Exploiting a Framework

for the Development of Segregation Rate Models

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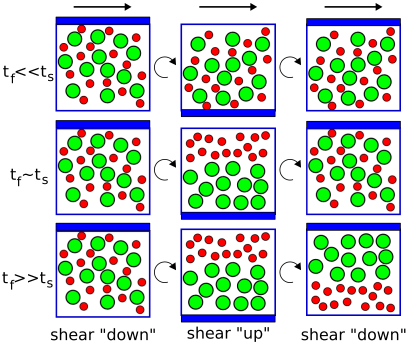
Quantitative Prediction of Segregation at Process Scale

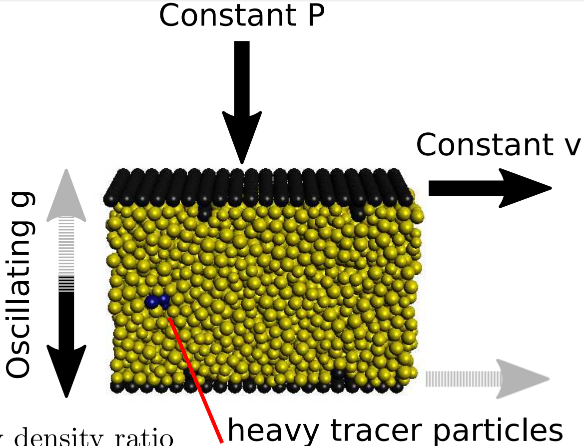
- Identify critical **material and process parameters** that control the *extent* of powder segregation
- Develop **quantitative models that predict** segregation and possible re-homogenization within a process train
- **Validate** models with appropriate experiments
- Demonstrate that the models are applicable to **full-scale** processes
- In scope:
  - Dense flows
  - Formulated (i.e. multicomponent) mixtures
- Additional considerations:
  - Cohesive powders
  - Particle shape effects

Competing Timescales



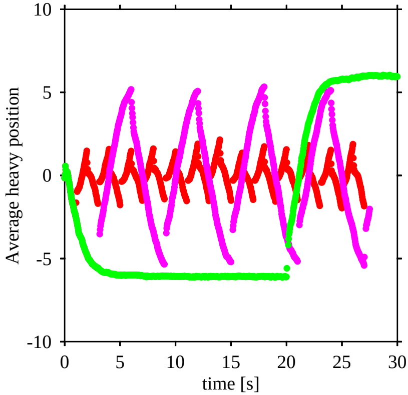
- If $t_{seg} \approx t_{forcing}$ balance of rates
  - We control $t_{forcing}$
  - Sensitive test of t_{seg} model
  - "Collapse" complex dynamic experiment onto "steady state" measurement

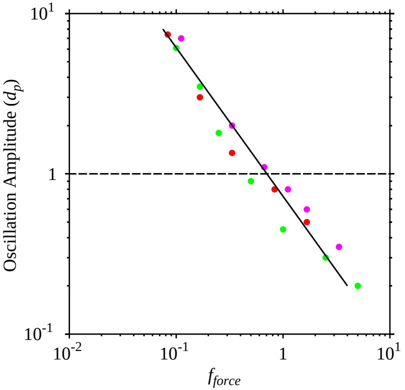
Shear Cell Simulations



- Vary density ratio
- Vary shear rate (velocity)
- (Mostly) Constant pressure BC

