Clustering
in Cohesive Granular Matter

Jean-Christophe Géminard

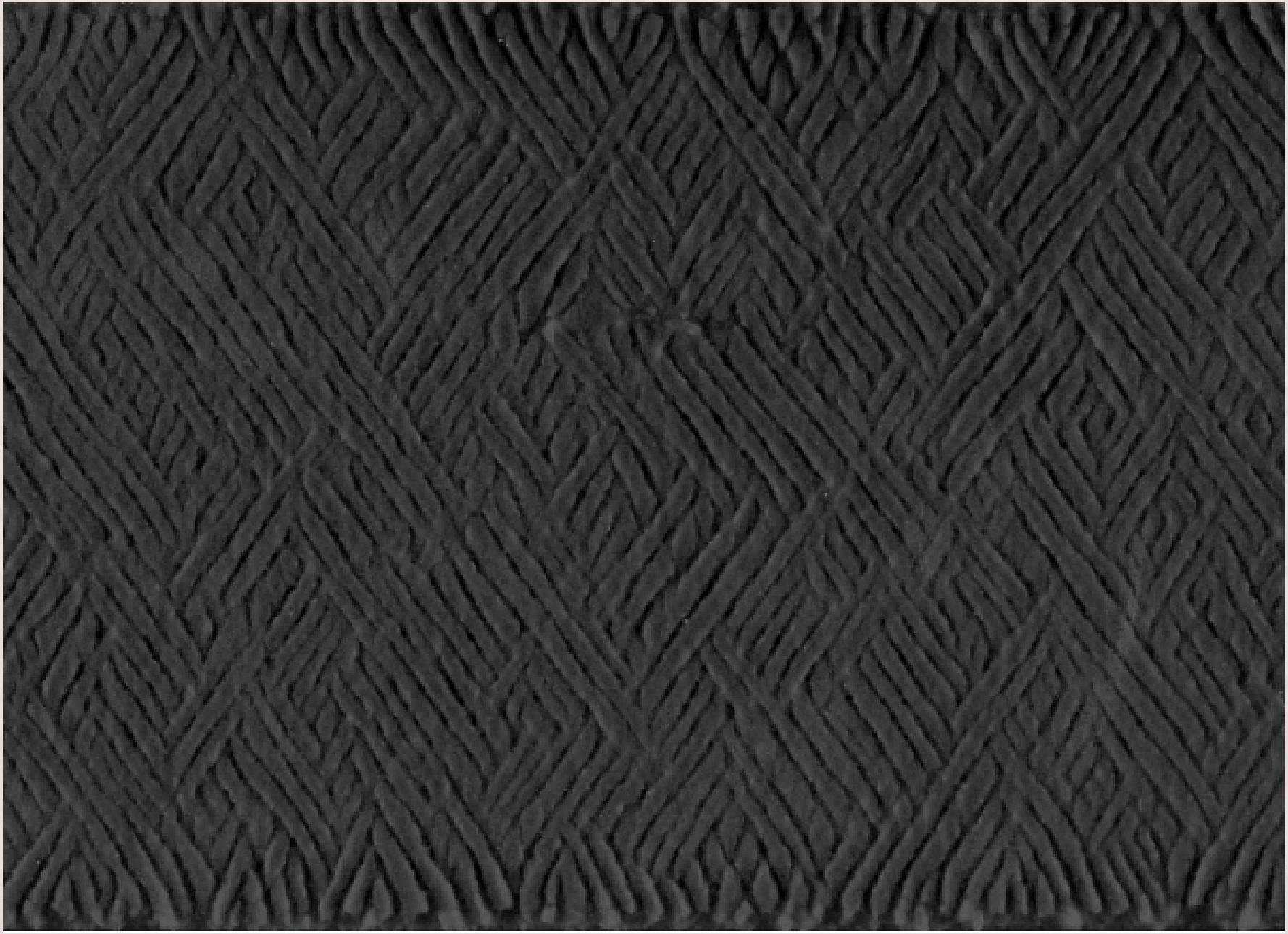
Laboratoire de Physique
Ecole Normale Supérieure de Lyon / CNRS / Université Claude Bernard
Université de Lyon

° F. Melo, H. Alarcón, J. Contreras, G. Lagubeau (NLPLAB, USACH, Chile).
° F. Tapia, P. Lidon, L. Champougny (L3, ENS-Lyon).

ENS DE LYON

cnrs

UNIVERSITÉ DE LYON



Content

Fracture experiments
- setup
- results
- mechanism

Cohesion
- characterization techniques
- dependence on grain size and relative humidity
- discussion

Clustering
- additional evidence
- “direct” observation

Content

Fracture experiments

- **setup**
- **results**
- **mechanism**

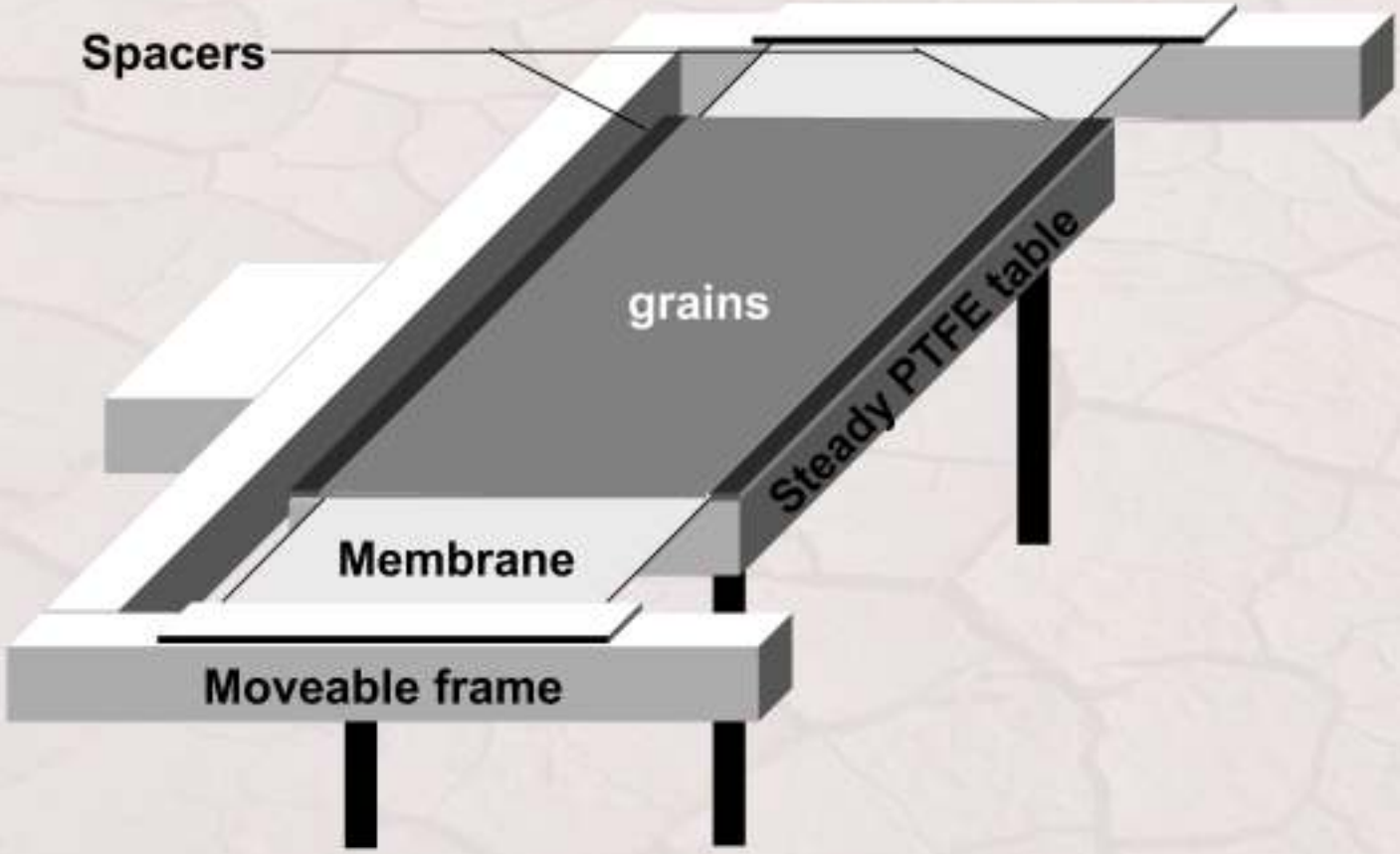
Cohesion

- characterization techniques
- dependence on grain size and relative humidity
- discussion

