

# Boundary interactions in three-dimensional gravity driven granular flows

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Patrick Richard, Université Eiffel

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# Introduction

Granular flows are encountered in geophysical contexts



# Introduction

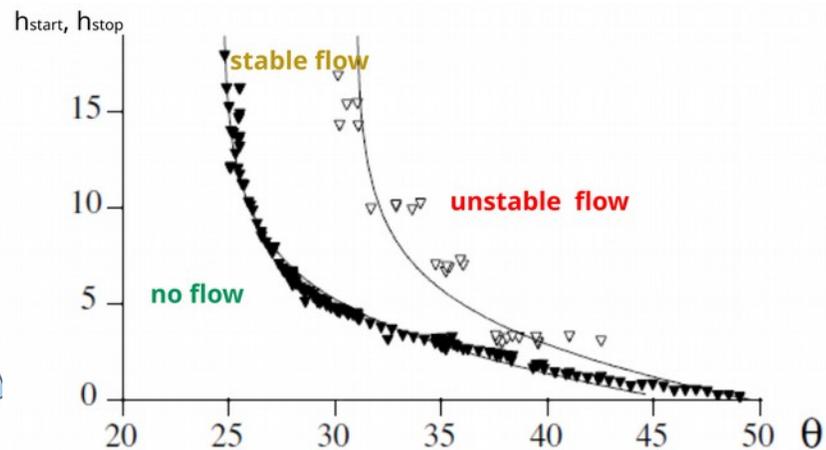
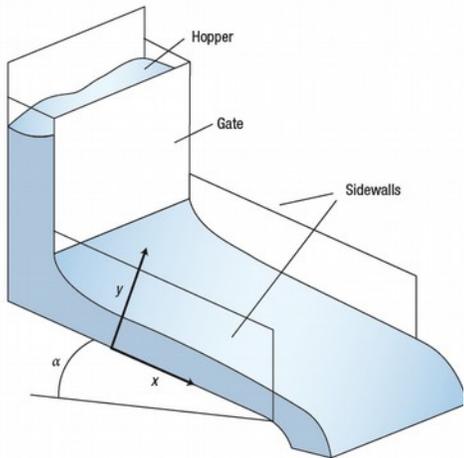
but also in industrial contexts ...



# Dense gravitational granular flows

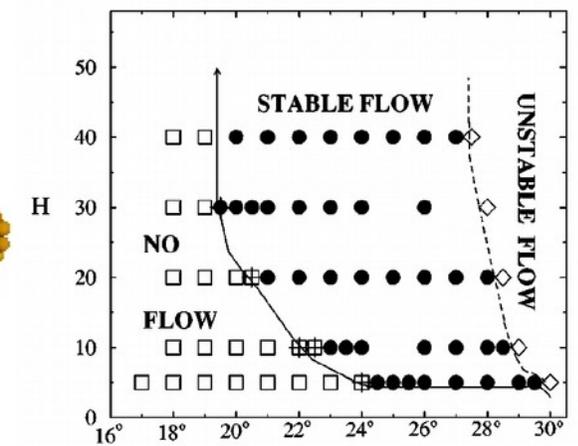
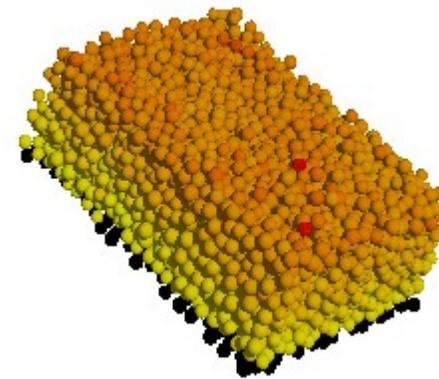
- **Steady and fully-developed flows**: no time dependence and invariance along the main flow direction
- **Simulations** : Periodic boundary conditions
- **No confinement** (or sidewalls very far apart)
- **Unidirectional flows** : 1D systems