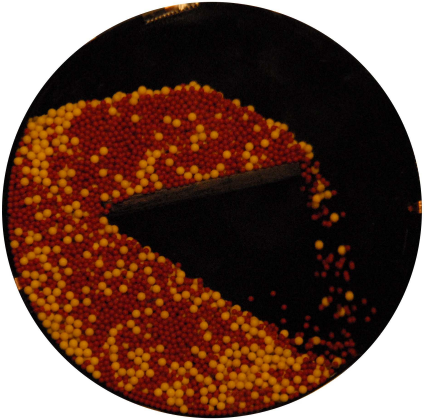
Shear Cell Results

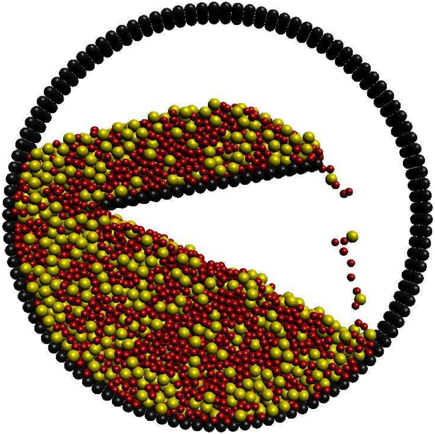




- "Asymptotic" segregation ↓ with ↑ forcing frequency
- Choose threshold segregation value to ID critical frequency
- Scaled segregation rate collapses onto single critical curve

Indirect Forcing in a Baffled Tumbler





- Changing the rotation rate changes $t_{forcing}$

Calculating the Effective/Critical Forcing Frequency

- Mean residence time → f (effective forcing frequency)

$$f = \frac{1}{\tau_{mean}} = \frac{\sqrt{\omega\dot{\gamma}}}{2\pi}, \text{ where } \dot{\gamma} = \left[\frac{g\sin(\beta_m - \beta_s)}{cd\cos(\beta_s)}\right]^{1/2}$$

Khakhar and Ottino, 2002

- Obtain critical frequency from theory to be tested, e.g.:
- Size segregation velocity

$$v_s = [K_S + (1-\phi)K_T](1-\bar{d})$$

$$\text{for fixed total concentration, } \phi,\ v_s = [K_\phi](1-\bar{d})$$

$$\text{where } K_\phi \propto \dot{\gamma}, \text{ thus, } f_{crit} \propto (1-\bar{d})\dot{\gamma}$$

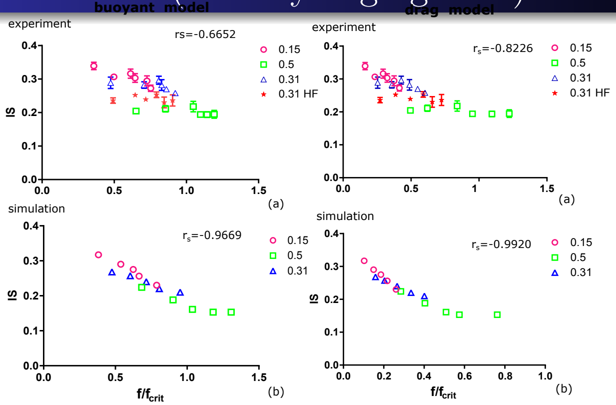
- Frequency ratio

$$\frac{f}{f_{crit}} \propto \frac{\sqrt{\omega}}{\sqrt{\dot{\gamma}}(1-\bar{d})}$$

$$\frac{f}{f_{crit}} = \frac{K_2\sqrt{\omega}(d_1\cos_{\beta_S})^{1/4}}{(1-\bar{d})[g\sin(\beta_m-\beta_s)]^{1/4}}$$

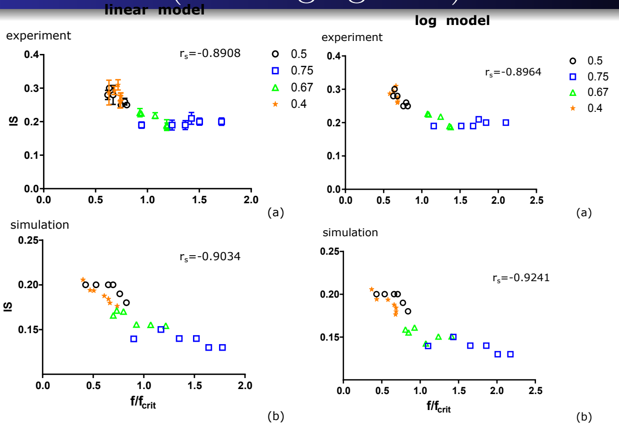
Hajra, Bhattacharya and McCarthy, *Powder Tech.*, 2012