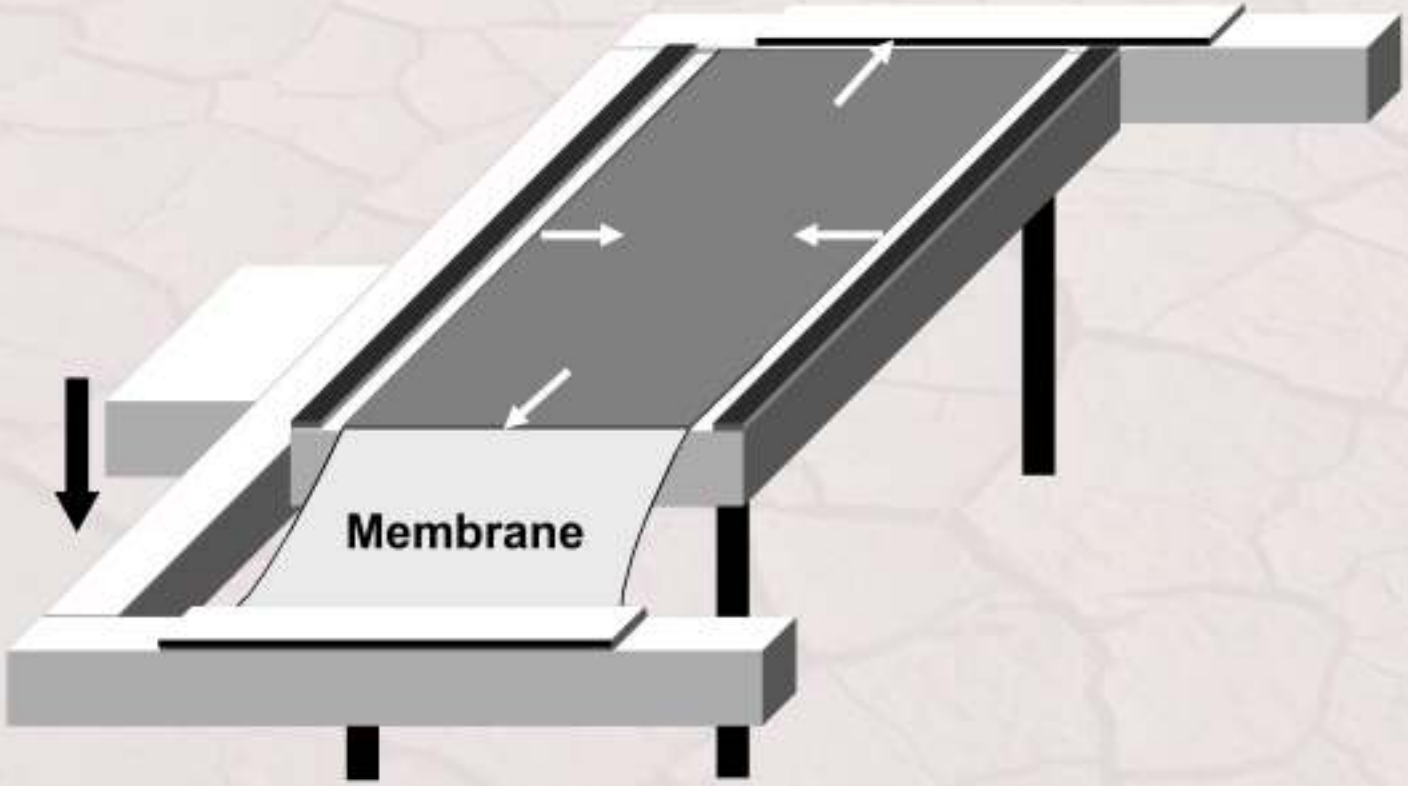
Clustering

- additional evidence
- “direct” observation

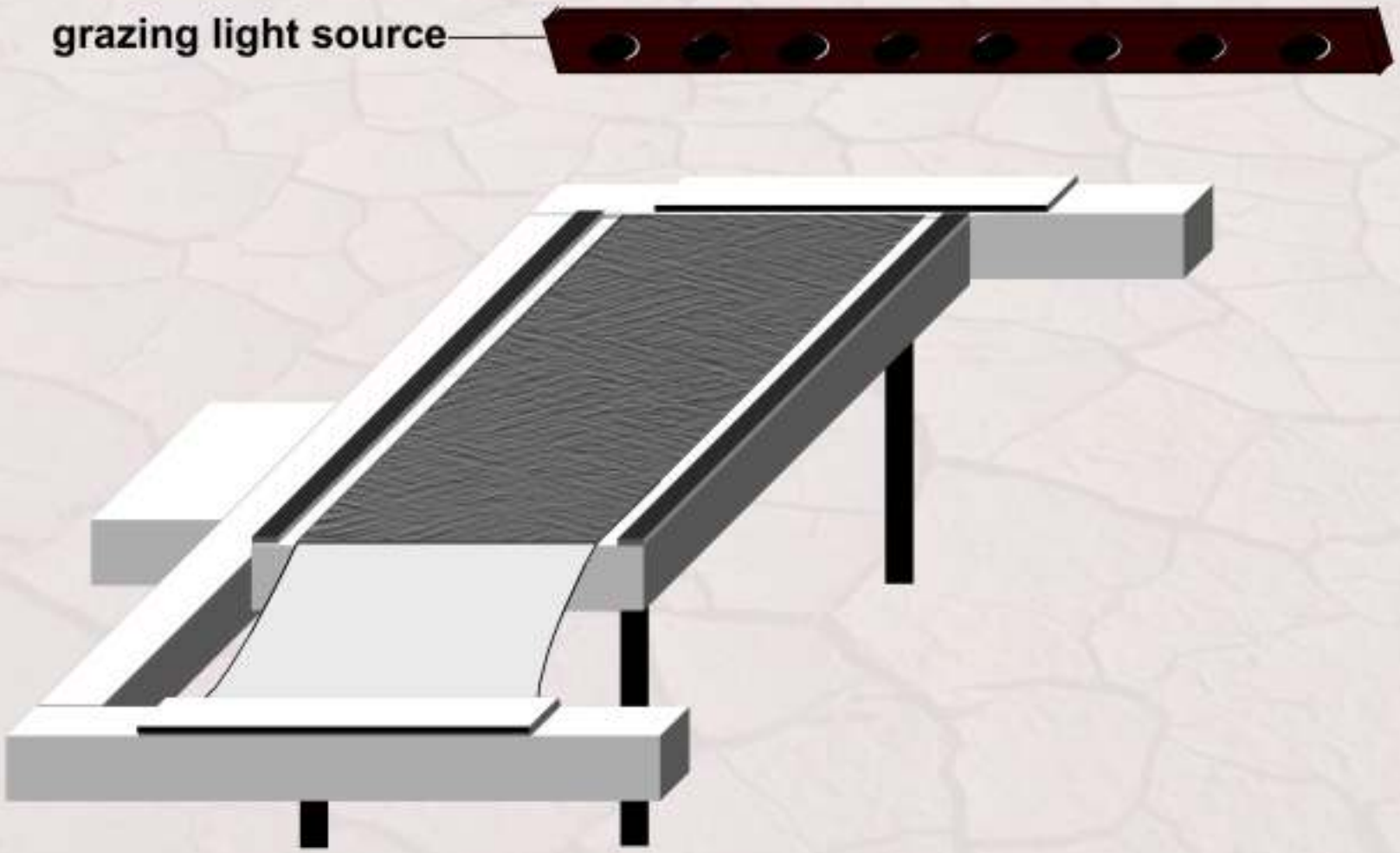
Setup

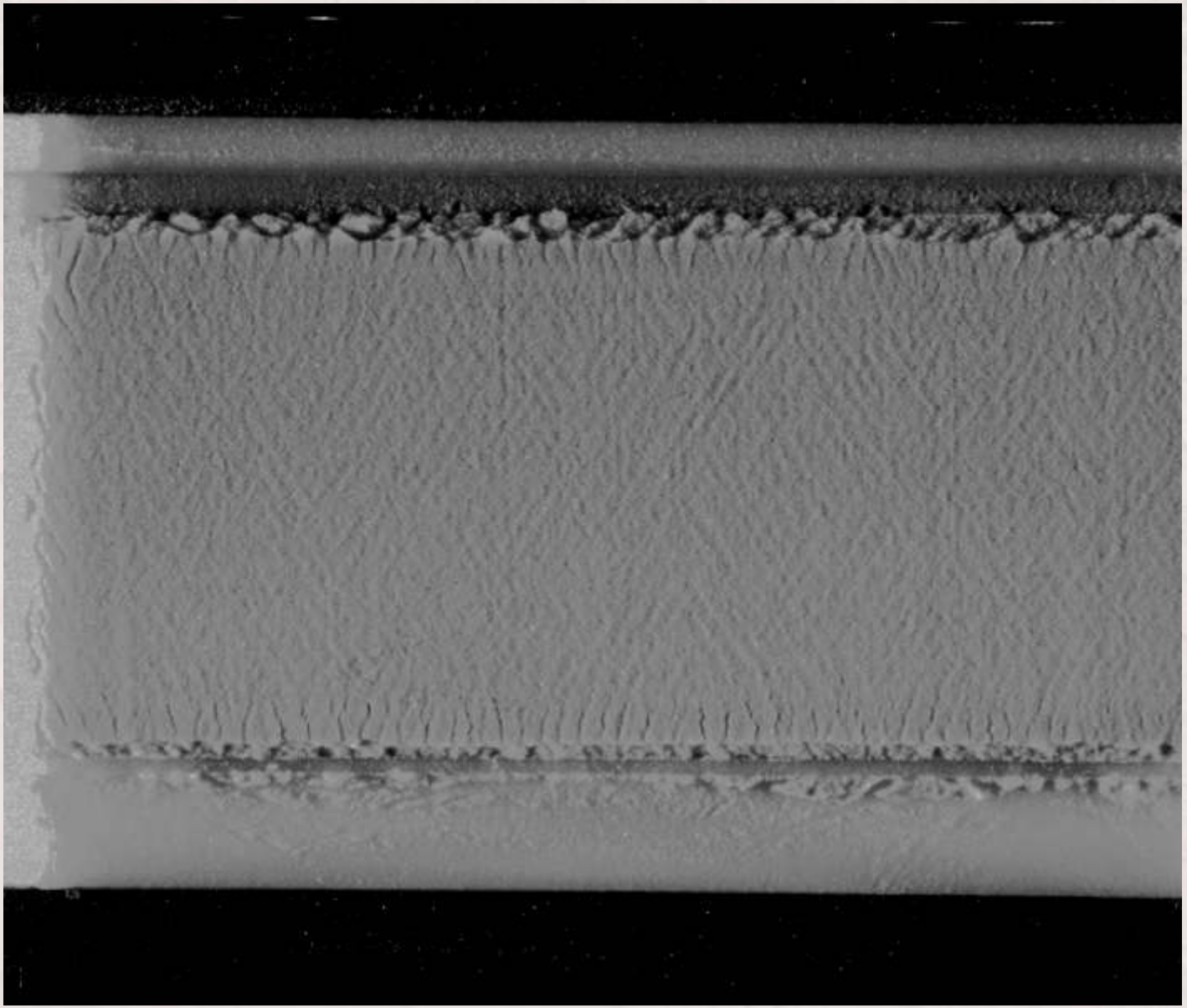


Setup

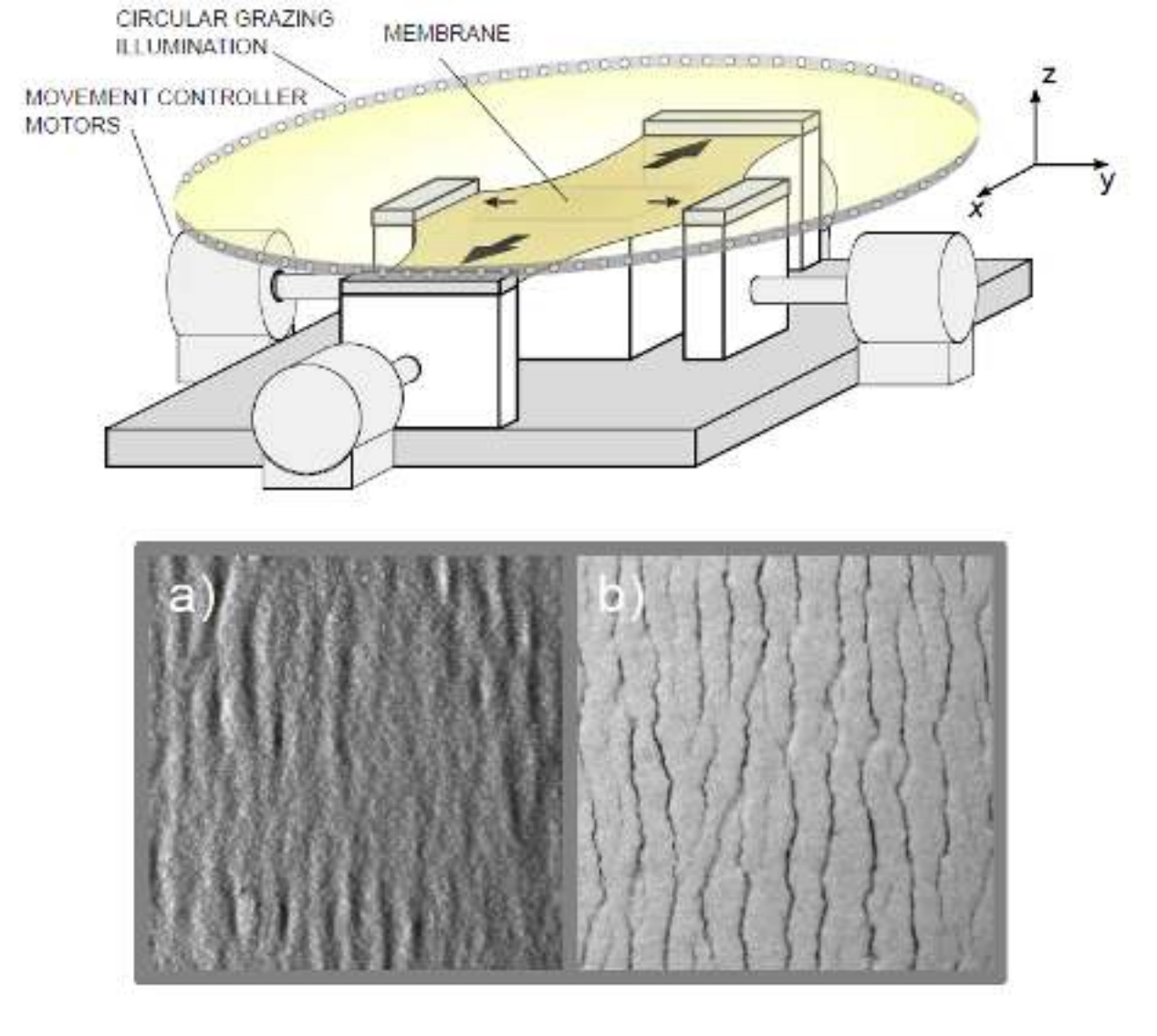


Setup



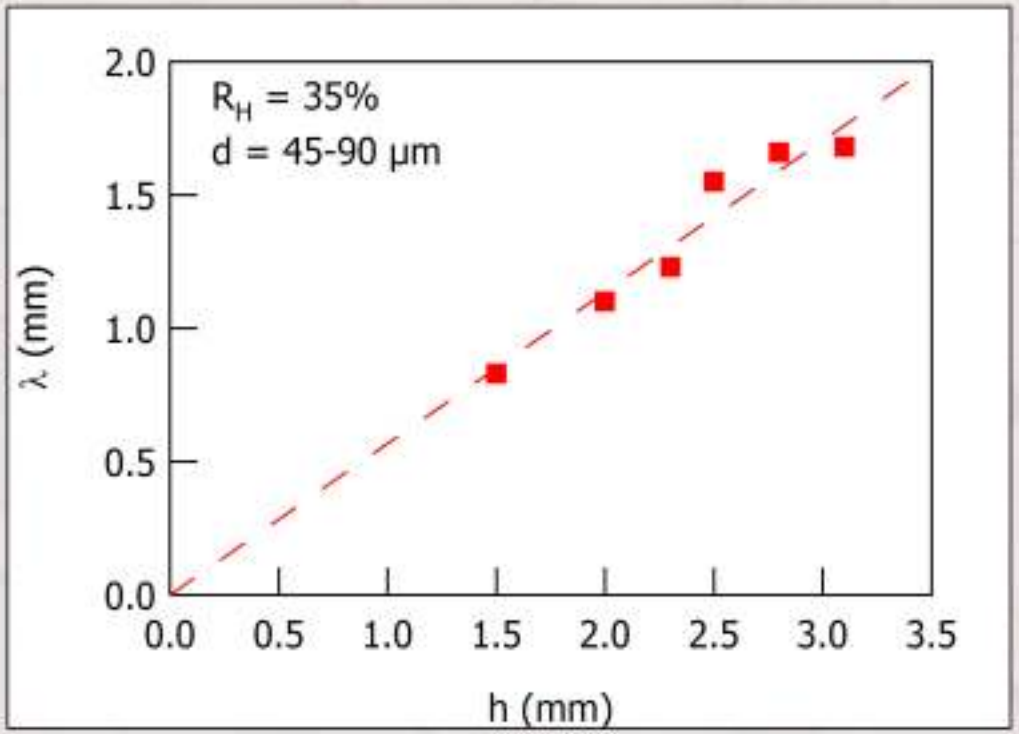


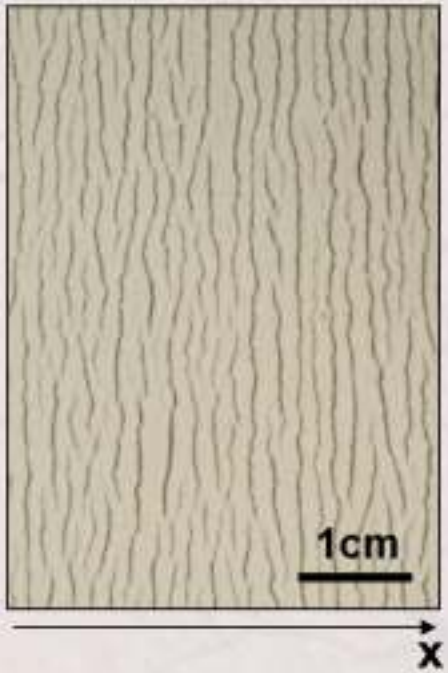
Setup


CIRCULAR GRAZING
ILLUMINATION
MEMBRANE
MOVEMENT CONTROLLER
MOTORS
z
y
x
a)
b)

Results

Thickness

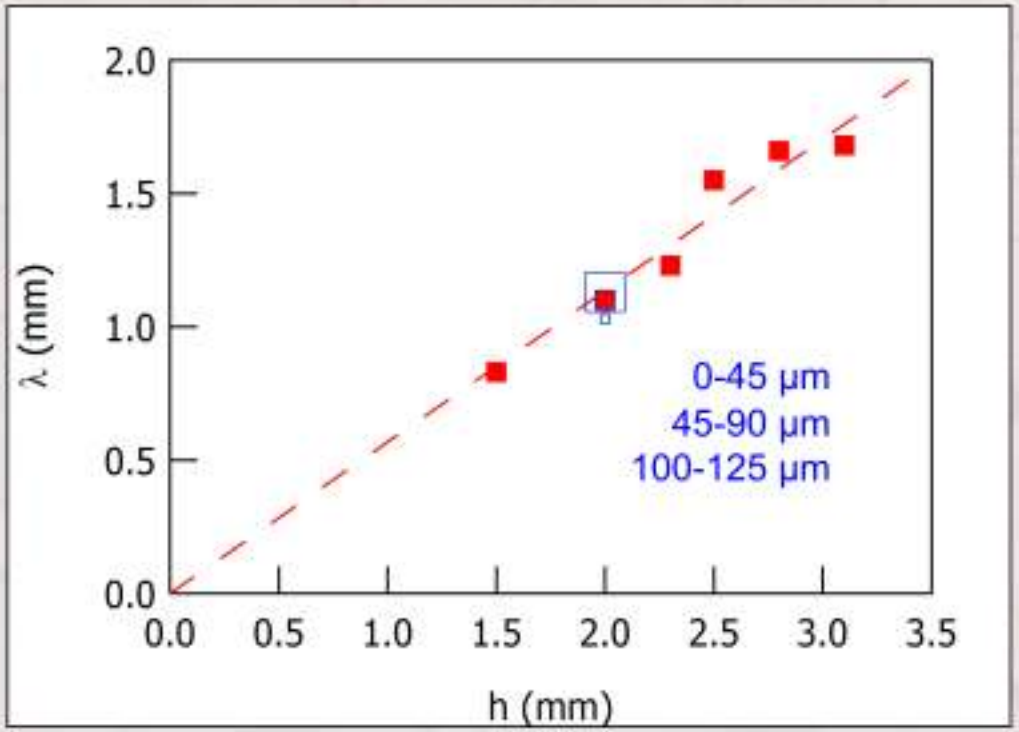


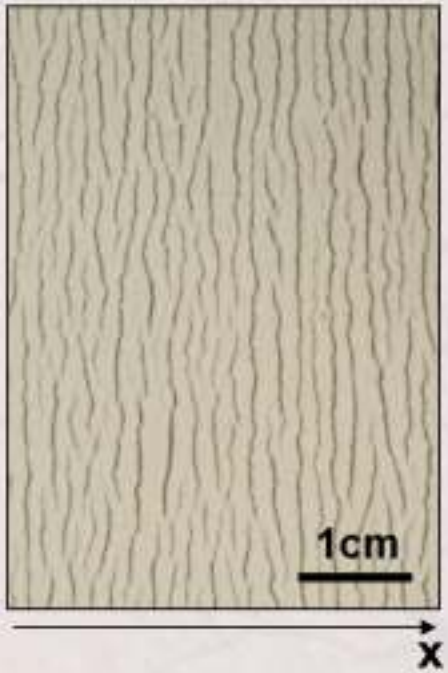


$$\lambda \propto h$$

Results

Grain size

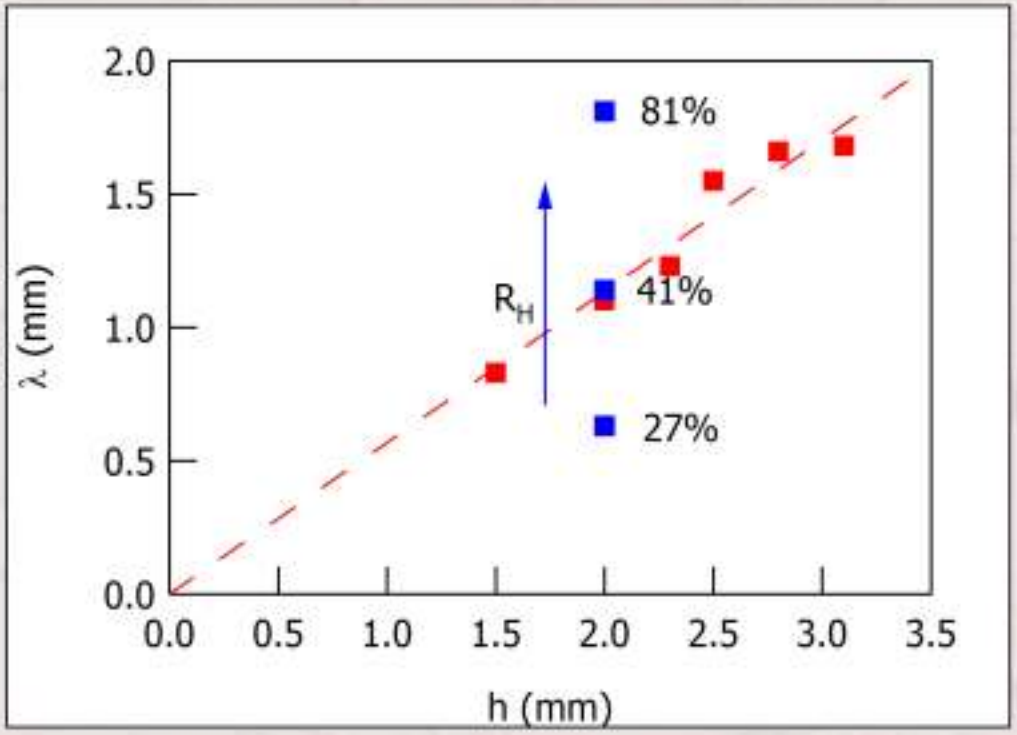


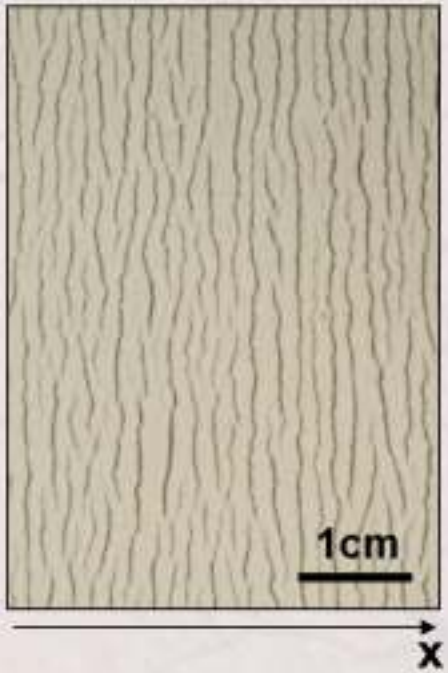


λ does not depend on d.