## Experiments



GDR MIDI, EPJE, 2001

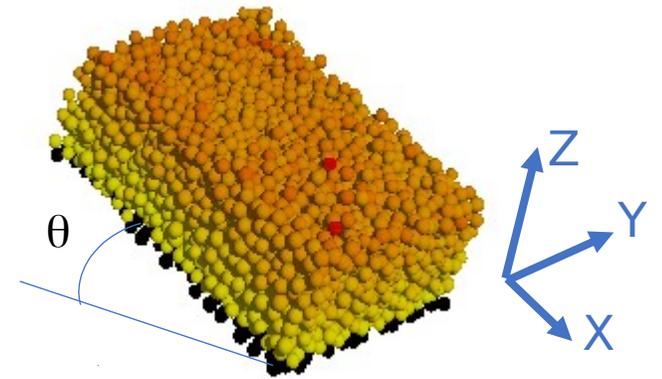
## Discrete simulations



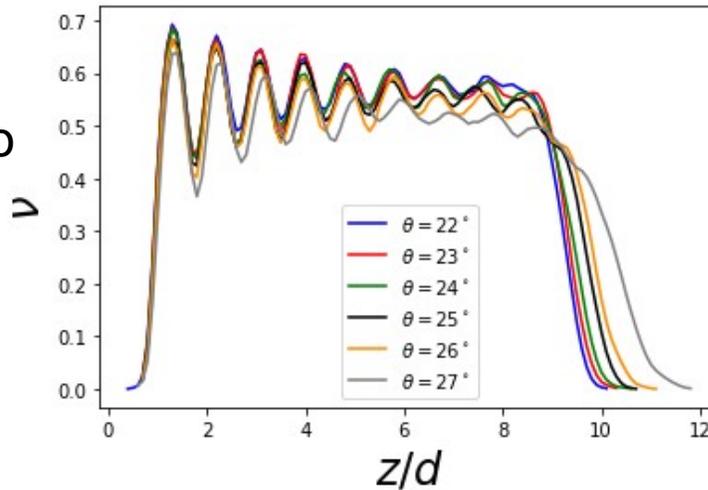
Silbert et al. PRE, 2001

# Dense gravitational granular flows

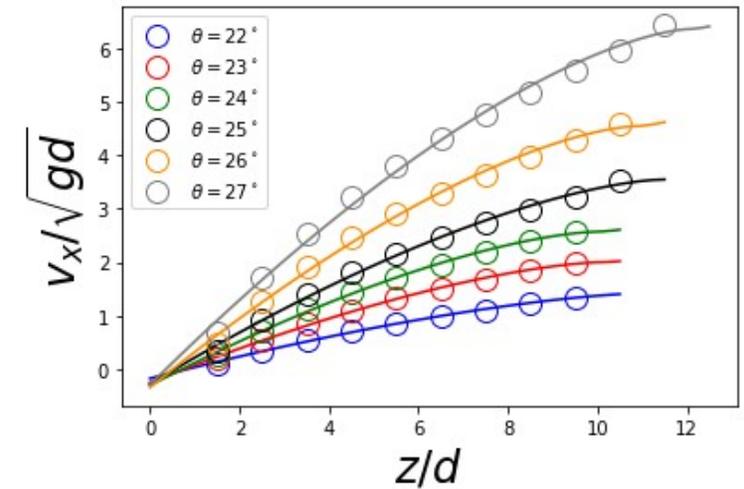
Discrete simulations provides a nice tools to document such flows



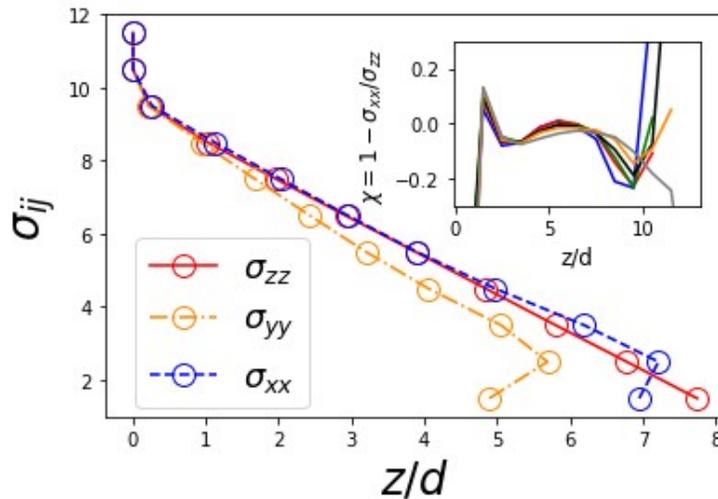
Volume fraction  
Layering close to  
the bumpy  
bottom



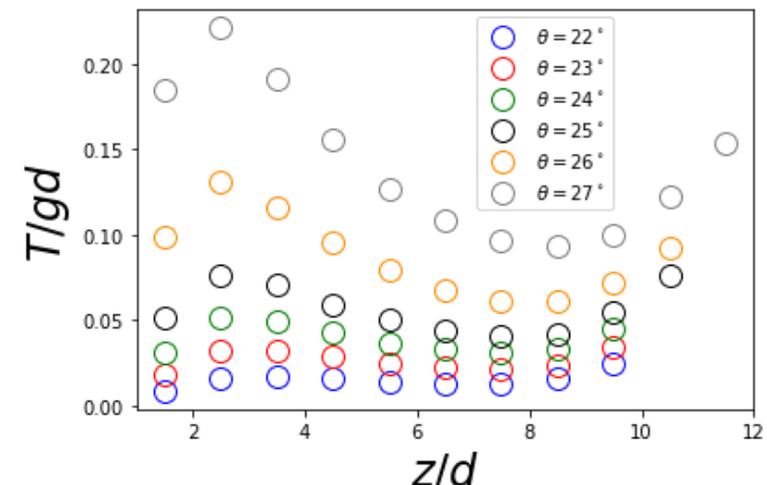
Streamwise velocity  
Bagnold scaling  $v_x$   
 $\propto z^{3/2}$



Stresses  
Hydrostatic  
 $\sigma_{zz}$   
 $\sigma_{xx} \approx \sigma_{zz}$   
 $\sigma_{yy} \ll \sigma_{zz}$



Granular  
temperature  
Energetic bottom



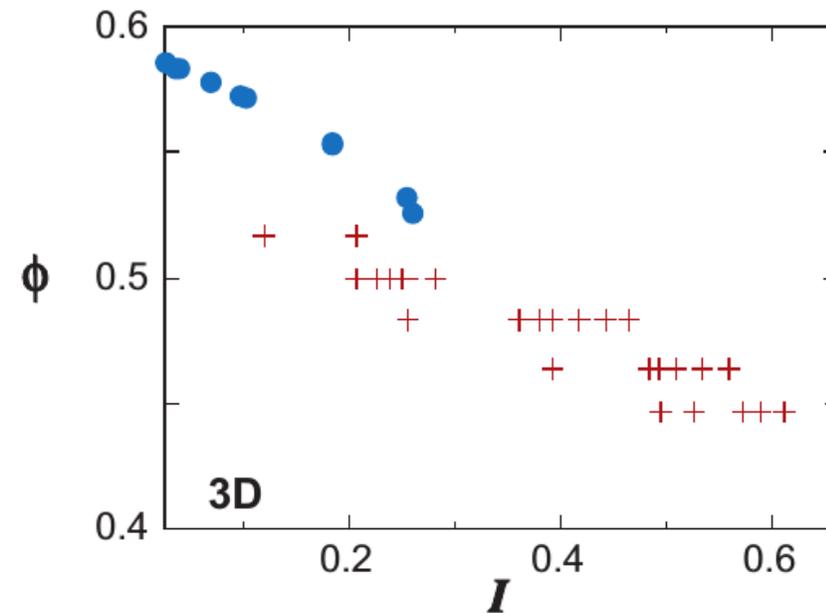
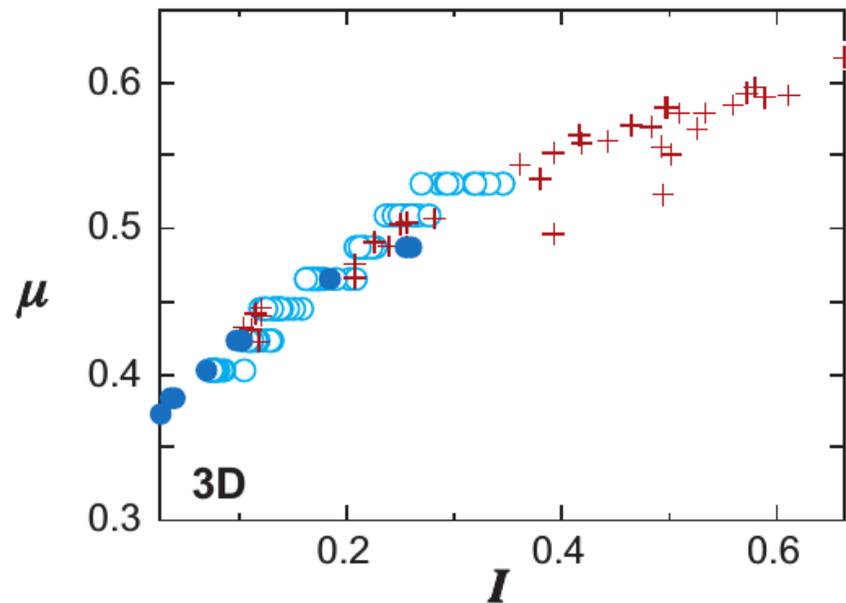
# $\mu(I)$ rheology for dense granular flows