Model Predictions (Density Segregation)



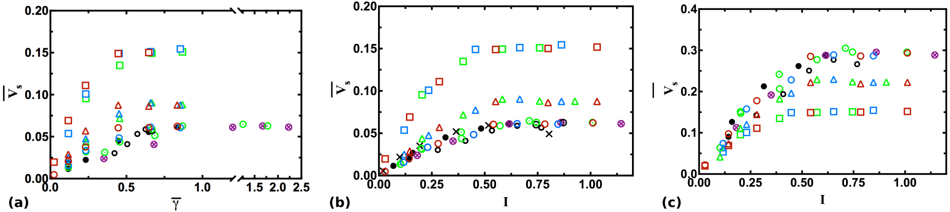
- Particle roughness suggested at AGM 2015
- "Proper" model will yield monotonic change in IS vs f/f_{crit}
- r_s for quantitative measure ($1 \rightarrow$ monotonic)

Model Predictions (Size Segregation)



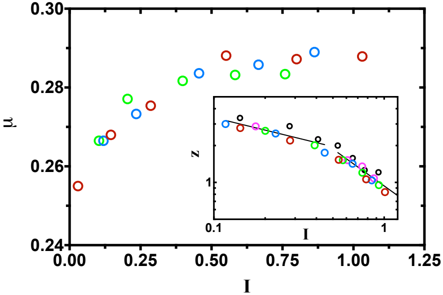
- "Proper" model will yield monotonic change in IS vs f/f_{crit}
- r_s for quantitative measure ($1 \rightarrow$ monotonic)

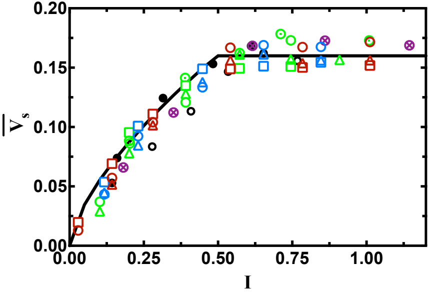
Density Segregation Under Varying Conditions



- Segregation under different confining pressure (or constant volume)
- Also varying shear rate, particle size, and density ratio
- Rheological quantity, $I = \dot{\gamma} d_p \sqrt{\frac{\rho}{P}}$, collapses data

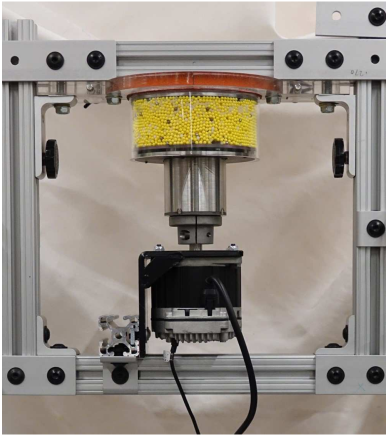
A Unified Model, Based on Rheology





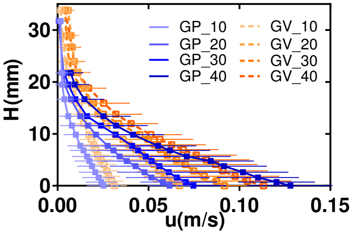
- Segregation saturation occurs at same location as frictional saturation
- Model based on coordination number fits **all** data

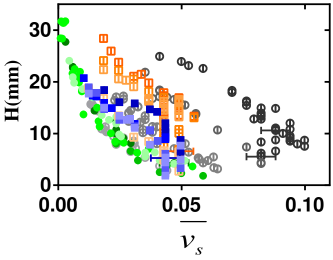
Experimental Validation of Density Model



- Experimental apparatus for continuous shearing
- Run with tracer particles that are visually tracked