Results

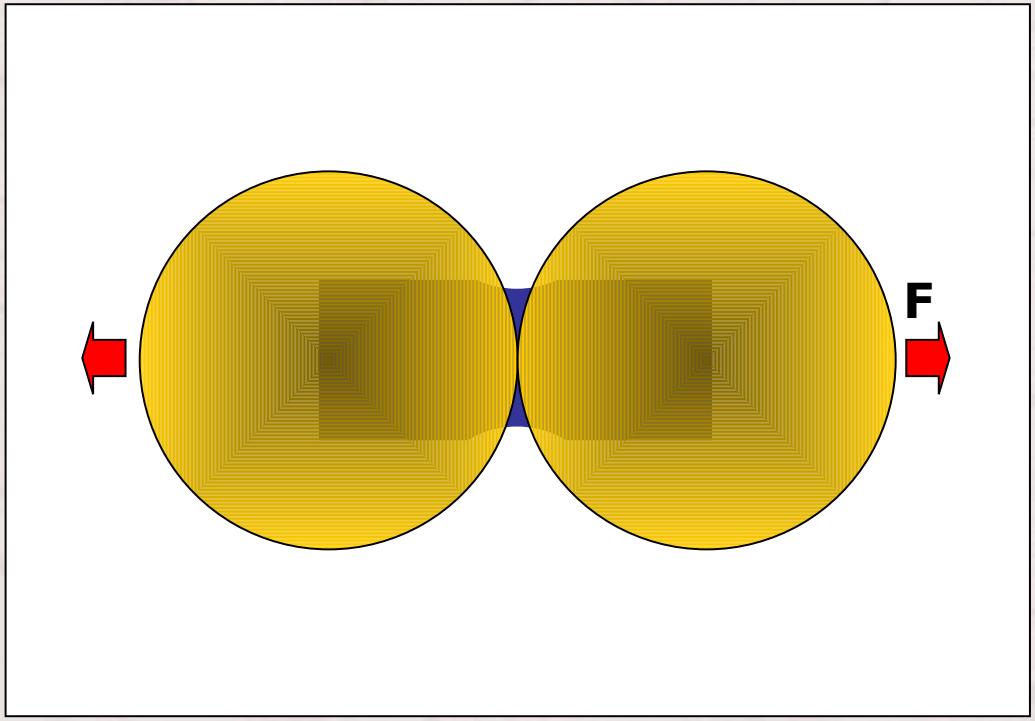
Relative humidity





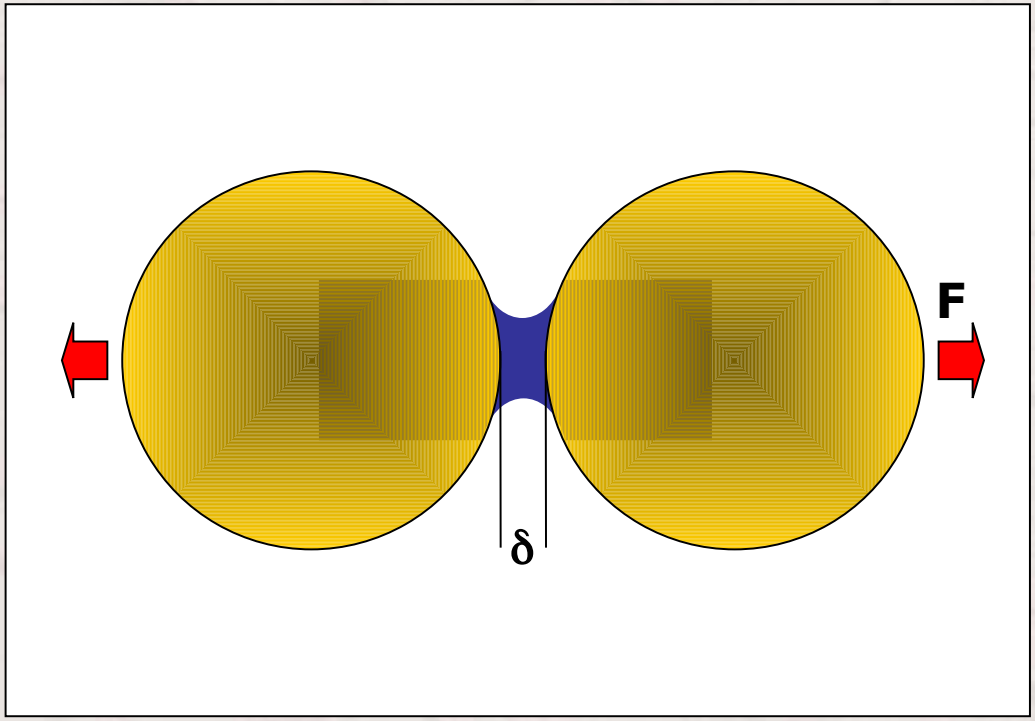
λ increases with R_H.

Mechanism



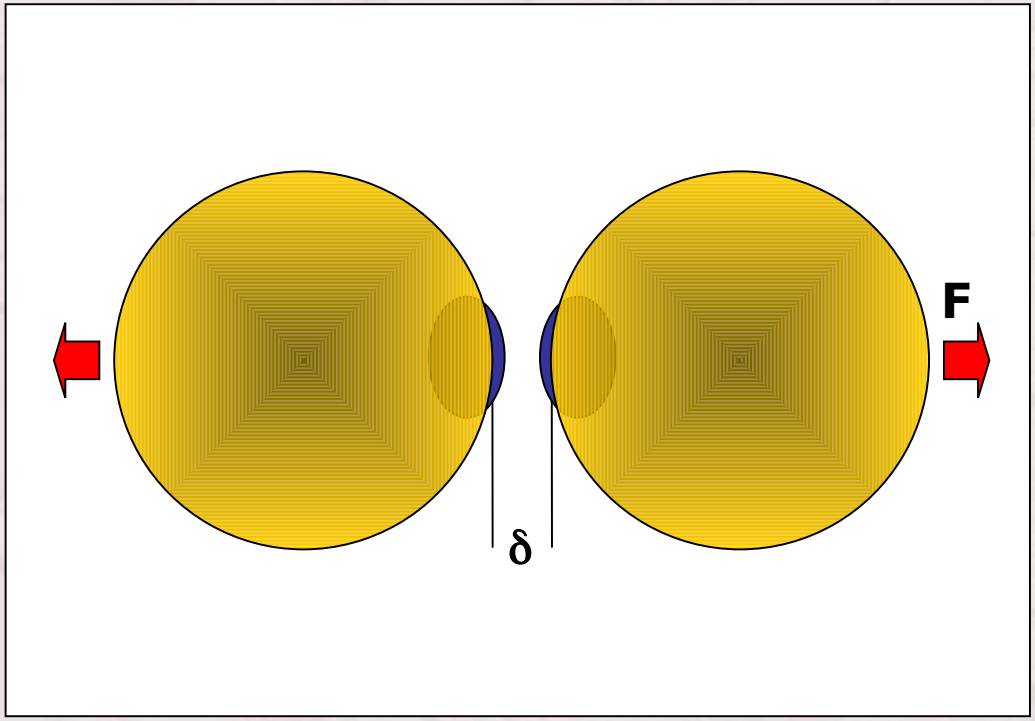
Liquid bridge

Mechanism



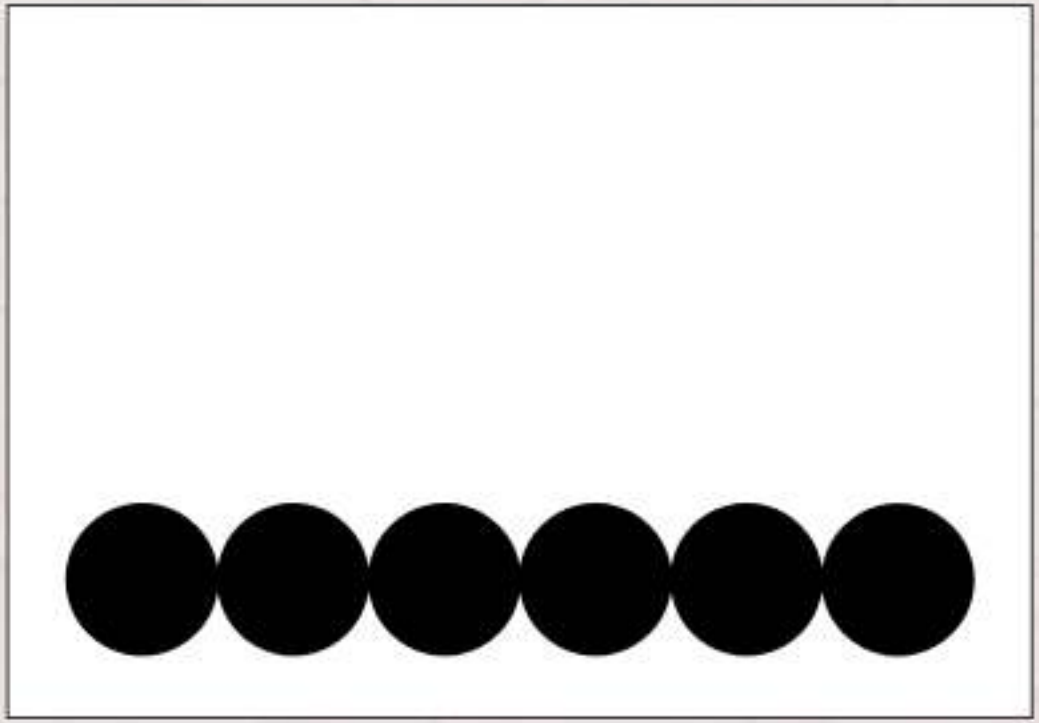
Liquid bridge

Mechanism

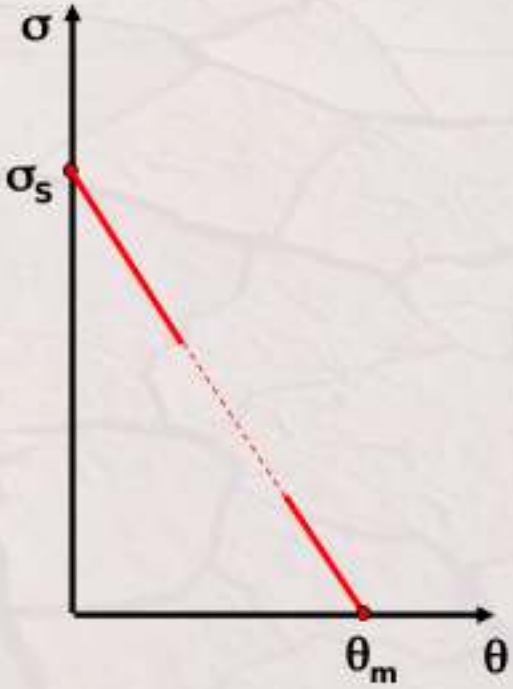


Liquid bridge

Mechanism

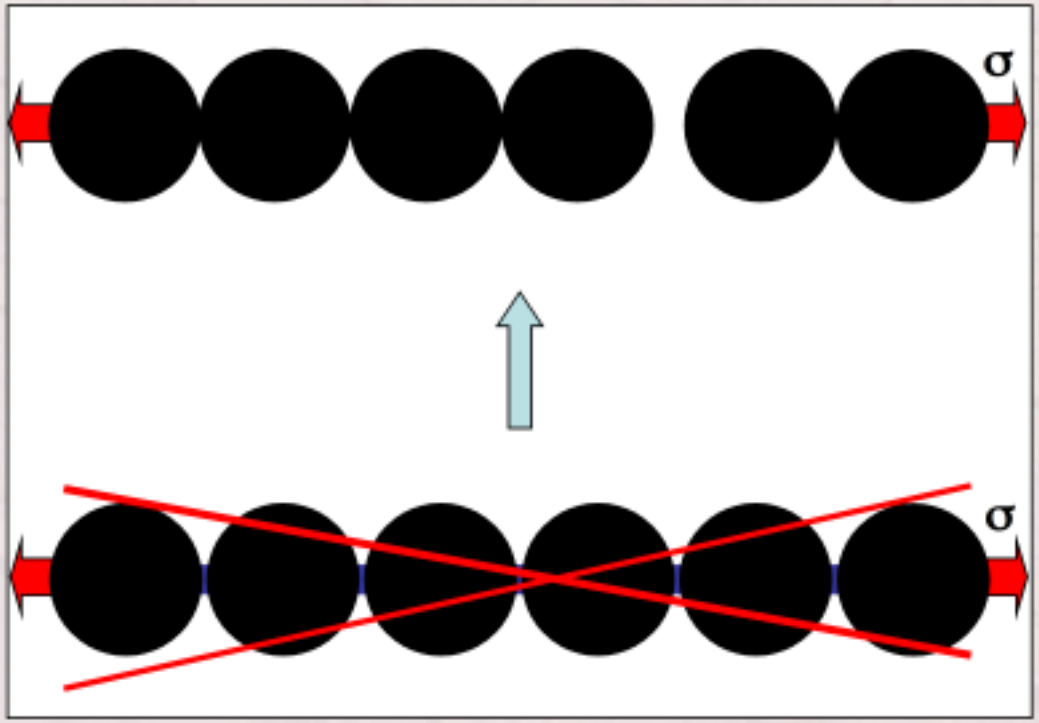


$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$

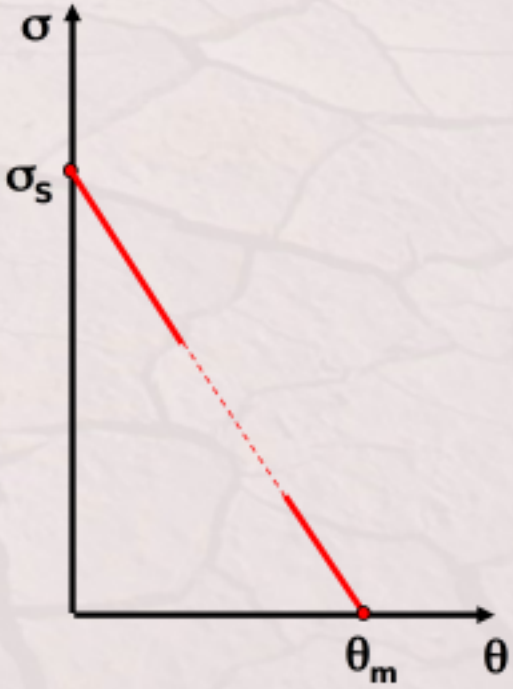


$$\theta \equiv u_{xx}$$

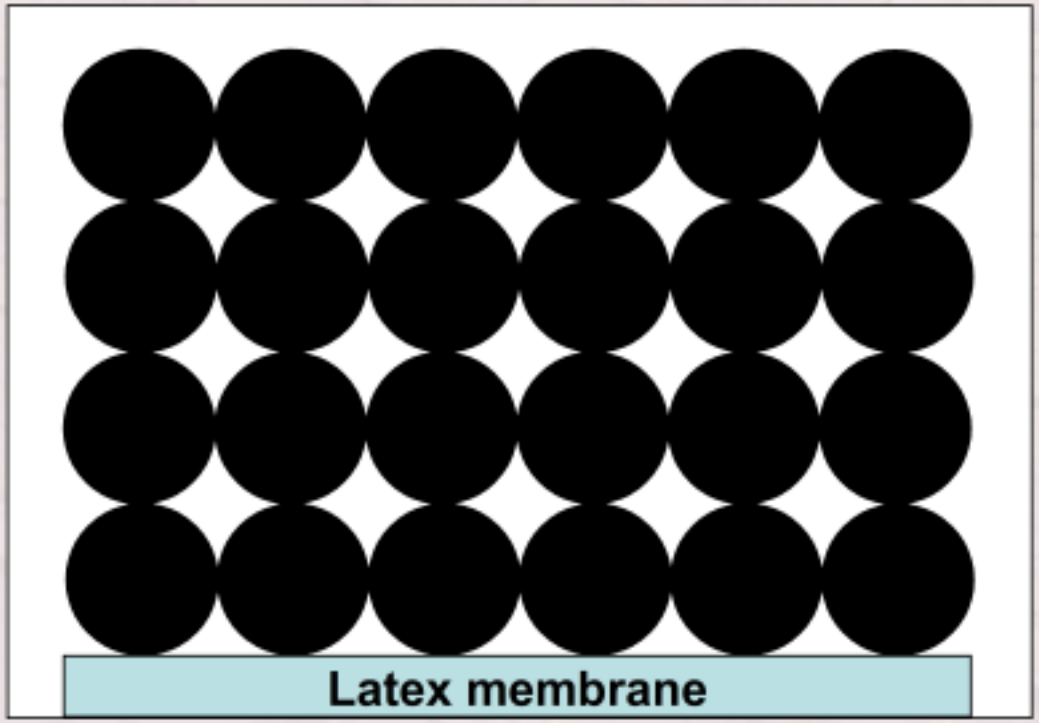
Mechanism



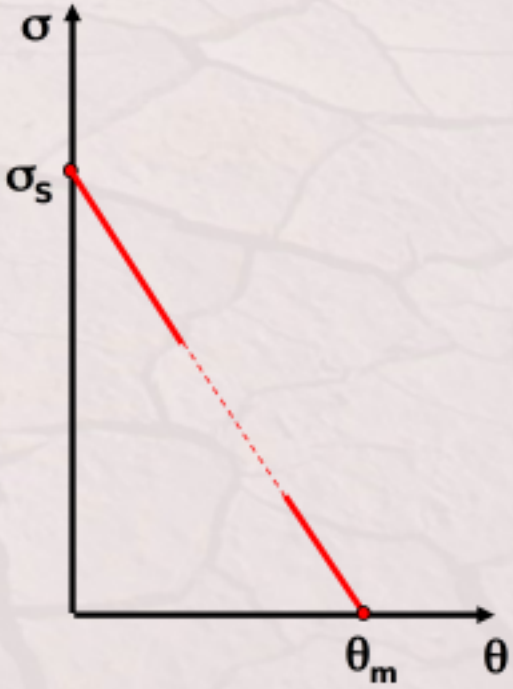
$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$