$\mu(I)$  rheology is relevant for describing dense granular flows (i.e., moderate slope)

Dimensionless inertial number  $I$  :  $I = |\dot{\gamma}|d/(P/\rho_s)^{0.5}$

Shear rate :  $\tau = \mu(I) P$  with  $\mu(I) = \mu_s + (\mu_2 - \mu_s)/(1 + I_0/I)$

Volume fraction :  $\phi = \phi(I)$  with  $\phi(I) = \phi_{\max} - \Delta \phi$



# Extended kinetic theory for dense granular flows

EKT is also capable of capture the essential features of dense granular flows

Granular Matter (2017) 19:56  
DOI 10.1007/s10035-017-0738-1

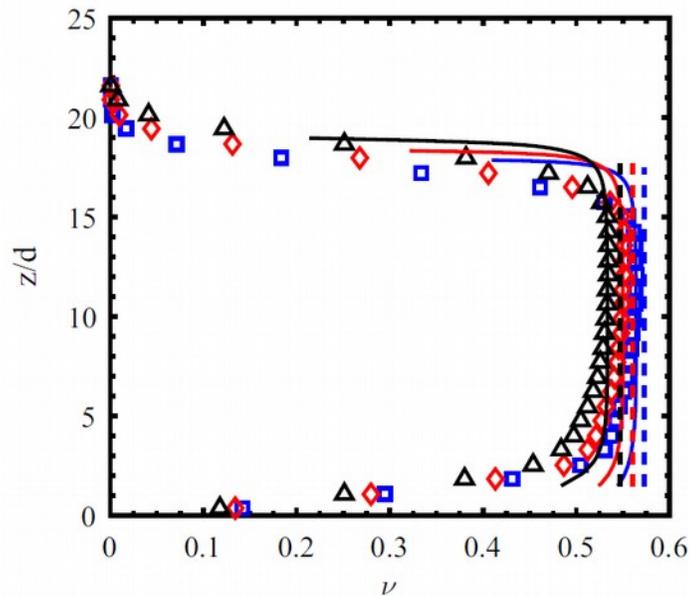
ORIGINAL PAPER

Extended kinetic theory applied to inclined granular flows:  
role of boundaries

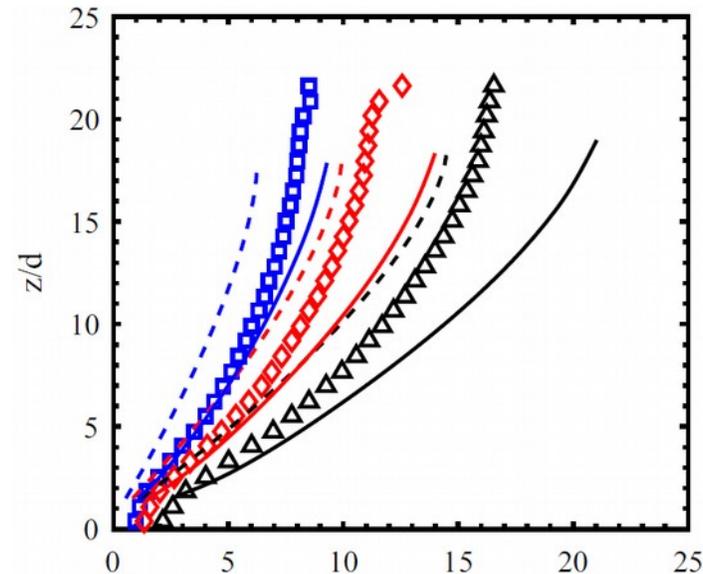
Devis Gollin<sup>1</sup> · Diego Berzi<sup>2</sup> · Elisabeth T. Bowman<sup>1</sup>

$$p = \rho_p f_1 T, \quad Q = -\rho_p d f_4 T^{1/2} T' - \rho_p d f_5 T^{3/2} v'$$
$$s = \rho_p d f_2 T^{1/2} u', \quad \frac{L}{d} = f_0 \frac{u'}{T^{1/2}}$$
$$\Gamma = \rho_p \frac{f_3}{L} T^{3/2},$$