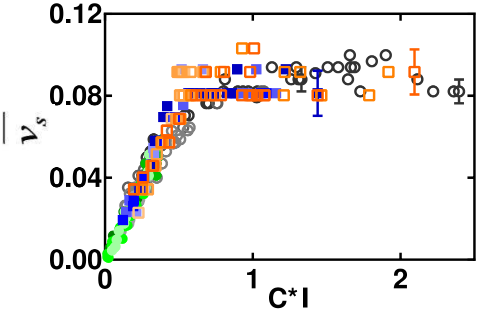
Experimental Validation





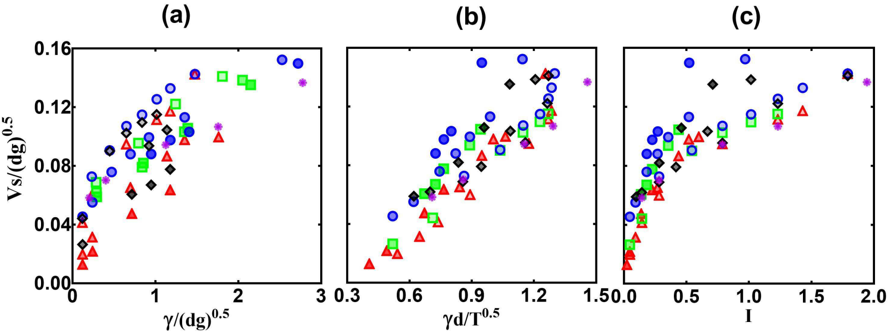
- Measurements of velocity vs height for varying conditions
- Matched to segregation measurements at same locations

Experimental Validation



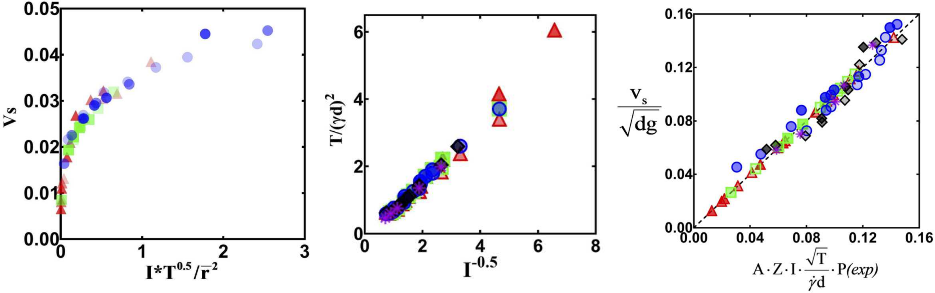
- Inhomogeneous shear means that the inertia number, I, varies with height
- Can easily measure v_s vs I for a range of conditions
- Results confirm novel segregation saturation model

Testing Size Segregation



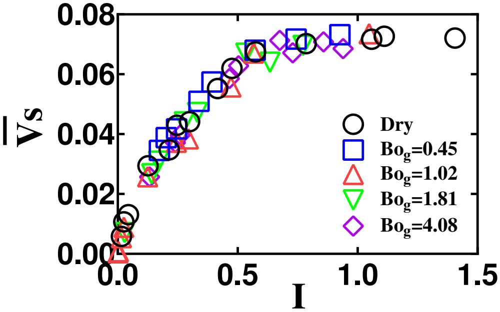
- No simple scaling of shear rate collapses size data
- Tried gravity, granular temperature, and inertia number

Impact of Rheology on Size Segregation



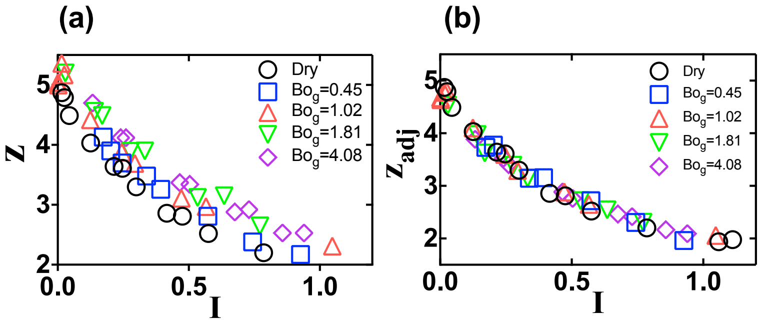
- Size segregation involves a more complex interplay between segregation and rheology
- Combining I and T captures both creation and finding of voids
- Novel observation: size ratio squared!