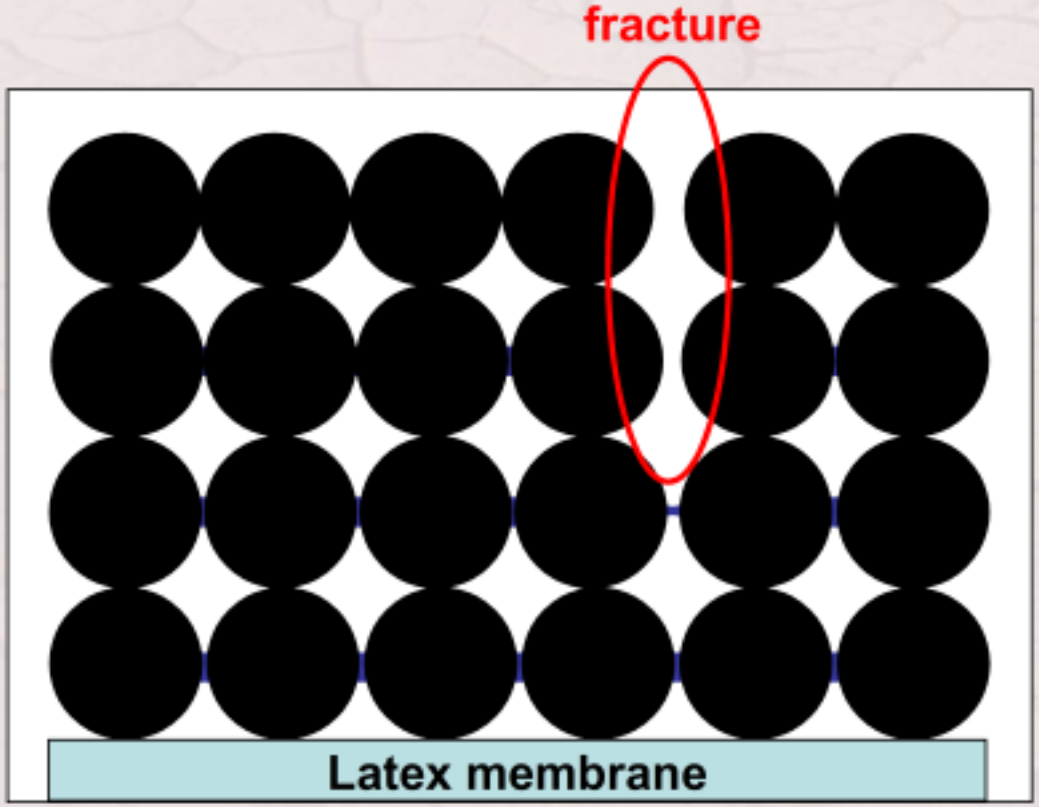
Mechanism



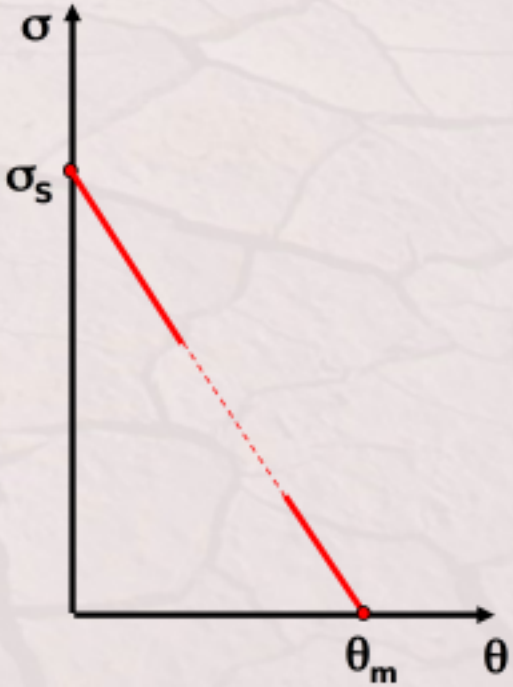
$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$



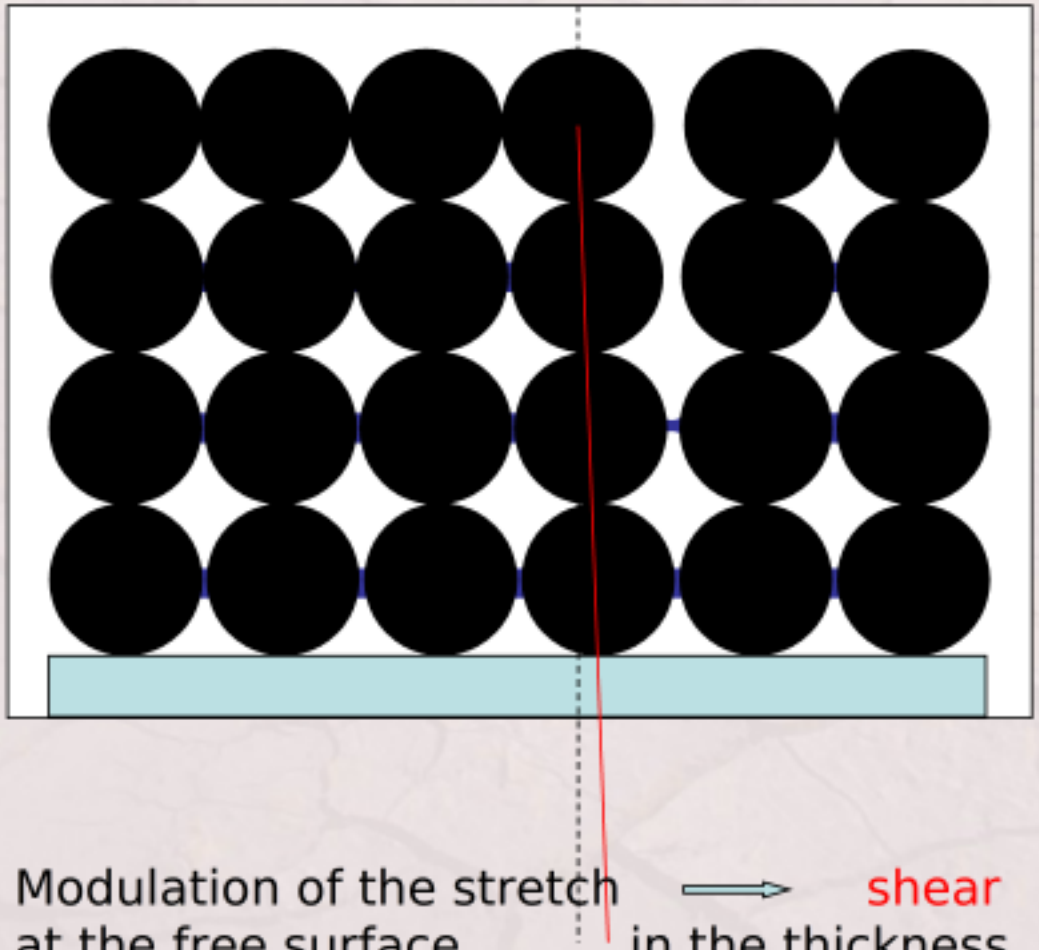
Mechanism



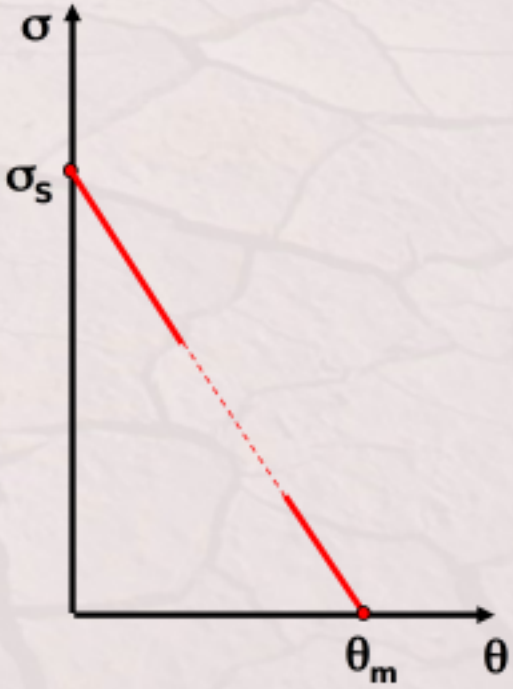
$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$



Mechanism



$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$



Modulation of the stretch at the free surface ⟹ shear in the thickness

Mechanism

$$E = \sigma_s\left(u_{xx} - \frac{u_{xx}^2}{2\theta_m}\right) + \frac{1}{2}Gu_{xz}^2$$

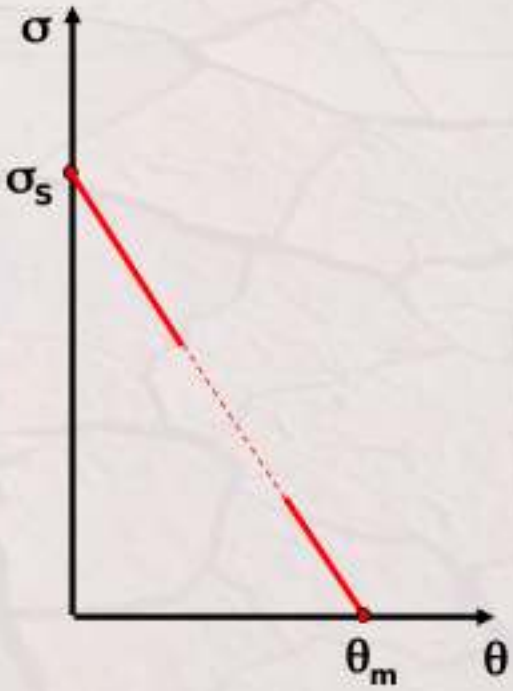
$$u_x = \theta x + f(z)\sin(kx)$$

$$u_{xx} + u_{zz} = 0$$

$$\frac{\partial}{\partial x}\left[\frac{\delta E}{\delta u_{xx}}\right] + \frac{\partial}{\partial z}\left[\frac{\delta E}{\delta u_{xz}}\right] = 0$$

$$\lambda \simeq 4\sqrt{2\sigma_s/G\theta_m}h$$

$$\sigma_{xx} = \sigma_s(1 - u_{xx}/\theta_m)$$



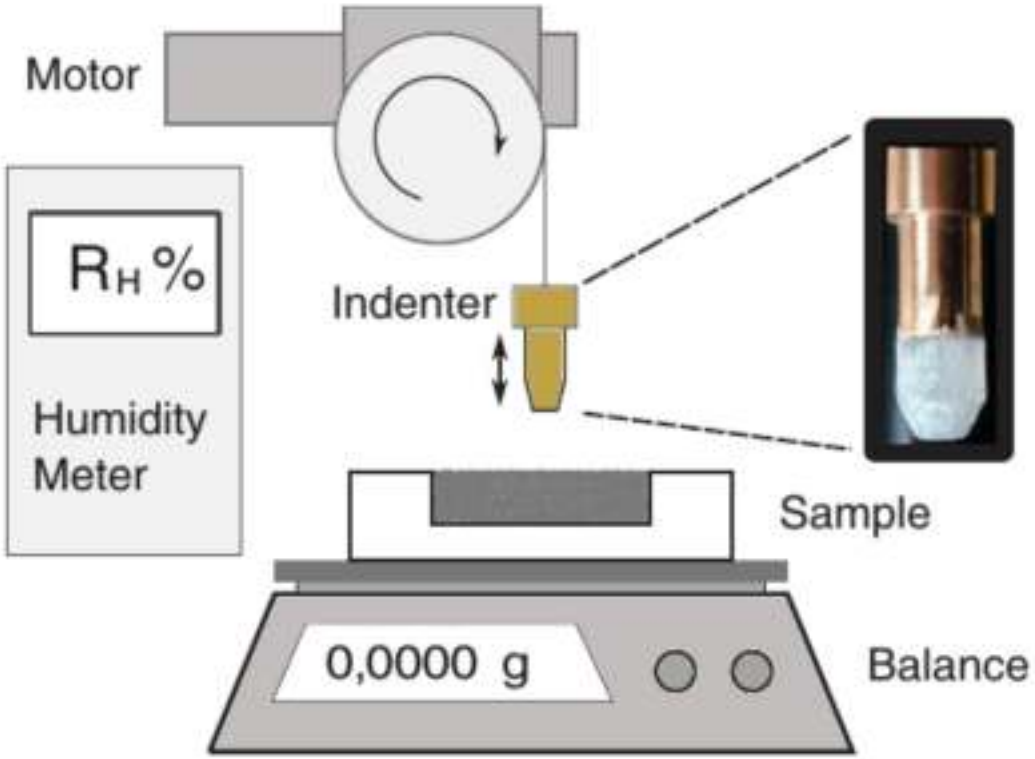
Content

Fracture experiments
- setup
- results
- mechanism

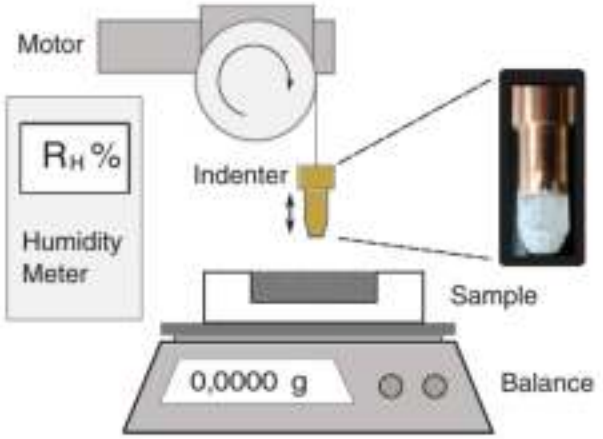
Cohesion
- **characterization techniques**
- **dependence on grain size and relative humidity**
- **interpretation**

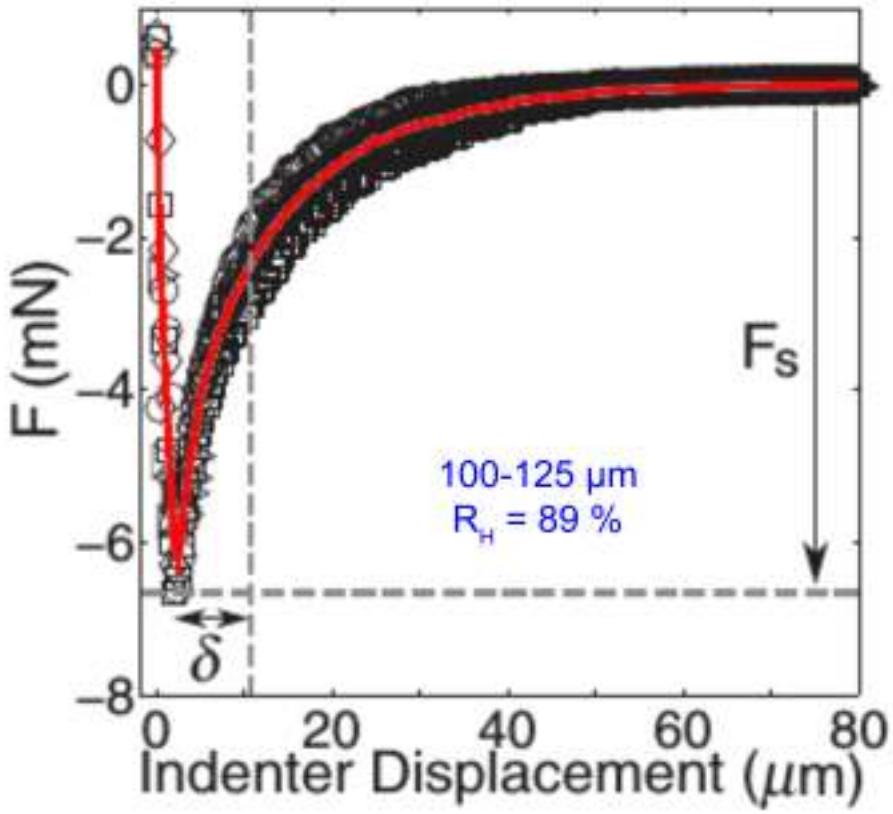
Clustering
- additional evidence
- “direct” observation