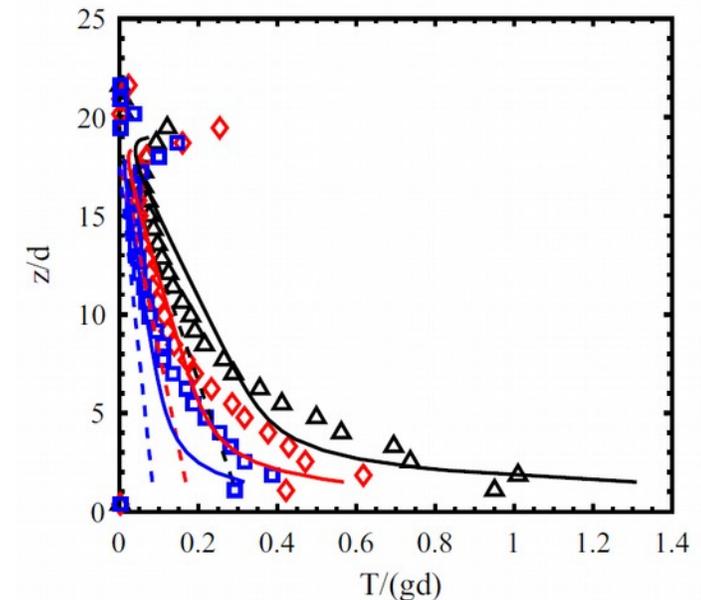
Packing fraction



Velocity

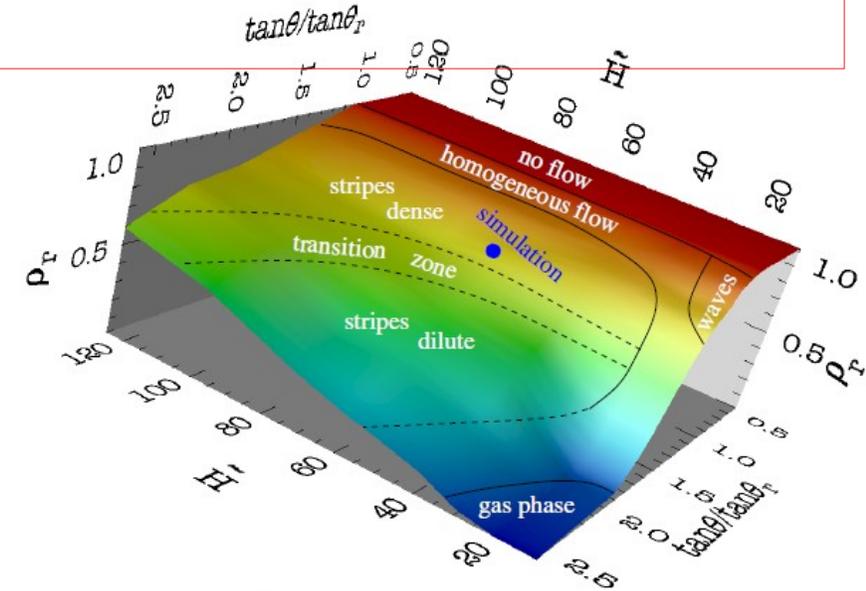
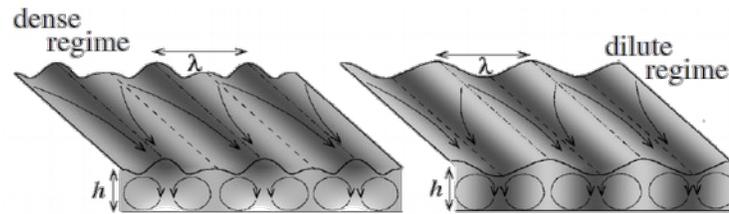
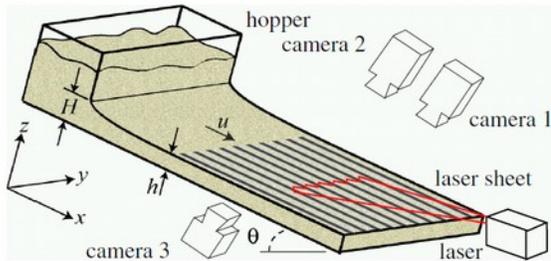


Temperature



# Instabilities in high-speed granular flows

On large slope (above  $30^\circ$ ) secondary flows appear together with spatial variation of the particle volume fraction



PRL 103, 178302 (2009)

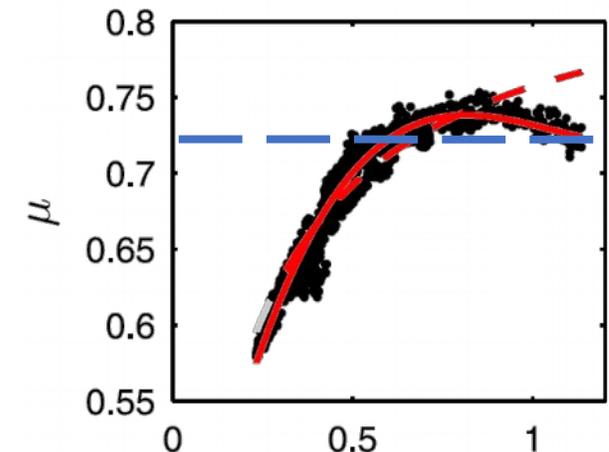
PHYSICAL REVIEW LETTERS

week ending  
23 OCTOBER 2009

## Patterns in Flowing Sand: Understanding the Physics of Granular Flow

Tamás Börzsönyi,<sup>1,2,\*</sup> Robert E. Ecke,<sup>2</sup> and Jim N. McElwaine<sup>3</sup>

- Secondary flows
- The system is no more one-directional
- The  $\mu(I)$  rheology breaks down

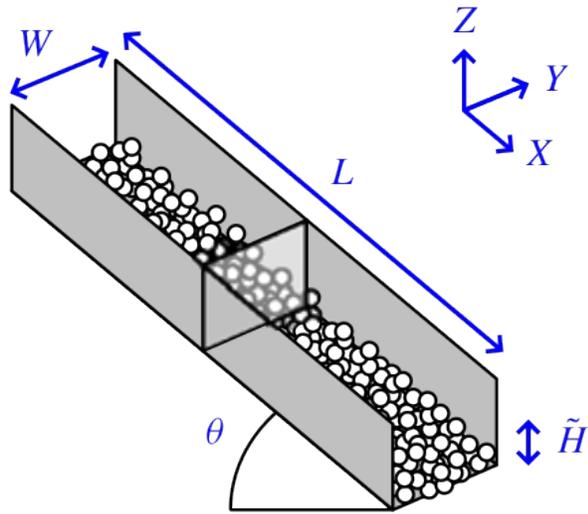


# Flows confined between lateral walls



- Some geophysical flows are laterally bounded or may form lateral levees
- Lab granular chute flow experiments are often bounded by lateral walls