Cohesive Segregation



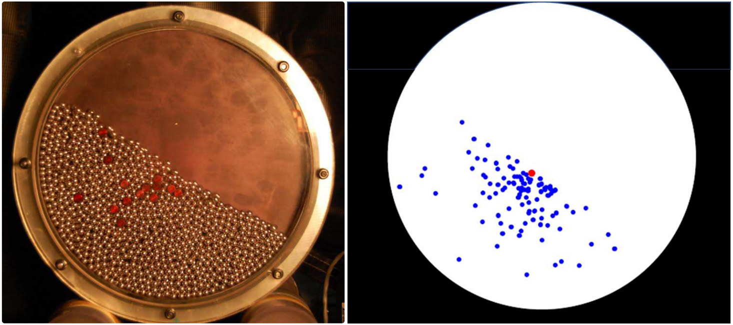
- $\bar{v}_s = \frac{z_{adj}(\bar{\rho}-1)}{6\beta\sqrt{\bar{\rho}}} I$ works for **both** cohesive and non-cohesive systems

Cohesive Segregation Works: How?



- $z_{adj} = z \cdot (1 - \frac{F_c}{F_z}) = z \cdot (1 - \frac{4\gamma}{\alpha P d_p})$
- Cohesion is important, but effective collisions still lead to segregation

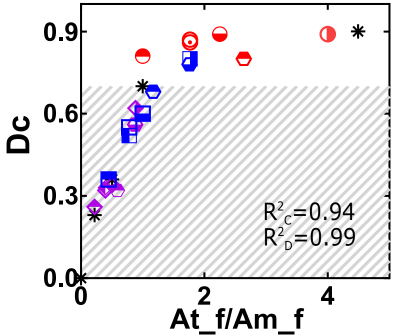
Experimental Exploration of Shape Segregation



(a) (b)

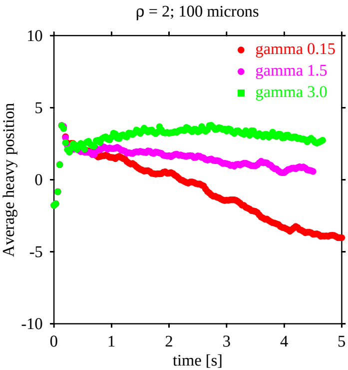
- Tracking the periodic observed location of tracers allow a measure of segregation based on "distance to center"
- Comparing to sphere-sphere systems → equivalent size parameter

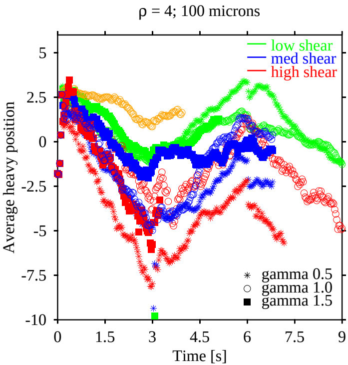
Effective Size of Cylinders/Discs



Shape descriptor #	Shortest L	Average L	"Lay down" A	"Spinning" A	Volume	**"Flowing" A**
Classification #	1D	1D	2D	2D	3D	2D
R^2 Cylinder	0.94	0.79	0.77	0.77	0.65	**0.94**
R^2 Disc	0.74	0.94	0.99	0.57	0.57	**0.99**

Using van der Waals Cohesion





- Continuing simulations
- Formal analysis to come

Transport Modeling (Density)

$$\frac{\partial c_i}{\partial t} + u\frac{\partial c_i}{\partial x} + w\frac{\partial c_i}{\partial z} + \frac{\partial v_s c_i}{\partial x} = \frac{\partial}{\partial z}\left(D\frac{\partial c_i}{\partial z}\right)$$

- Route to "scale up" of models to relevant-scale usage
- Our model combines rheology and segregation; perfect for transport equations