Tensile stress

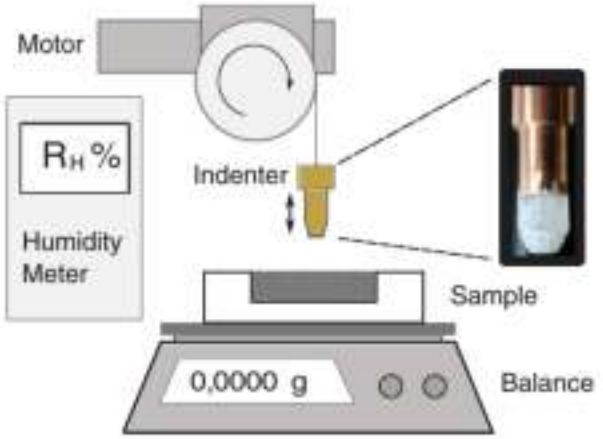


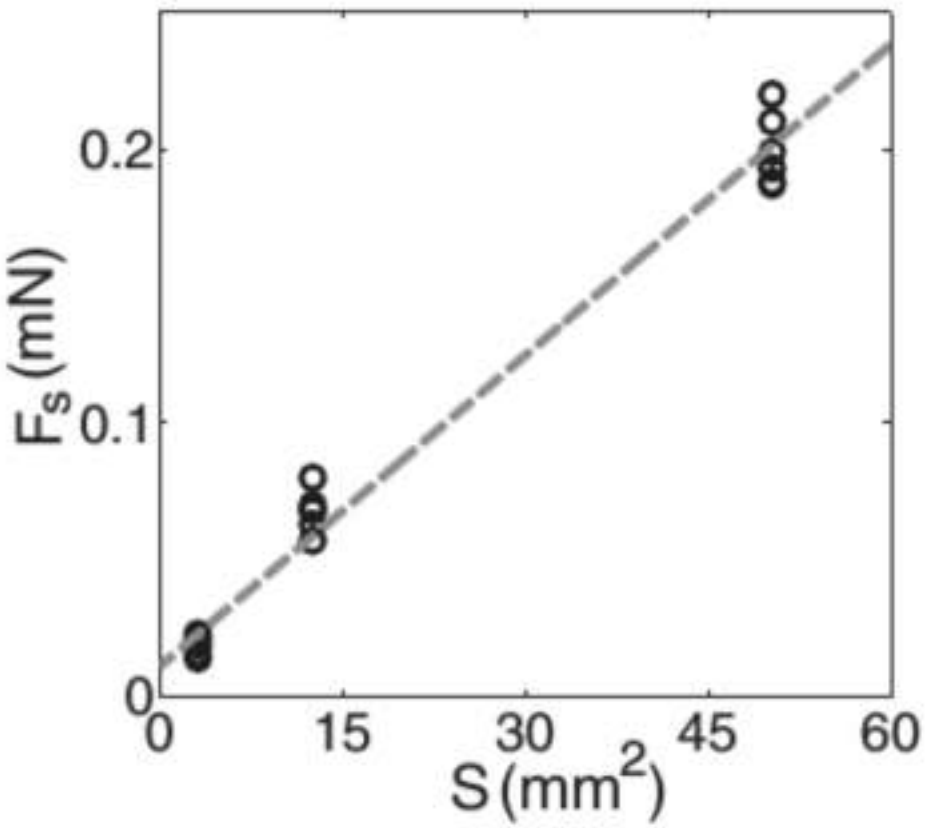
Tensile stress



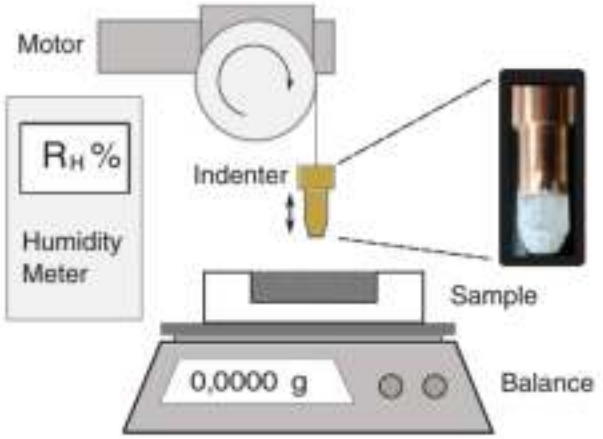


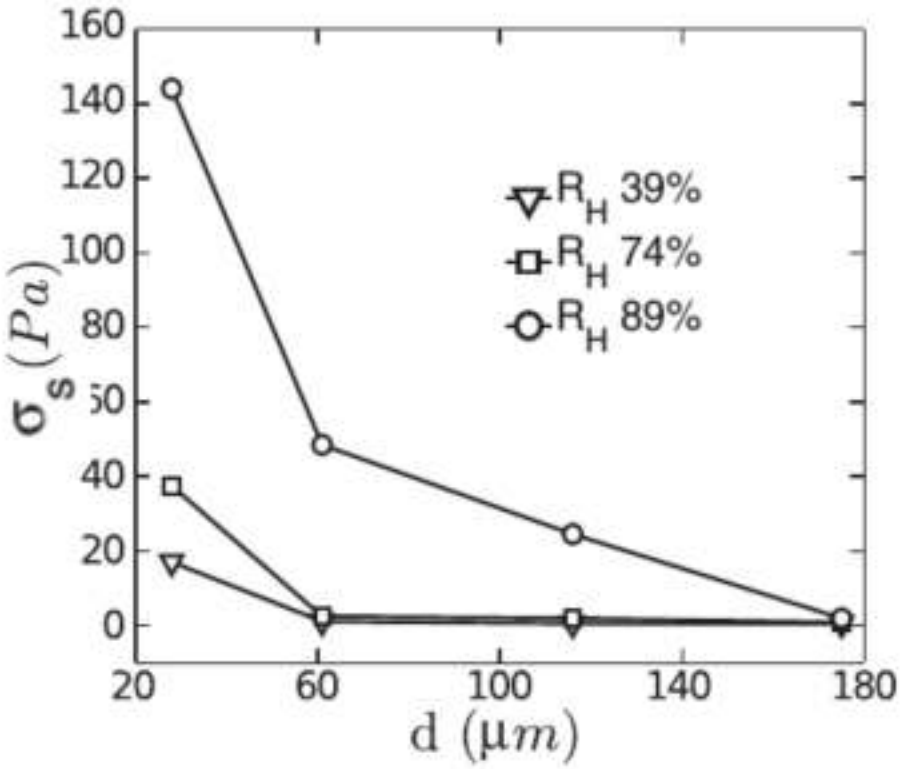
Tensile stress



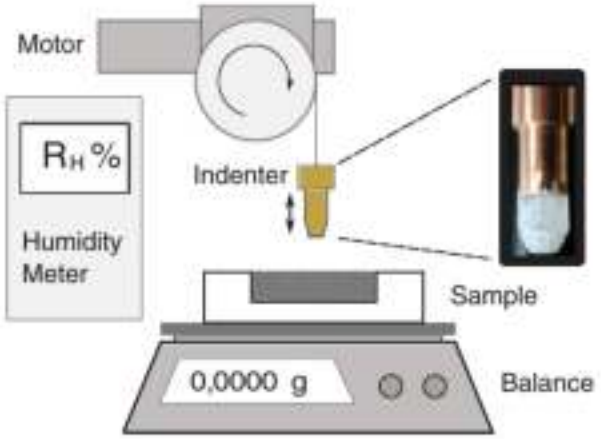


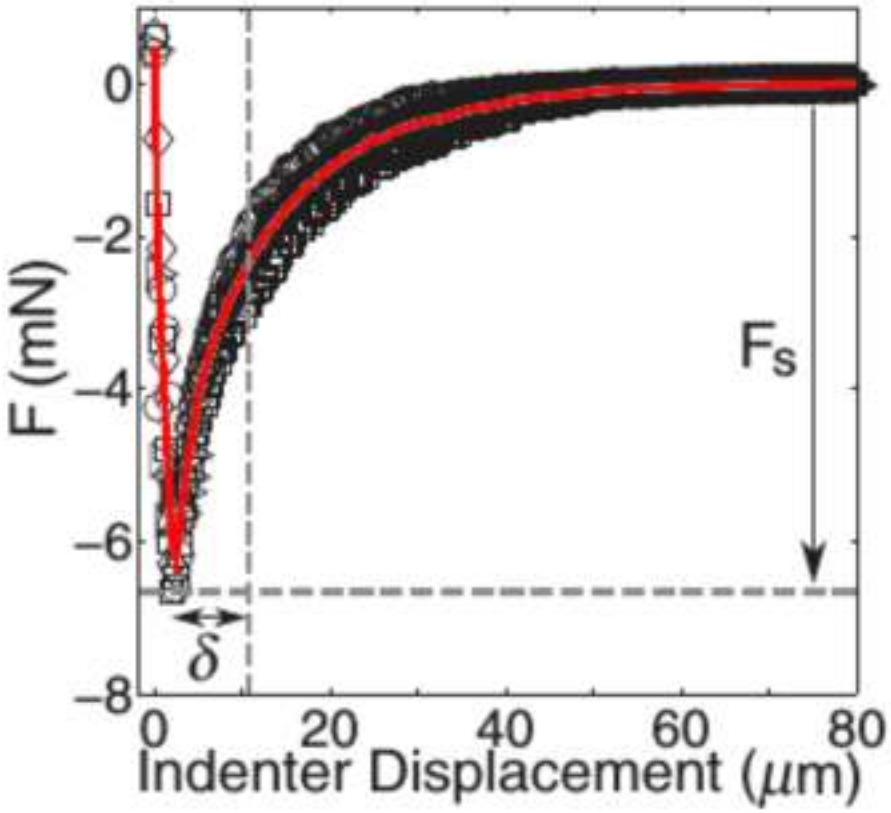
Tensile stress



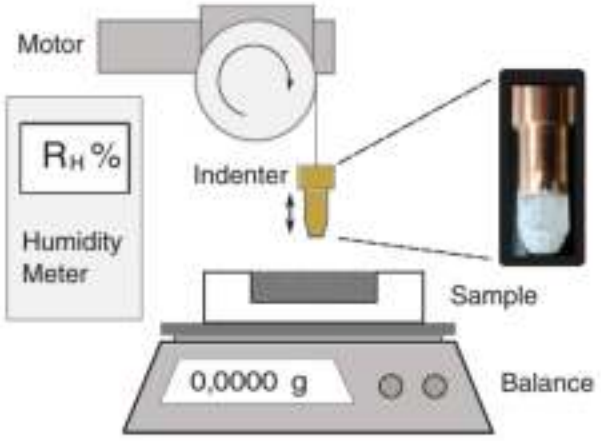


Critical strain



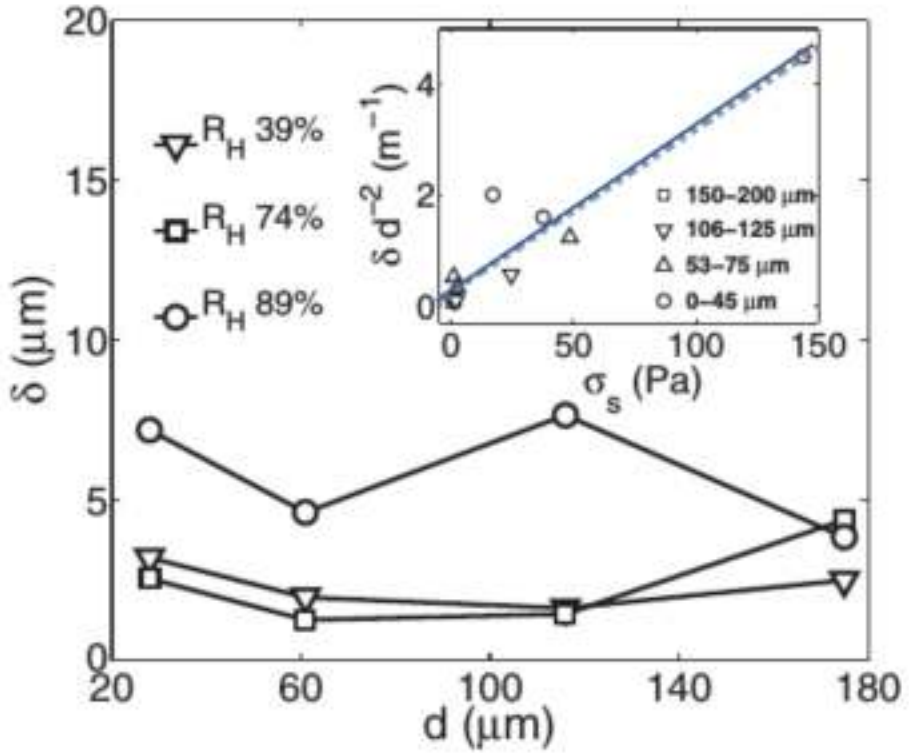


Critical strain



$$\delta \approx d^2\sigma_s/\gamma_f$$

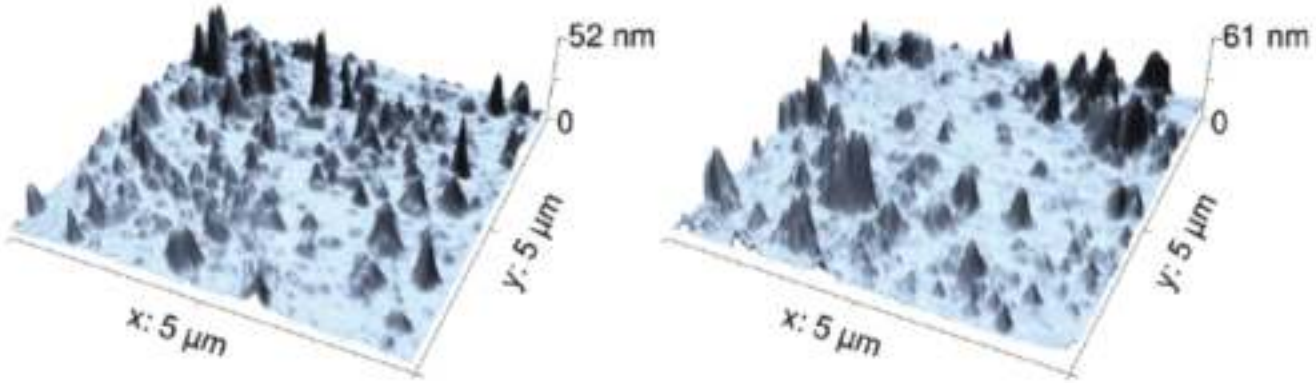
$$\gamma_f \approx 10\,\mathrm{mJ/m^2}$$



Critical strain

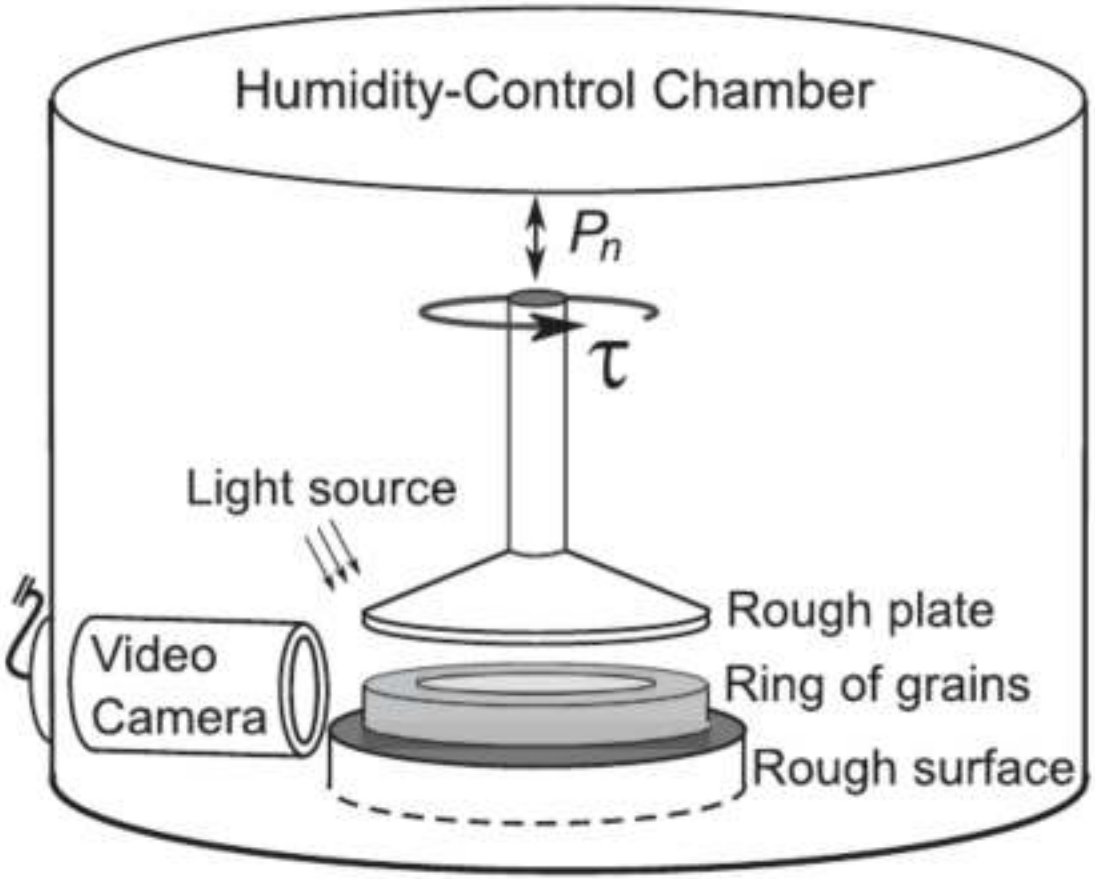
$\sigma_s \ll \pi\gamma/d$ δ is almost independent of the particle size

roughness is playing an important role

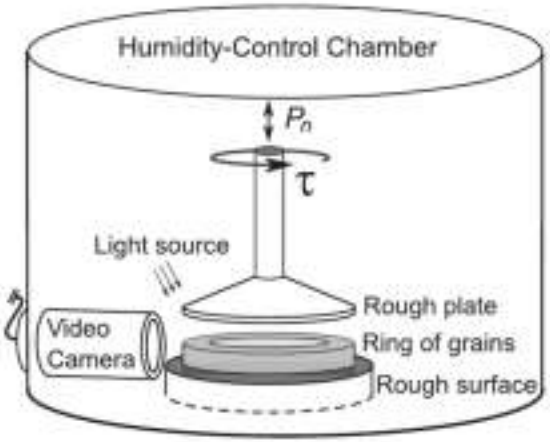


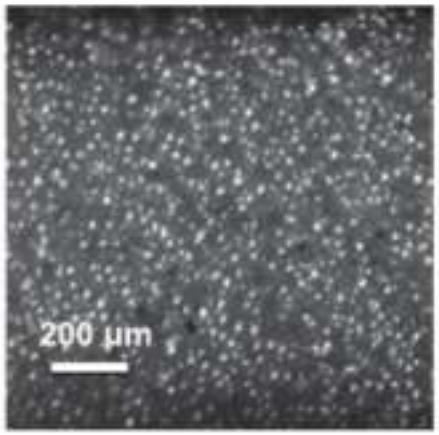
d (μm)	0–45	53–75	105–125	150–200
l_R (nm)	70	50	100	70
l_W (nm)	270	365	680	460