# DEM simulations of granular flows in a confined geometry



**Chute set-up** : Granular flows on flat frictional base confined between flat frictional walls (Brodu et al, PRE 2013, JFM 2015)

-**Periodic boundary conditions** in the stream-wise direction

-**Dimensions** :  $L_x = 20D$ ,  $L_z = 100D$

-**Control Parameters** : Inclinaison angle ( $10^0 < \theta < 60^0$ ), Particle hold-up ( $2D < H < 20D$ ), Gap width ( $W = 68D$ )

-**Material parameters**

Grain/grain interaction:  $e_{gg} = 0.972$ ,  $\mu_{gg} = 0.33$

Grain/wall interaction:  $e_{gw} = 0.8$ ,  $\mu_{gw} = 0.593$

*J. Fluid Mech.* (2015), vol. 769, pp. 218–228. © Cambridge University Press 2015  
doi:10.1017/jfm.2015.109

**New patterns in high-speed granular flows**

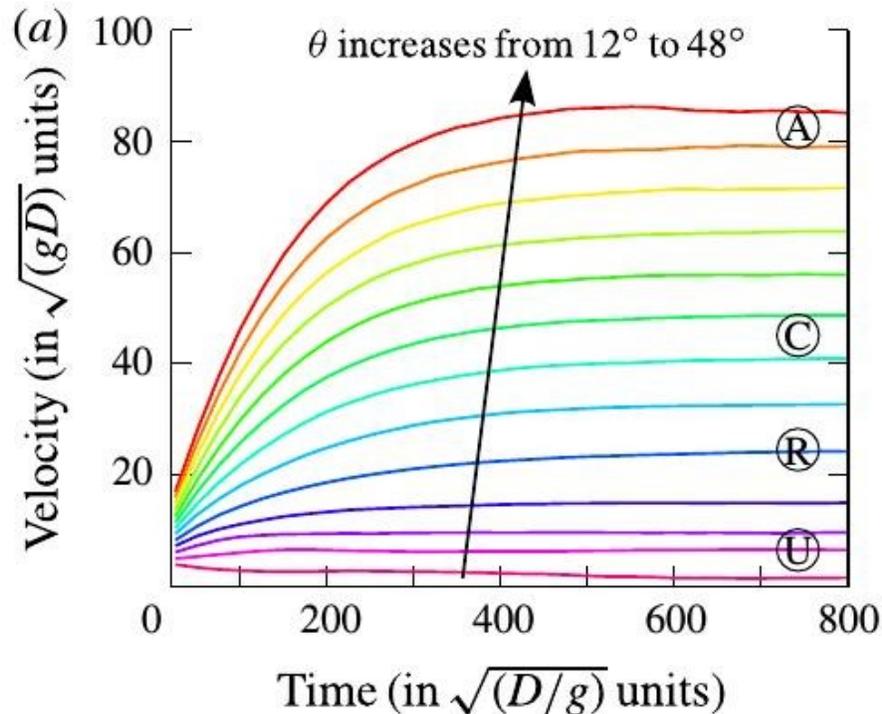
Nicolas Brodu<sup>1,†</sup>, Renaud Delannay<sup>1</sup>, Alexandre Valance<sup>1</sup> and Patrick Richard<sup>2</sup>

# Confined geometry: Steady state

-Stationary flows at arbitrary high inclination angles

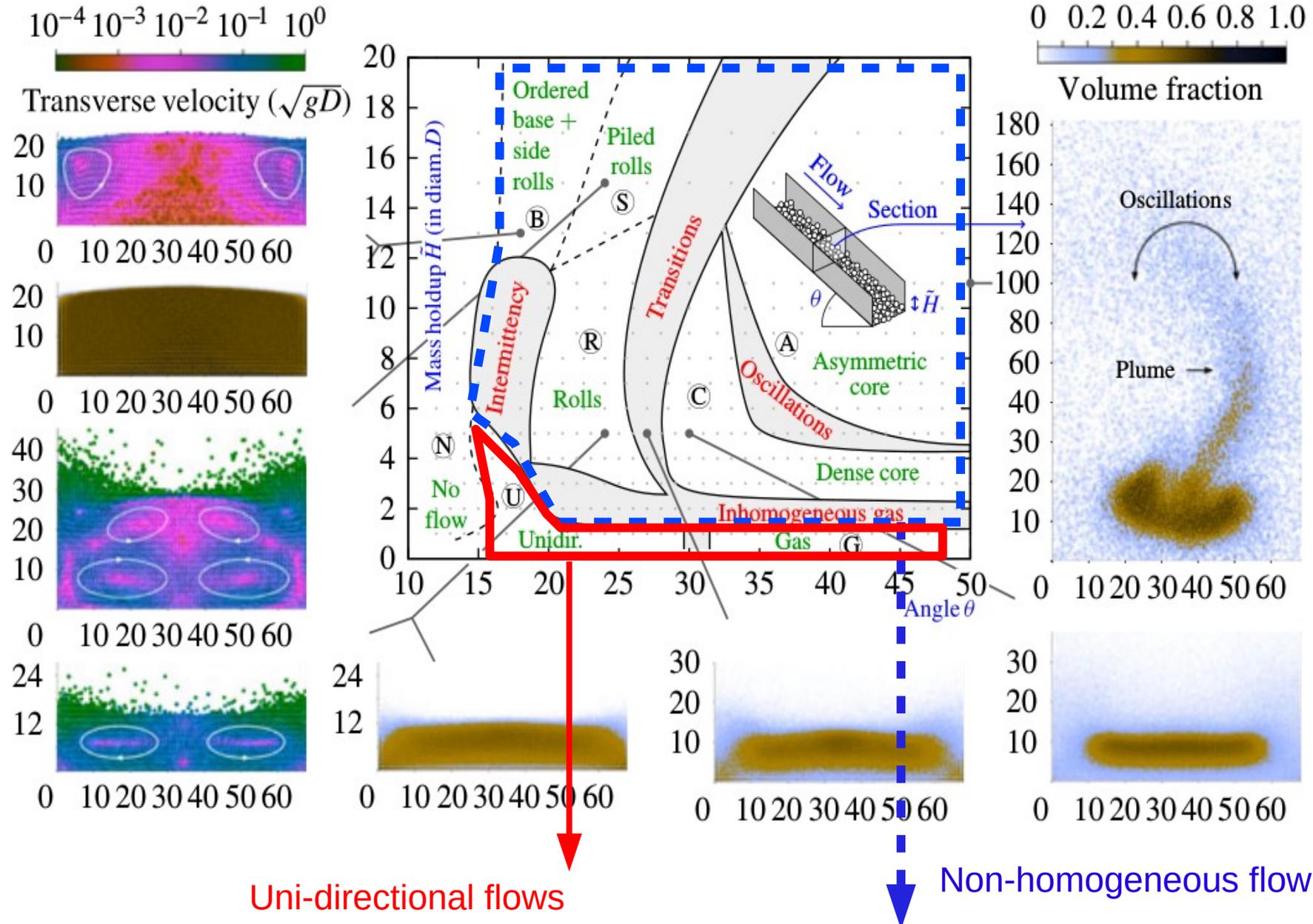
-Lateral friction allows to achieve a force balance at any angles :

