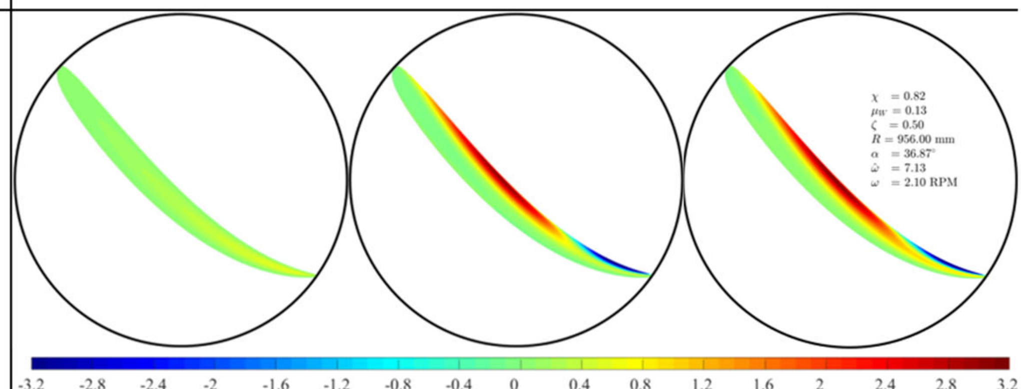
R → 4R
ω → $\omega/(4^{9/8})$
Fill Frac (ζ) = 0.5



Scale-Up of Shear-&-Advection Mixing

Scaling Rule	Shear + Advection = Total
R = 239 mm ω = 3.00 RPM Fill Frac (ζ) = 0.5	
R → 2R ω → ω/($2^{9/8}$) Fill Frac (ζ) = 0.5	
R → 4R ω → ω/($4^{9/8}$) Fill Frac (ζ) = 0.5	

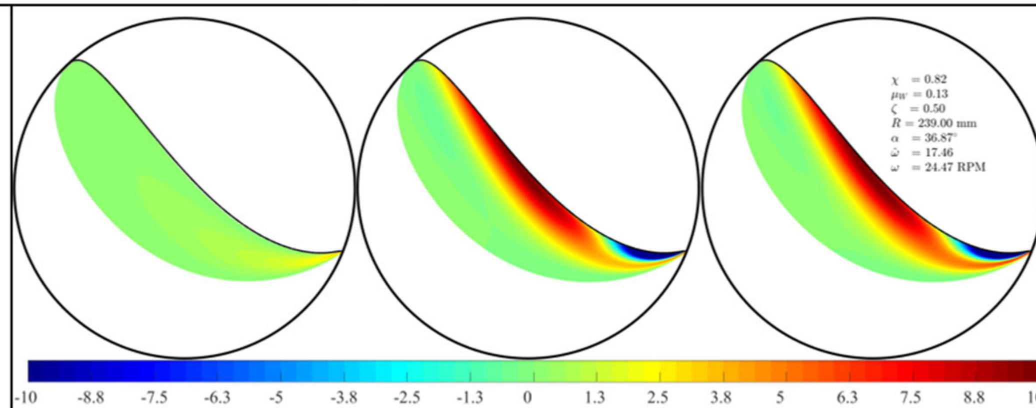
Scale-Up of Advective Dominated Mixing

Scaling Rule	Shear + Advection = Total
R = 239 mm ω = 10.00 RPM Fill Frac (ζ) = 0.5	
R → 2R ω → ω/($2^{9/8}$) Fill Frac (ζ) = 0.5	
R → 4R ω → ω/($4^{9/8}$) Fill Frac (ζ) = 0.5	

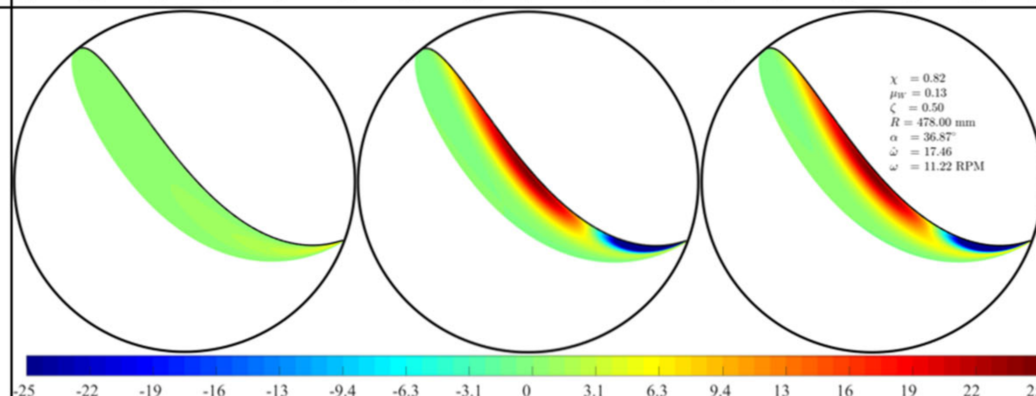
Scale-Up of Fully Cascading Mixing

Scaling Rule | **Shear + Advection = Total**

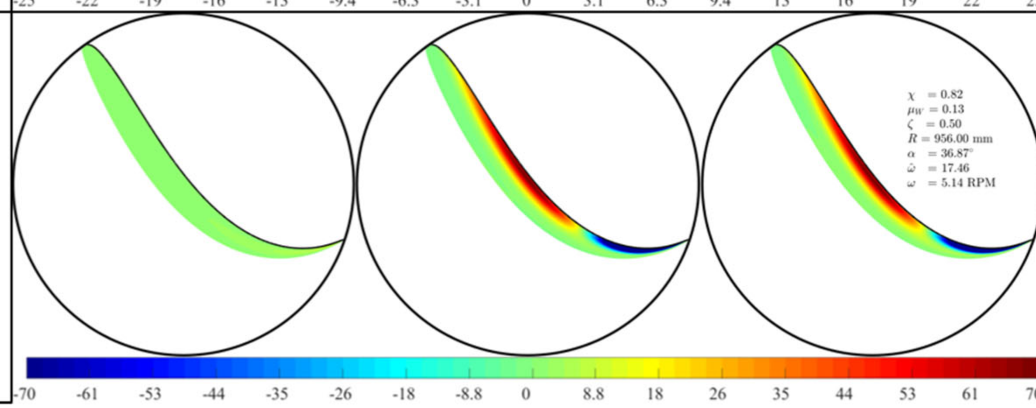
R = 239 mm,
ω = 24.47 RPM
Fill Frac (ζ) = 0.5



R → 2R
ω → $\omega/(2^{9/8})$
Fill Frac (ζ) = 0.5



R → 4R
ω → $\omega/(4^{9/8})$
Fill Frac (ζ) = 0.5



Conclusions & Future Work

- A robust granular flow model of rotating drum flows was developed using the linearized viscoplastic Inertial rheology
- The accurate velocity field predictions have hitherto not been reported in the literature, and for such a wide range of flow regimes
- Shear & Advective Energy dissipation signatures are useful in identifying the dominant mixing mechanisms
- The Entrainment Number allows us to scale-up the dominant mixing mechanisms across the rolling-to-cascading flow regimes

- Future (ongoing) work will include:
- An exploration of other rheologies, including those that incorporate wet systems (Peclet & Visco-Inertial)
- A full tensorial analysis that facilitates comparison between measurements (via PEPT) and simulations (via DEM) is currently underway
- Mixtures beyond bi-disperse systems will be studied using PEPT & DEM

Thank You!