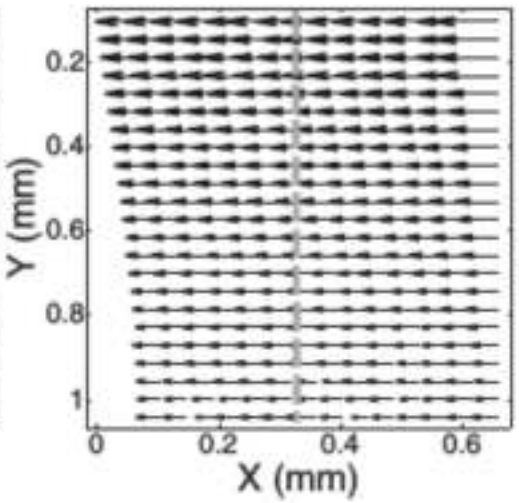
Shear modulus

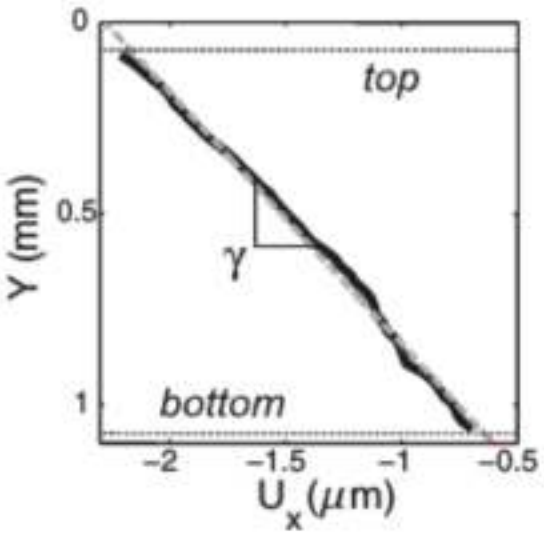


Shear modulus

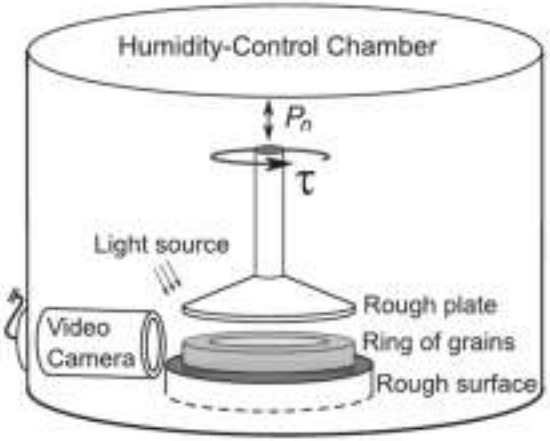


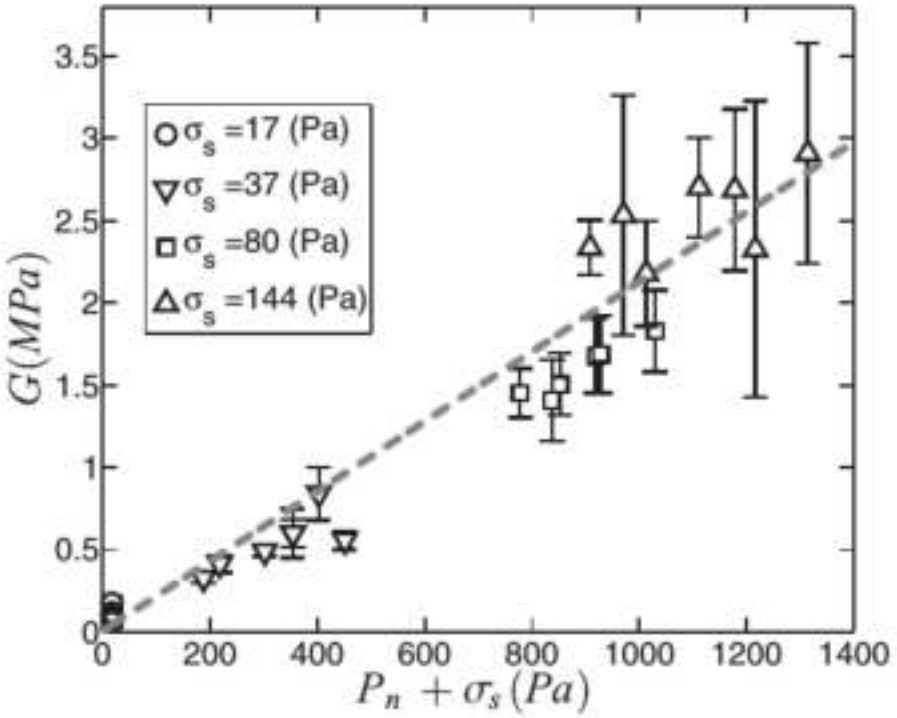




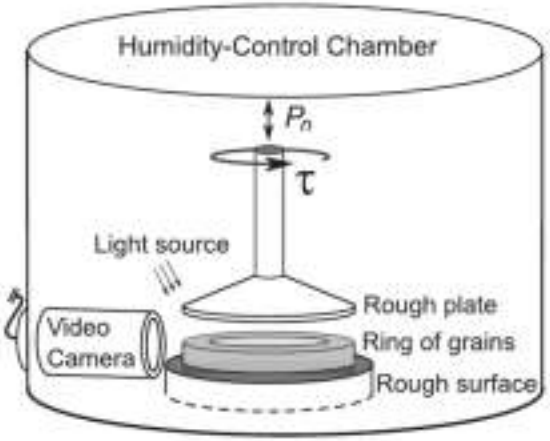


Shear modulus





Shear modulus

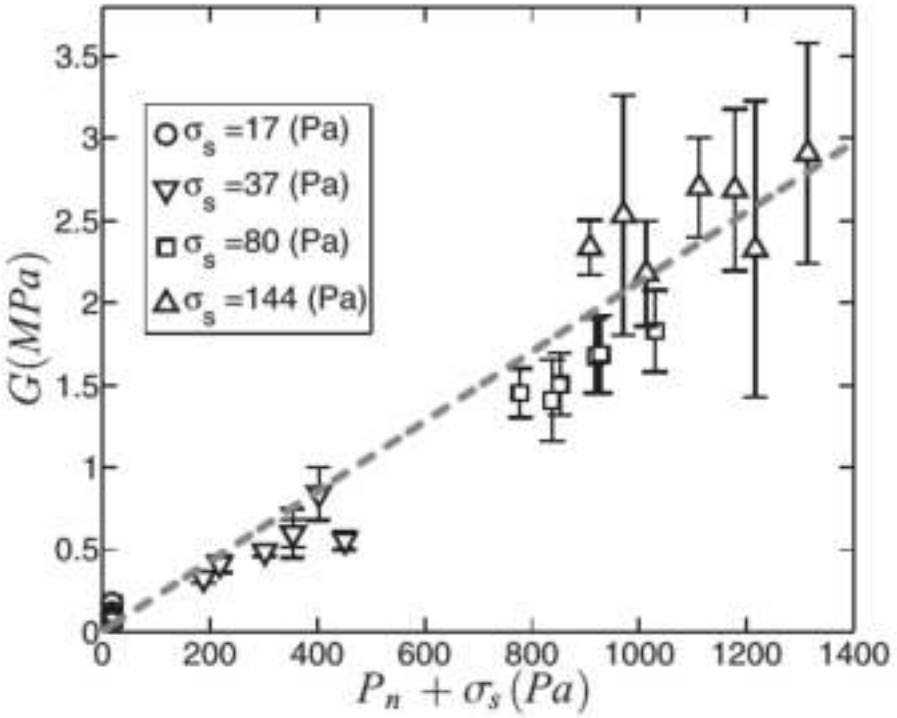


$$F_s + N_n = J A_r$$

$$F_{\mathrm{el}} \approx E A_r \Delta\delta / l_R$$

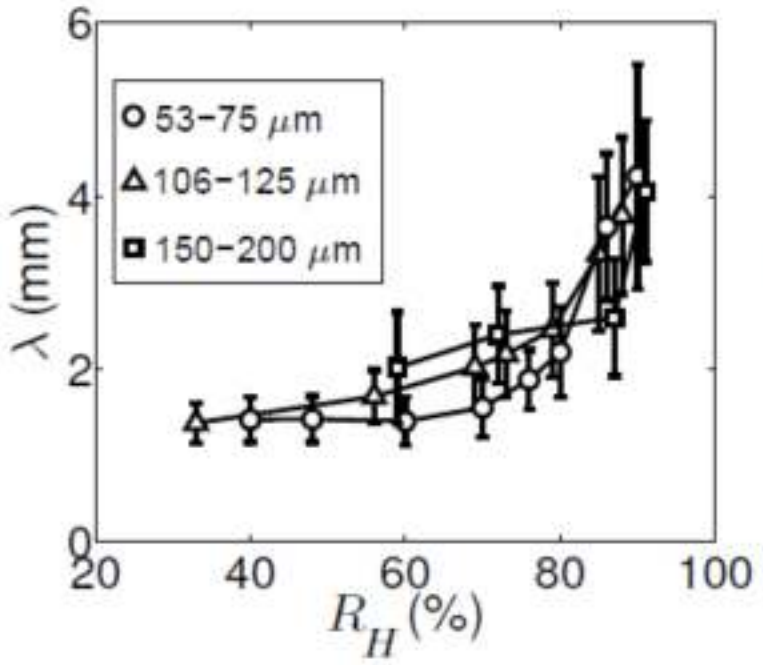
$$E_{\mathrm{eff}} \propto (F_{\mathrm{el}}/d^2)/(\Delta\delta/d)$$

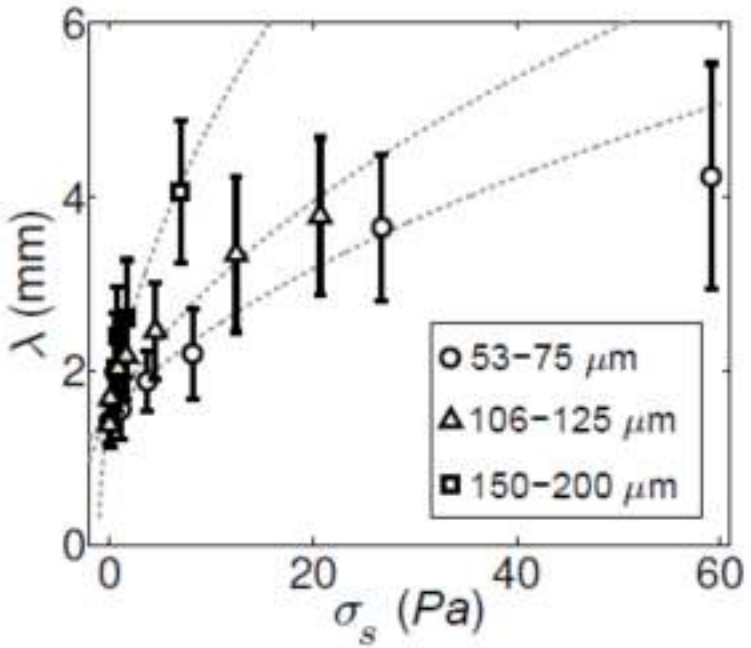
$$G \approx \frac{Ed}{6Jl_R}(\sigma_s + P_n)$$



$$E/J \approx 40$$

Discussion



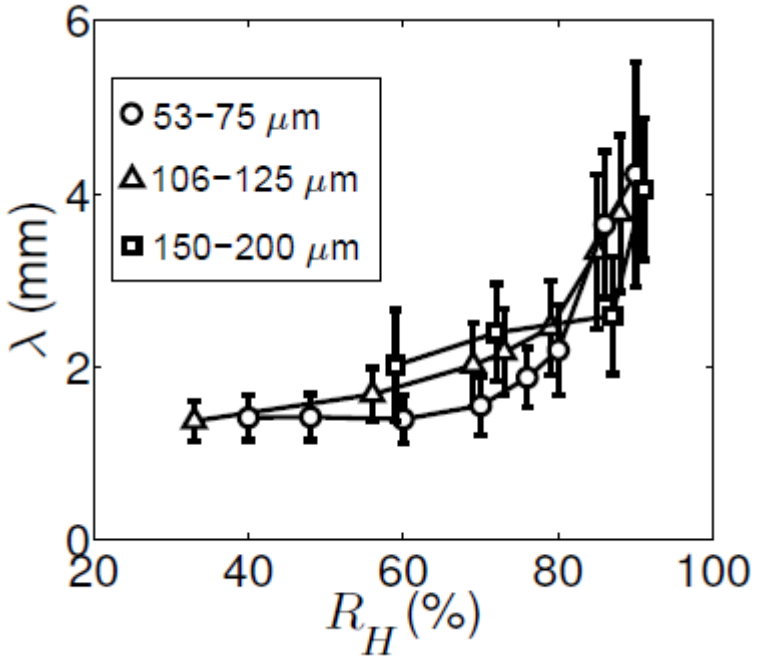


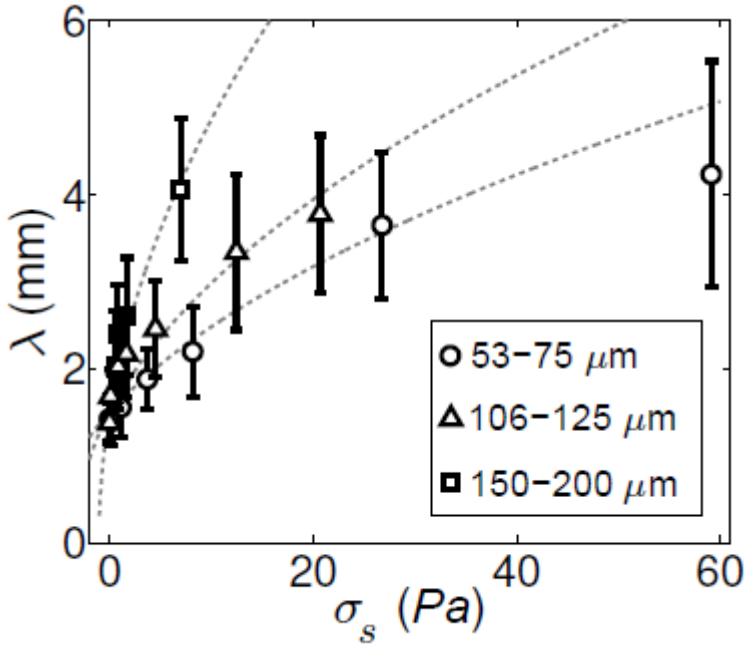
$$\lambda \simeq 4\sqrt{2\sigma_s / G\theta_m h}$$

$$\theta_m \approx \delta / L_c$$

$$L_c \approx \alpha_s d/2$$

Discussion





$$\lambda \simeq 4\sqrt{6J\alpha_s/Eh}$$

$$\alpha_s \approx 1 + \sigma_s d^2/\beta l_R \qquad \beta = (0.44 \pm 0.03)\ \mathrm{mJ/m^2}$$

cluster size ranges from d to $10d$

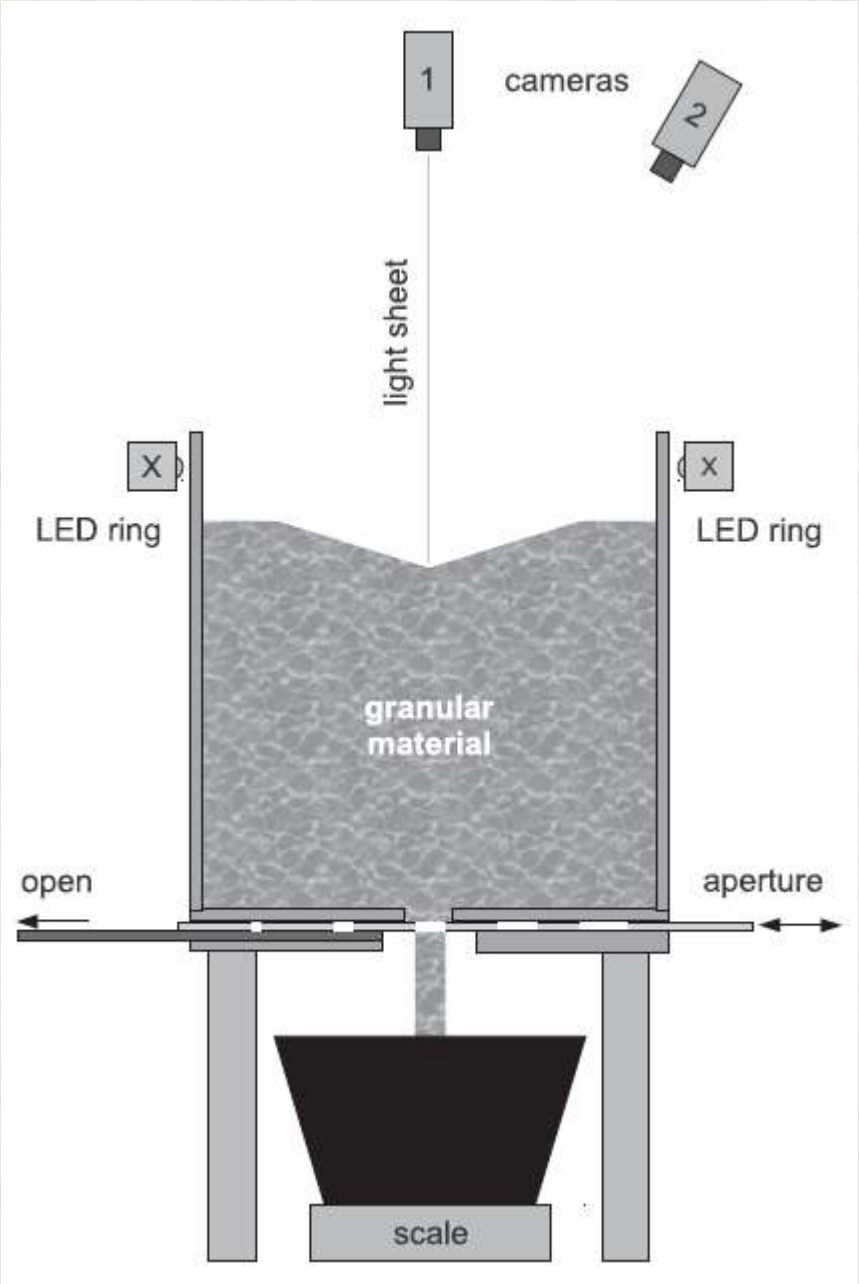
Content

Fracture experiments
- setup
- results
- mechanism

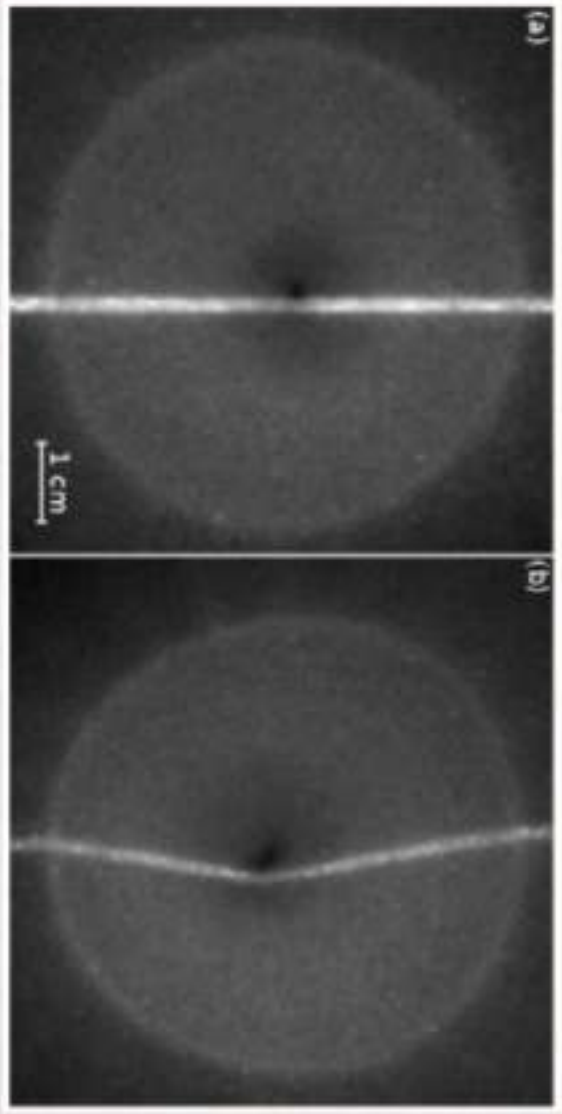
Cohesion
- characterization techniques
- dependence on grain size and relative humidity
- discussion