$$\tan \theta = \mu_{base} + \mu_{wall} \frac{Z}{W} \Rightarrow \text{The height } Z \text{ of the flow self adjusts}$$



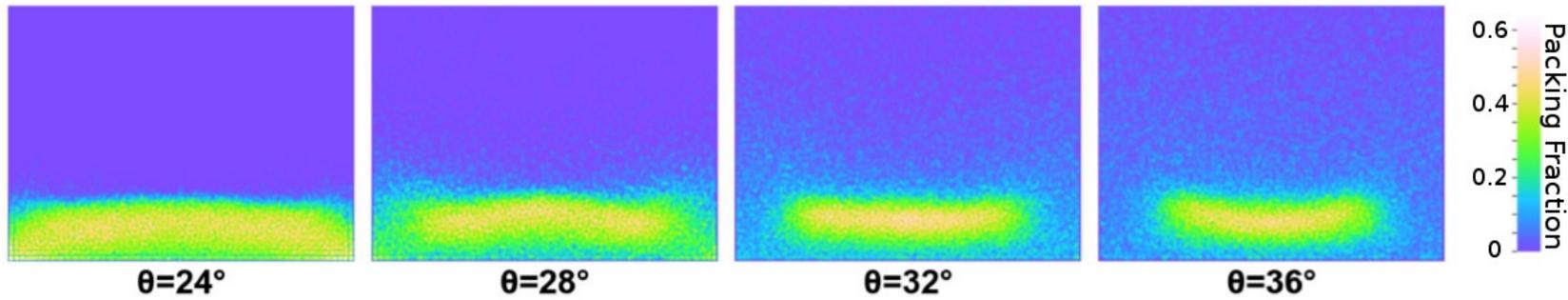
**W=68D (flow width)**  
**H=5D (flow height)**

# Confined geometry: Steady states



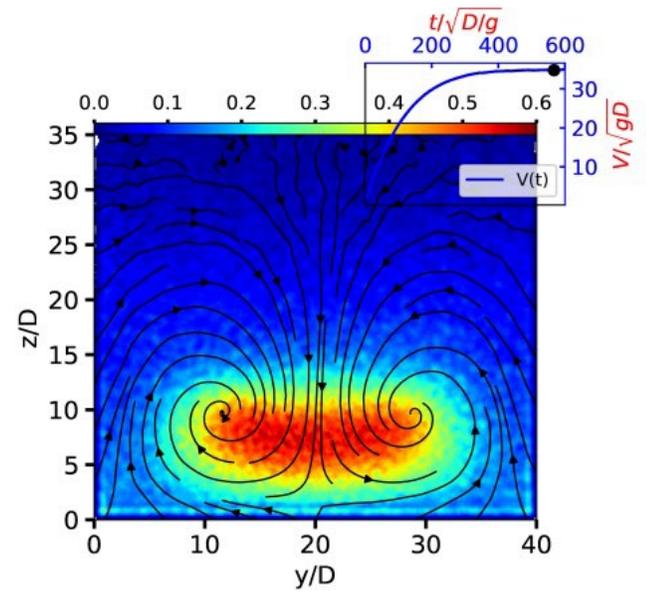
# Confined geometry: The supported flow regime

Packing fraction map: increasing angle (H=5D and W=68D)



(Brodu et al. JFM 2015)

