Exploiting a Framework
for the Development of Segregation Rate Models

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June, 2019
Quantitative Prediction of Segregation at Process Scale

- Identify critical **material and process parameters** that control the *extent* of powder segregation
- Develop **quantitative models that predict** segregation and possible re-homogenization within a process train
- **Validate** models with appropriate experiments
- Demonstrate that the models are applicable to **full-scale** processes
- **In scope:**
  - Dense flows
  - Formulated (i.e. multicomponent) mixtures
- **Additional considerations:**
  - Cohesive powders
  - Particle shape effects
Competing Timescales

- If $t_{seg} \approx t_{forcing}$ balance of rates
  - We control $t_{forcing}$
  - Sensitive test of $t_{seg}$ model
  - “Collapse” complex dynamic experiment onto “steady state” measurement
Shear Cell Simulations

- Vary density ratio
- Vary shear rate (velocity)
- (Mostly) Constant pressure BC

Constant P

Oscillating g

heavy tracer particles

Constant v
Shear Cell Results

- “Asymptotic” segregation ↓ with ↑ forcing frequency
- Choose threshold segregation value to ID critical frequency
- Scaled segregation rate collapses onto single critical curve
Indirect Forcing in a Baffled Tumbler

- Changing the rotation rate changes $t_{forcing}$
Calculating the Effective/Critical Forcing Frequency

- Mean residence time $\rightarrow f$ (effective forcing frequency)
  \[ f = \frac{1}{\tau_{\text{mean}}} = \frac{\sqrt{\omega \dot{\gamma}}}{2\pi}, \text{ where } \dot{\gamma} = \left[ \frac{g \sin(\beta_m - \beta_s)}{cd \cos(\beta_s)} \right]^{1/2} \]
  Khakhar and Ottino, 2002

- Obtain critical frequency from theory to be tested, e.g.:
  \[ v_s = [K_S + (1 - \phi)K_T](1 - \bar{d}) \]
  \[ \text{for fixed total concentration, } \phi, \ v_s = [K_\phi](1 - \bar{d}) \]
  where $K_\phi \propto \dot{\gamma}$, thus, $f_{\text{crit}} \propto (1 - \bar{d})\dot{\gamma}$

- Frequency ratio
  \[ \frac{f}{f_{\text{crit}}} \propto \frac{\sqrt{\omega}}{\sqrt{\dot{\gamma}(1 - \bar{d})}} \]
  \[ \frac{f}{f_{\text{crit}}} = \frac{K_2 \sqrt{\omega}(d_1 \cos \beta_s)^{1/4}}{(1 - \bar{d})[g \sin(\beta_m - \beta_s)]^{1/4}} \]
Model Predictions (Density Segregation)

- Particle roughness suggested at AGM 2015
- “Proper” model will yield monotonic change in IS vs $f/f_{crit}$
- $r_s$ for quantitative measure ($1 \rightarrow$ monotonic)

(a) Experiment for buoyant model

(b) Experiment for drag model

(a) Simulation for buoyant model

(b) Simulation for drag model
Model Predictions (Size Segregation)

- "Proper" model will yield monotonic change in IS vs $f/f_{crit}$
- $r_s$ for quantitative measure ($1 \rightarrow$ monotonic)
Segregation under different confining pressure (or constant volume)

Also varying shear rate, particle size, and density ratio

Rheological quantity, \( I = \dot{\gamma}d_p \sqrt{\frac{\rho}{P}} \), collapses data
Segregation saturation occurs at same location as frictional saturation

- Model based on coordination number fits all data
Experimental Validation of Density Model

- Experimental apparatus for continuous shearing
- Run with tracer particles that are visually tracked
Experimental Validation

- Measurements of velocity vs height for varying conditions
- Matched to segregation measurements at same locations
Inhomogeneous shear means that the inertia number, $I$, varies with height.

- Can easily measure $v_s$ vs $I$ for a range of conditions.
- Results confirm novel segregation saturation model.
Testing Size Segregation

- No simple scaling of shear rate collapses size data
- Tried gravity, granular temperature, and inertia number
Impact of Rheology on Size Segregation

- Size segregation involves a more complex interplay between segregation and rheology
- Combining $I$ and $T$ captures both creation and finding of voids
- Novel observation: size ratio squared!
Cohesive Segregation

\[ \bar{u}_s = \frac{z_{adj} (\bar{\rho} - 1)}{6\beta\sqrt{\rho}} I \]

works for both cohesive and non-cohesive systems
Cohesive Segregation Works: How?

\[ z_{adj} = z \cdot \left(1 - \frac{F_c}{F_z}\right) = z \cdot \left(1 - \frac{4\gamma}{\alpha P d_p}\right) \]

- Cohesion is important, but effective collisions still lead to segregation
Experimental Exploration of Shape Segregation

(a) Tracking the periodic observed location of tracers allow a measure of segregation based on “distance to center”
(b) Comparing to sphere-sphere systems → equivalent size parameter
Effective Size of Cylinders/Discs

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<th>Shortest L</th>
<th>Average L</th>
<th>“Lay down” A</th>
<th>“Spinning” A</th>
<th>Volume</th>
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</table>

$R_c^2 = 0.94$

$R_D^2 = 0.99$
Using van der Waals Cohesion

- Continuing simulations
- Formal analysis to come
Transport Modeling (Density)

\[ \frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z} + \frac{\partial v sc_i}{\partial x} = \frac{\partial}{\partial z} \left( D \frac{\partial c_i}{\partial z} \right) \]

- **Route to “scale up” of models to relevant-scale usage**
- **Our model combines rheology and segregation; perfect for transport equations**