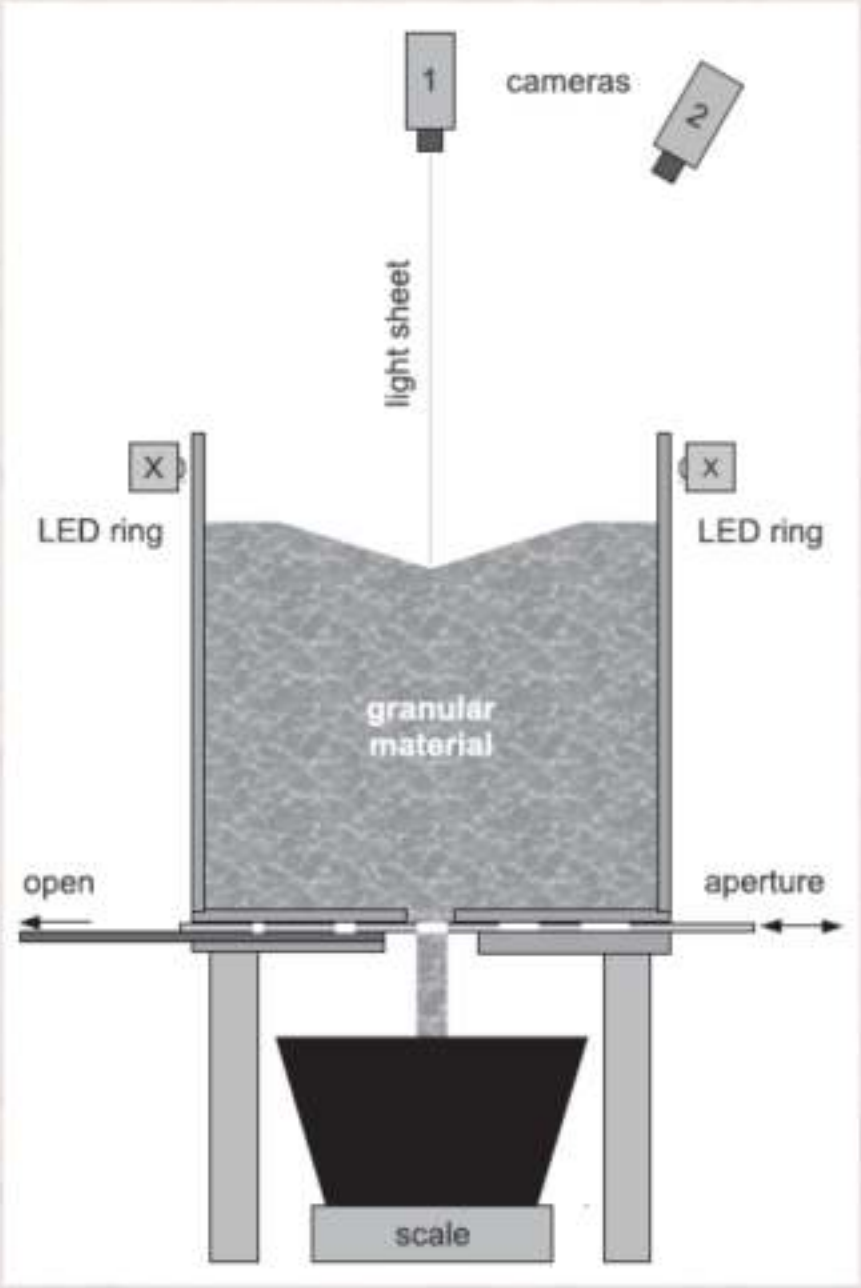
Clustering
- **additional evidence**
- **“direct” observation**