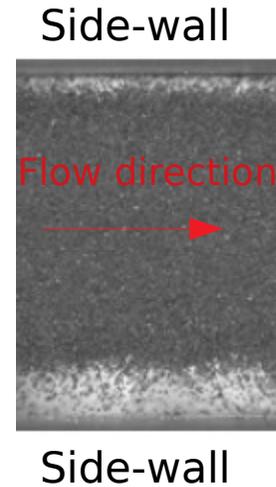
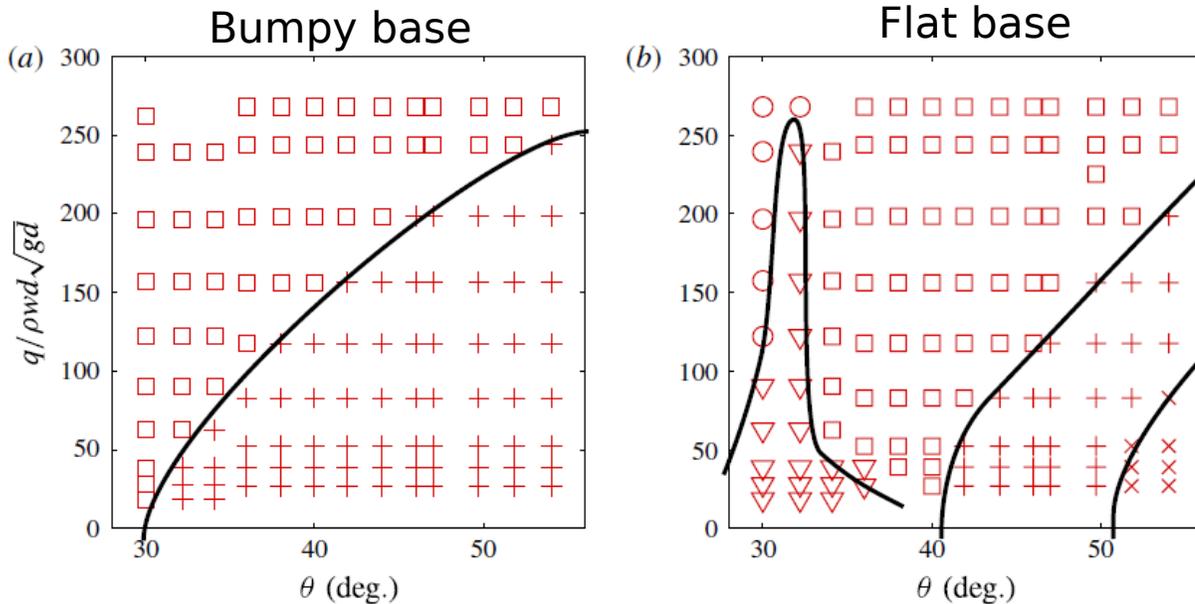
Secondary flows:



# Confined geometry: Experiments

Holyoake & McElwaine experiments (JFM 2012):

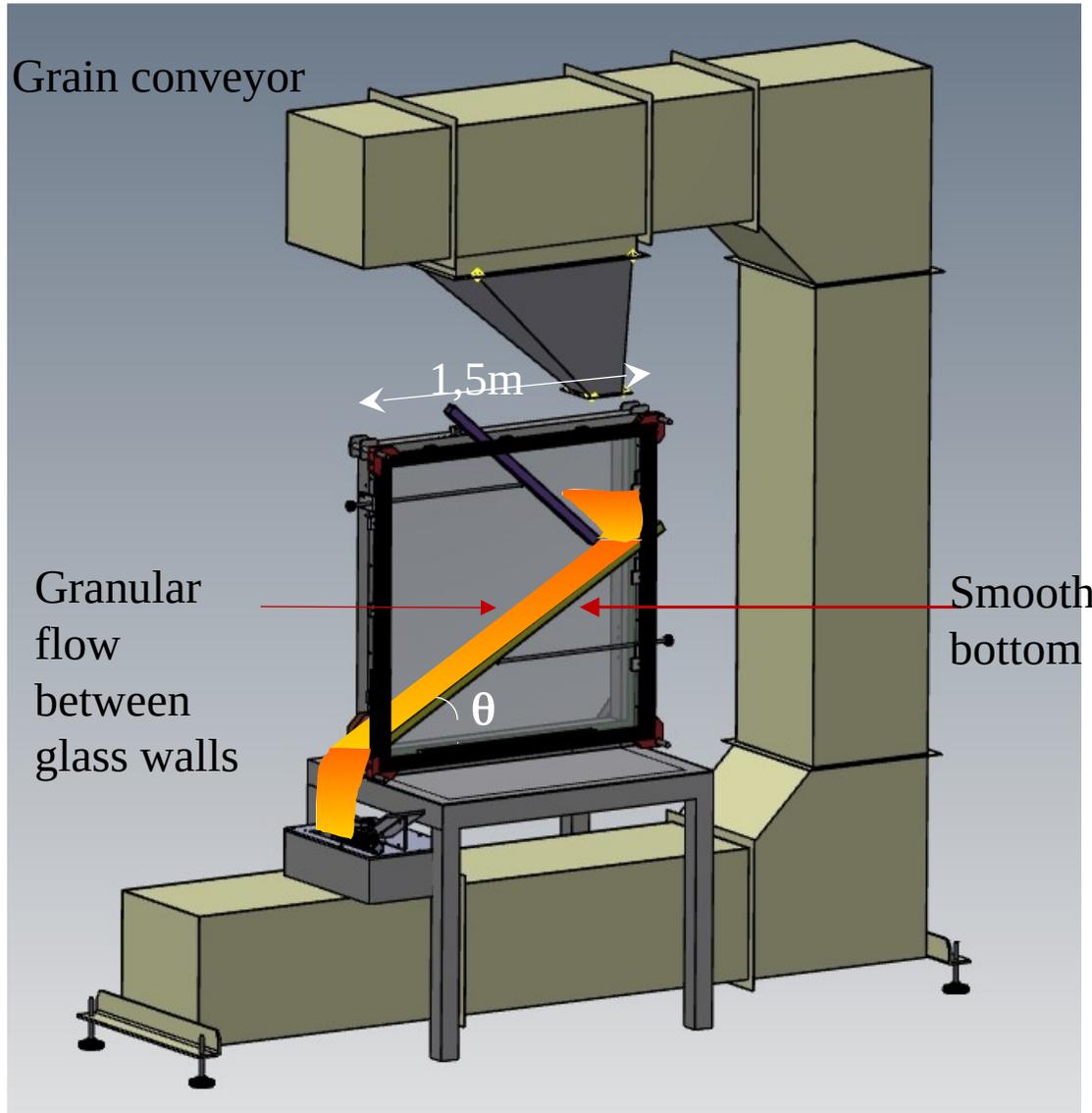
- Long chute with sidewalls
- Important mass flow rate
- Steep slopes



Top view: Evidence of existence of supported flows

( ) Constant velocity flows (□) accelerating, dense flows;  
 (+) flows with separation at walls; (x) low-density flows; (○) superstable heap formation