Silo discharge

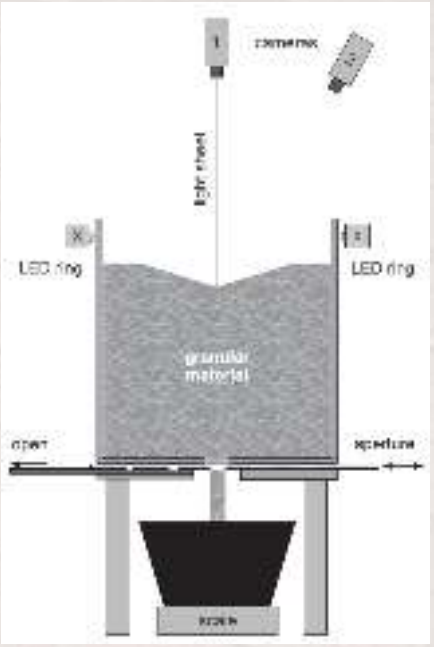


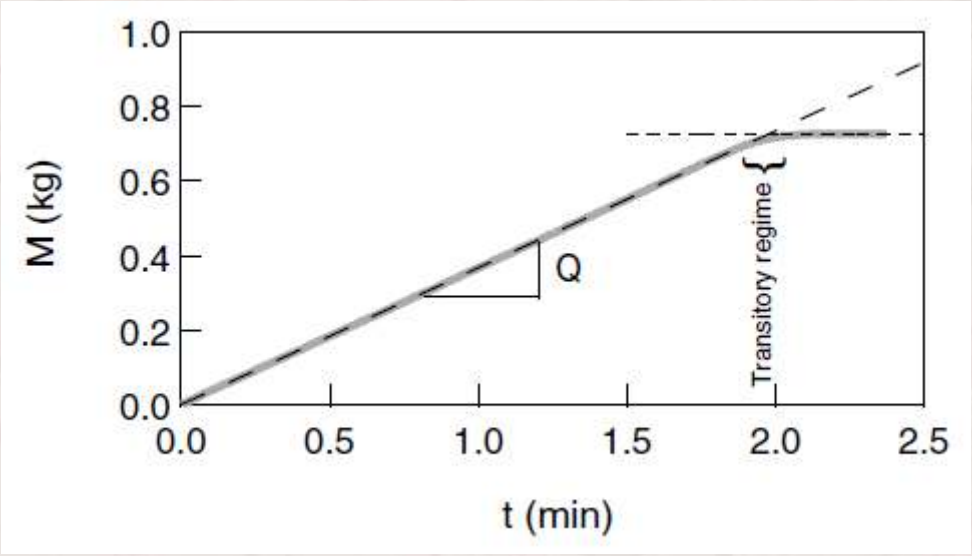
Silo discharge





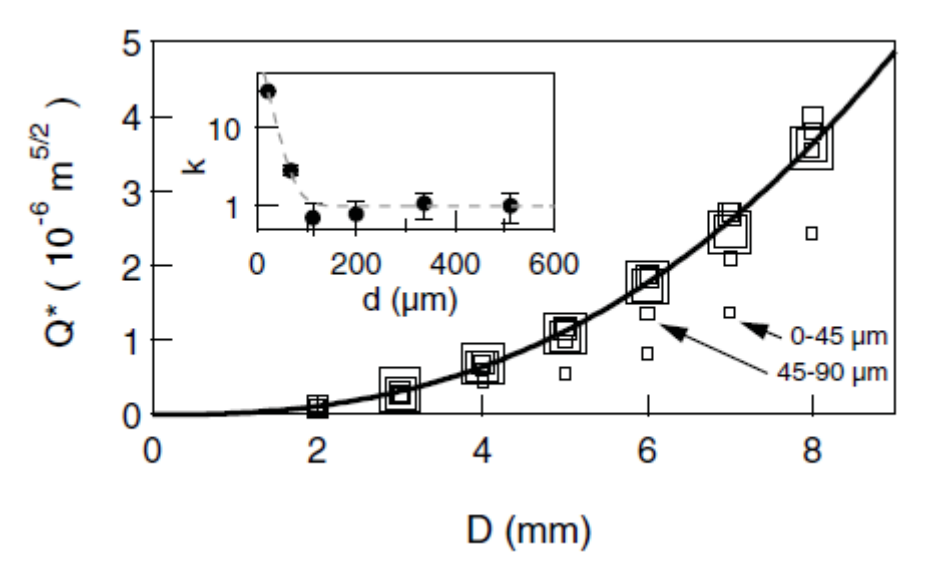
Silo discharge

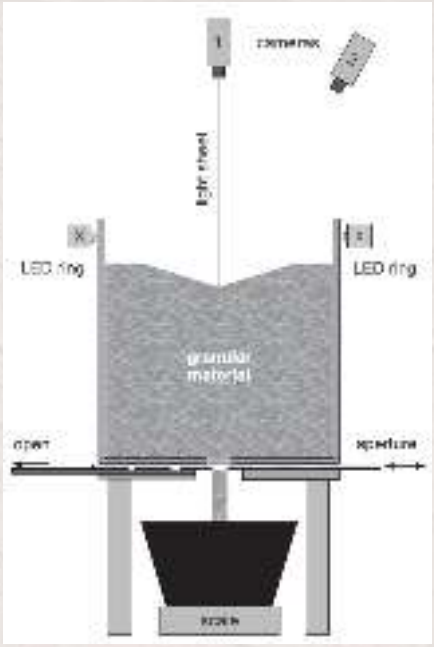




Constant flow rate

Silo discharge

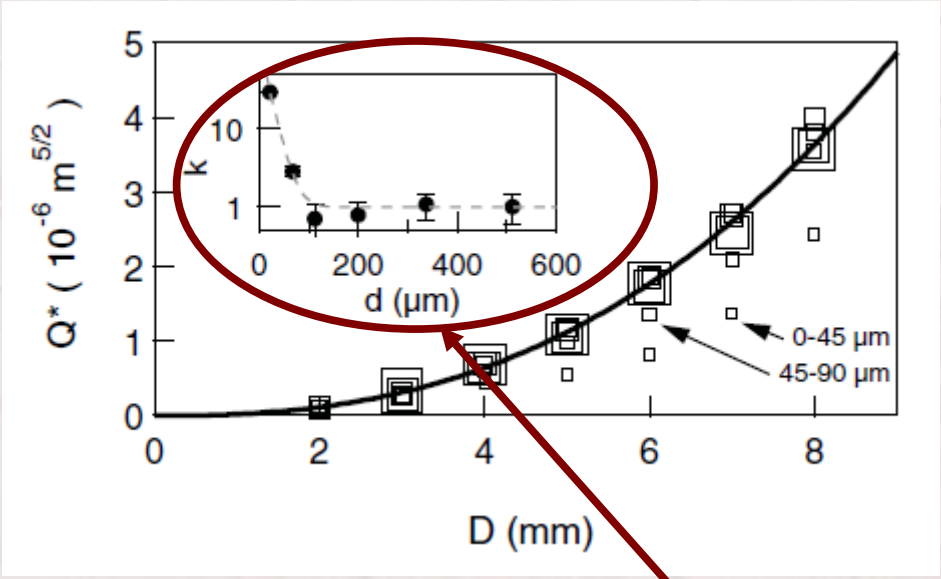


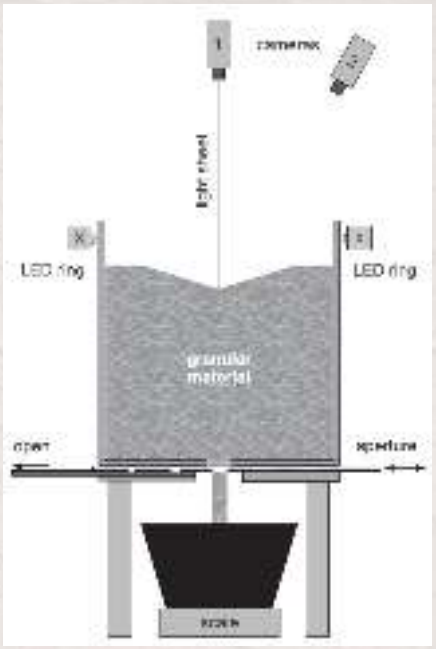


$$Q = A\,\rho\,g^{\frac{1}{2}}(D - k\,d)^{\frac{5}{2}}$$

Beverloo law

Silo discharge



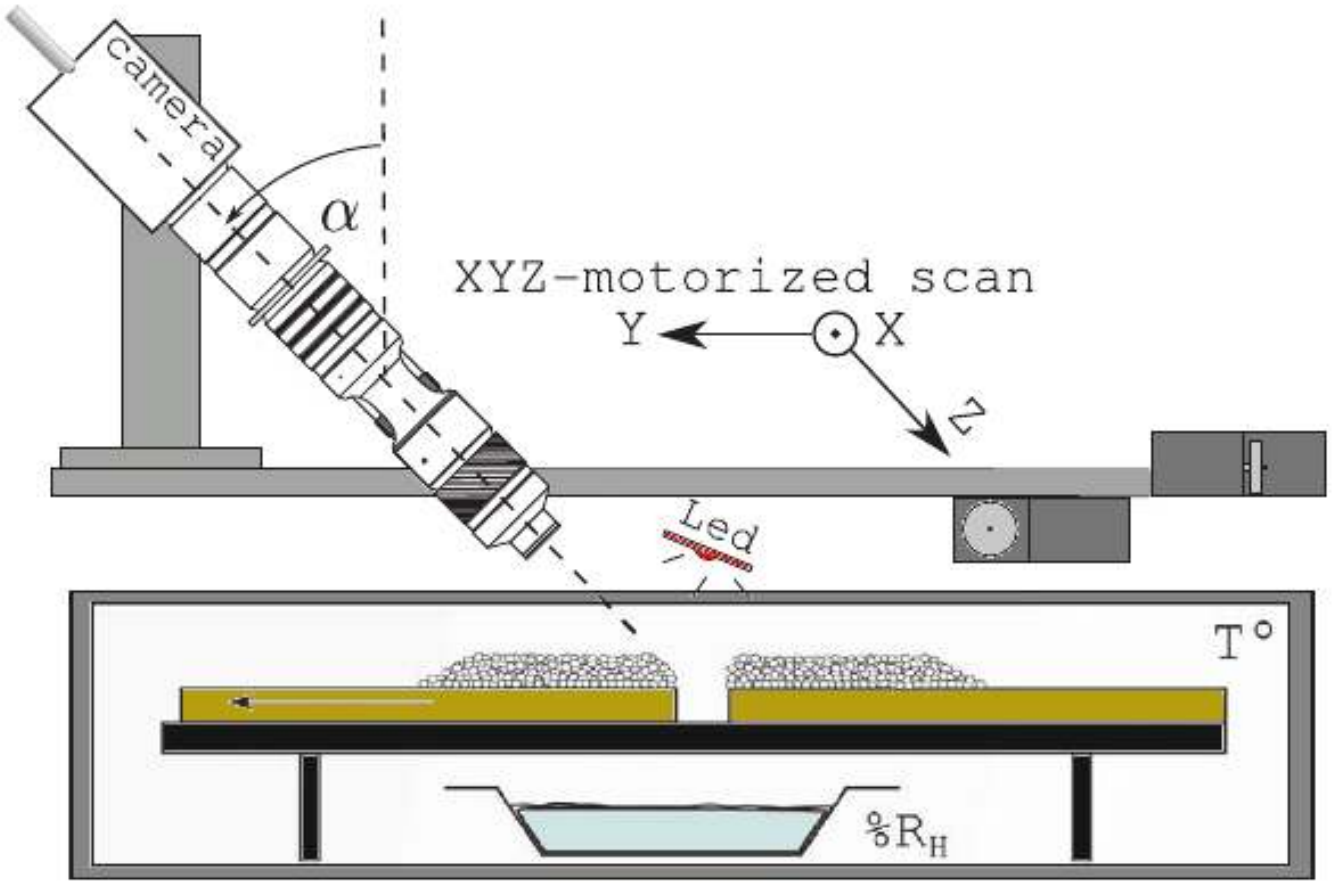


$$Q = A\,\rho\,g^{\frac{1}{2}}(D - k\,d)^{\frac{5}{2}}$$

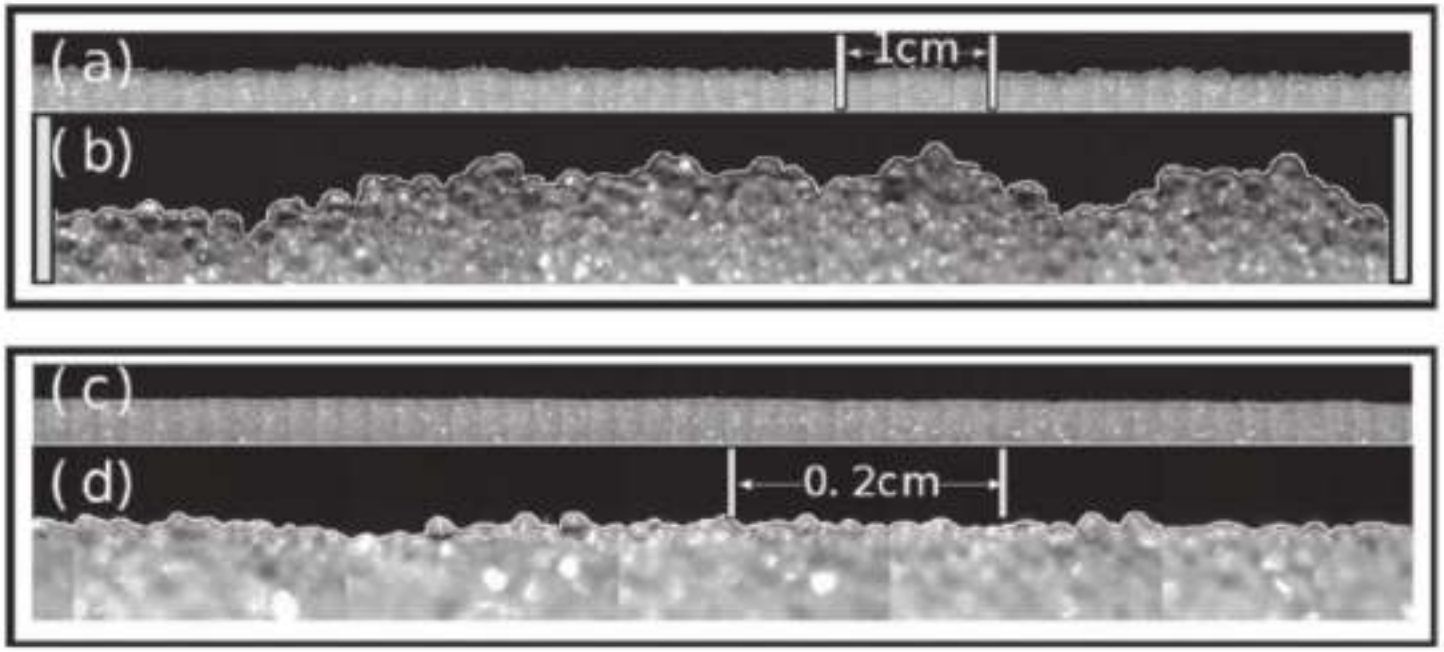
Beverloo law

The cohesion leads to an increase in the effective grains size

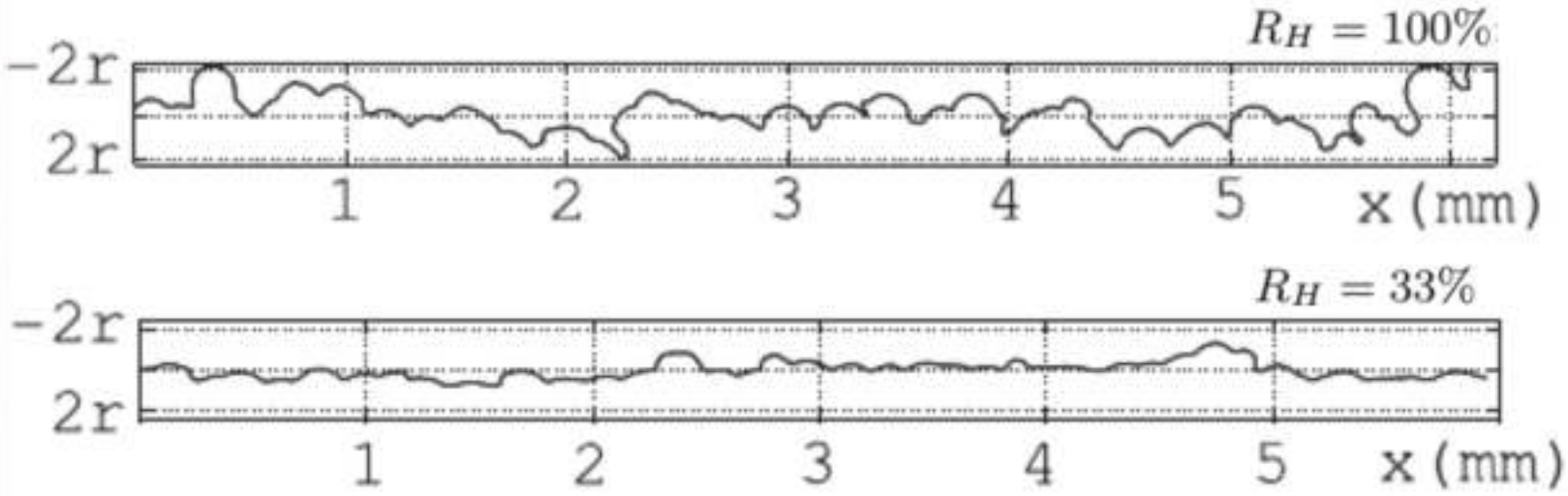
« direct » observation



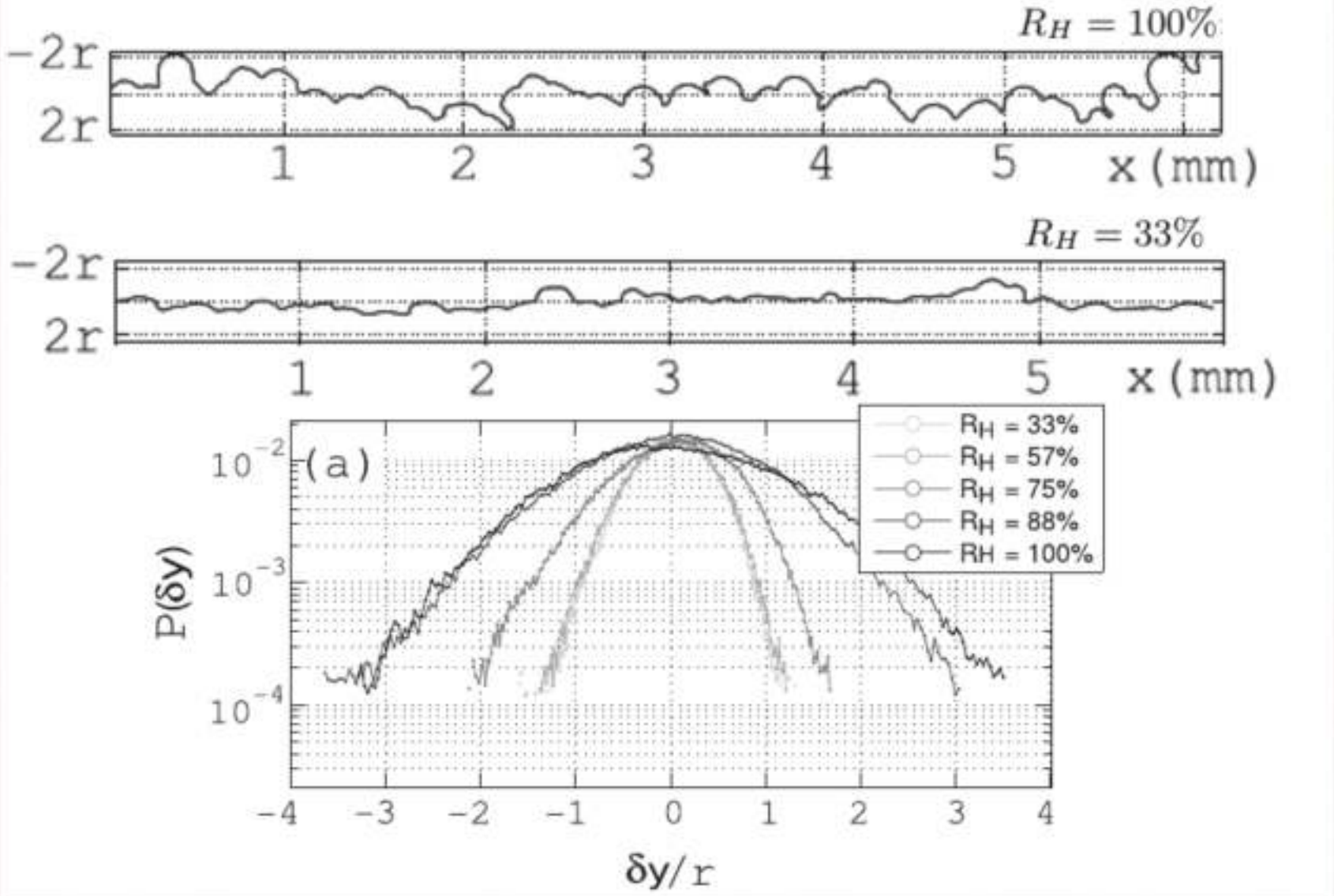
« direct » observation



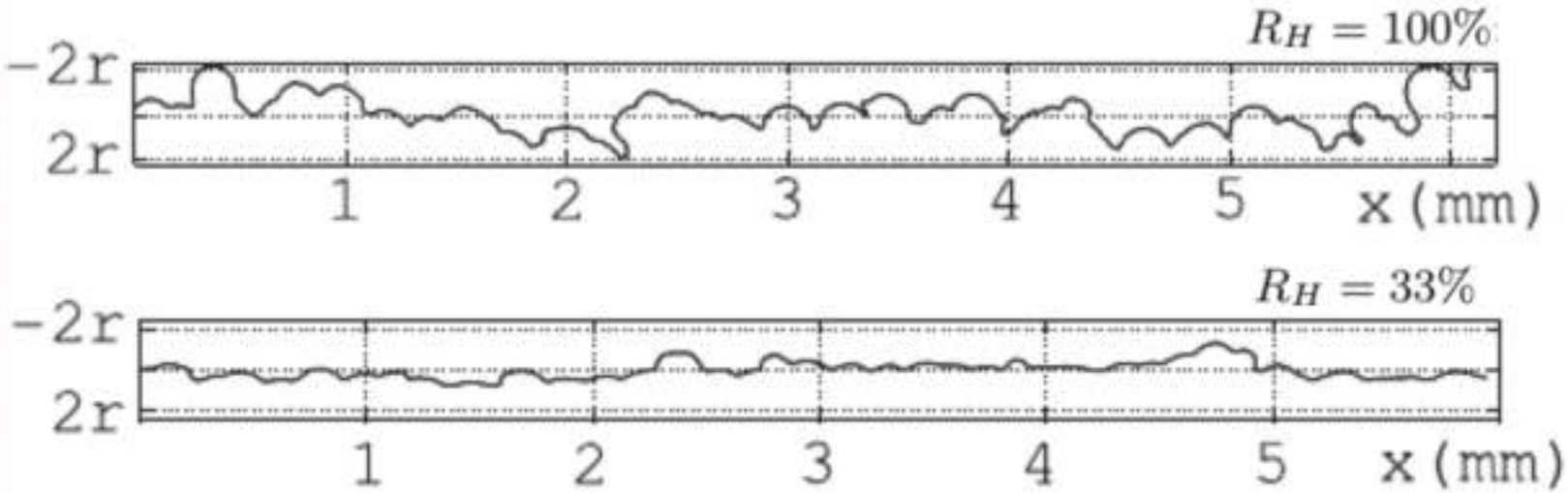
« direct » observation

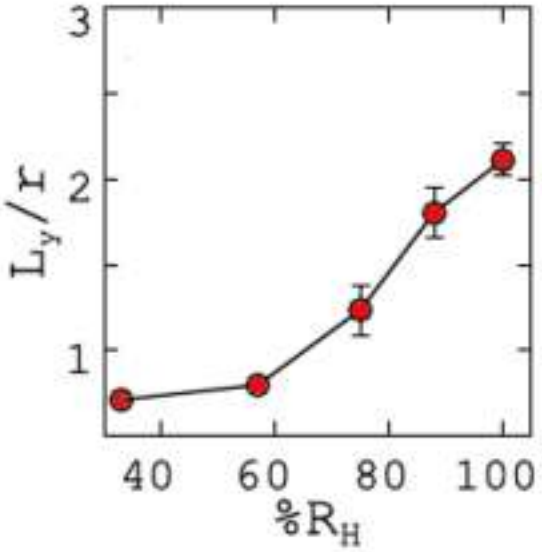


« direct » observation

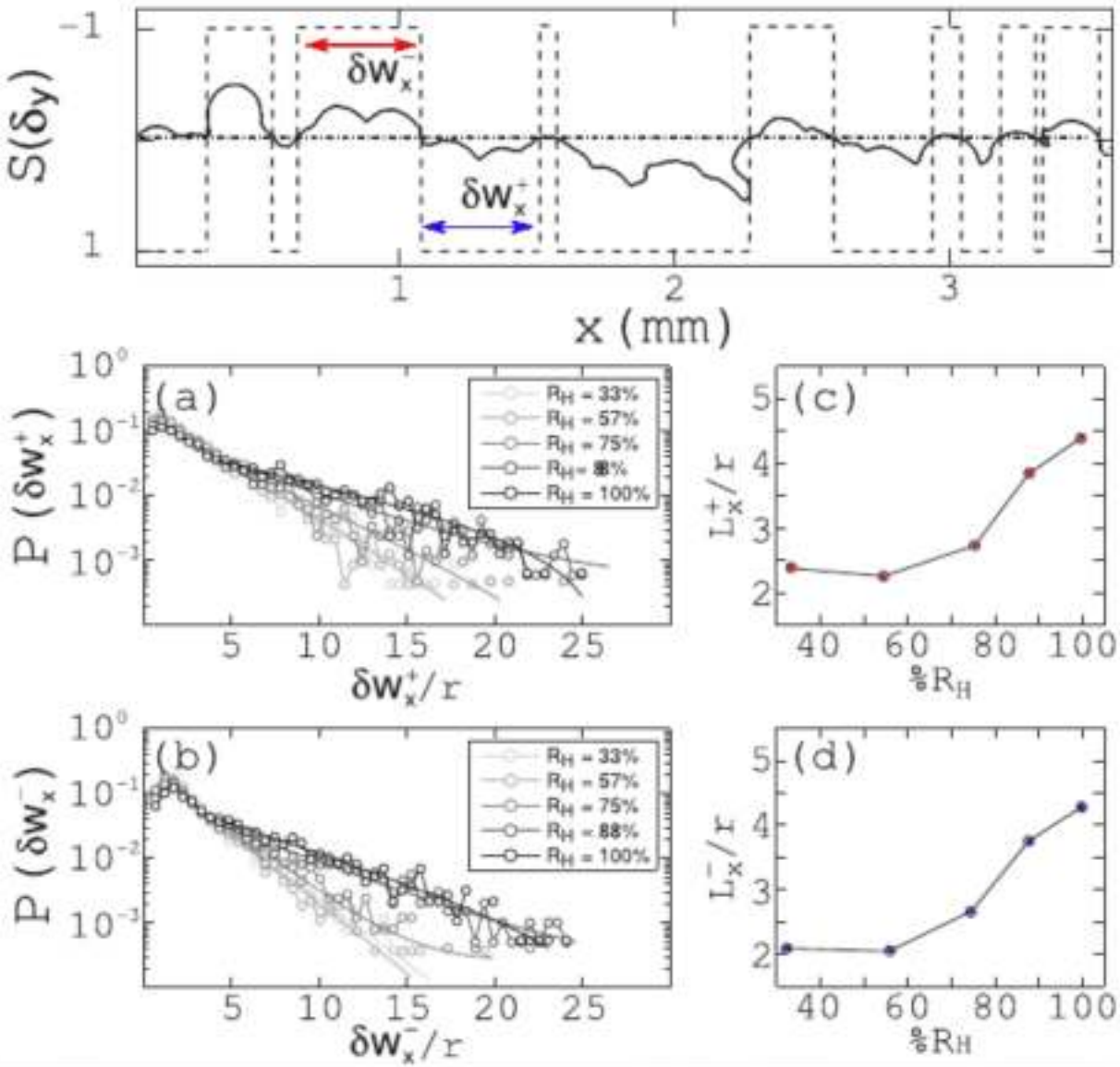


« direct » observation

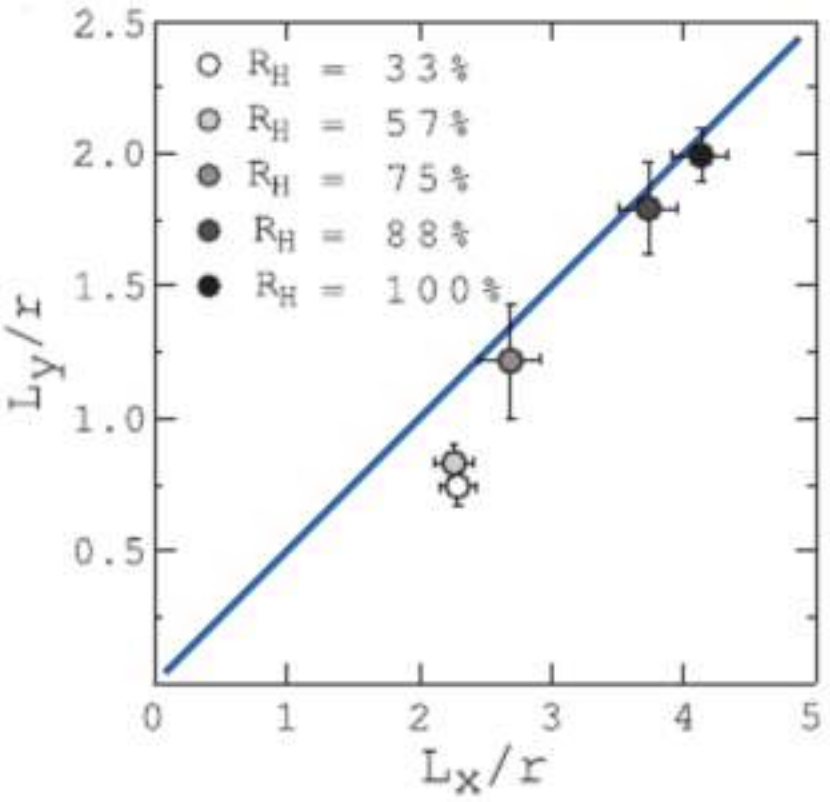




« direct » observation



« direct » observation



Conclusions

- Detailed analysis of crack patterns reveals clustering
- The analysis also points out the role played by roughness
- Clusters are also attested by the fracture path
- Even for ‘simple’ grains, we miss a precise theoretical description of the mechanical properties of the material
- Generalization to other physical origins of cohesion ?