# Chute flow experiments in Rennes



## Main :

- High flow rate
- steady state flow obtained thanks to the conveyor belt

## Control parameters :

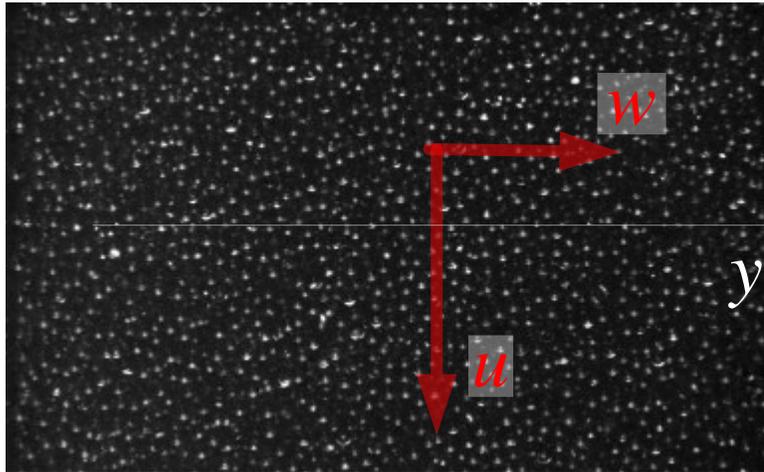
- Flow rate  $Q$
- Inclinaison angle  $\theta$
- gap width  $W$

## Particles :

- glass beads
- diamètre  $0.5 \text{ mm} < d < 2 \text{ mm}$
- densité  $\rho = 2500 \text{ kg/m}^3$

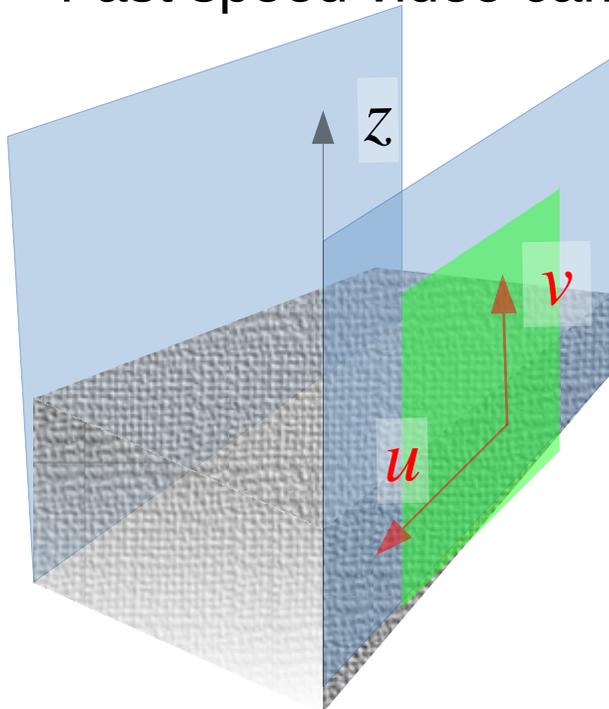
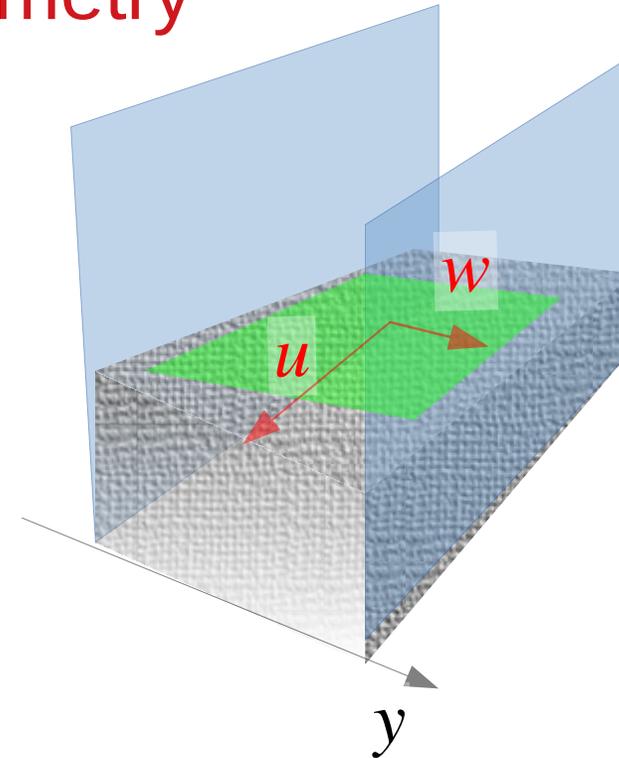


# Particle tracking velocimetry

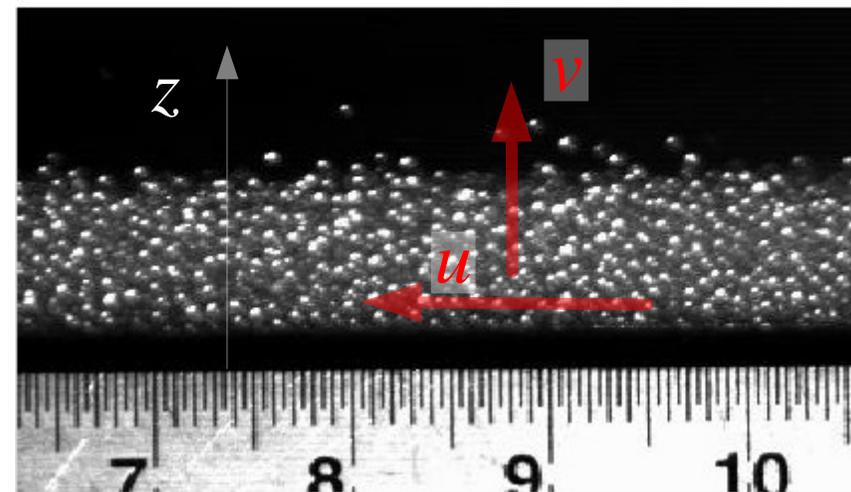


Fast speed video camera : 10 000 frames/s !

Top view ←

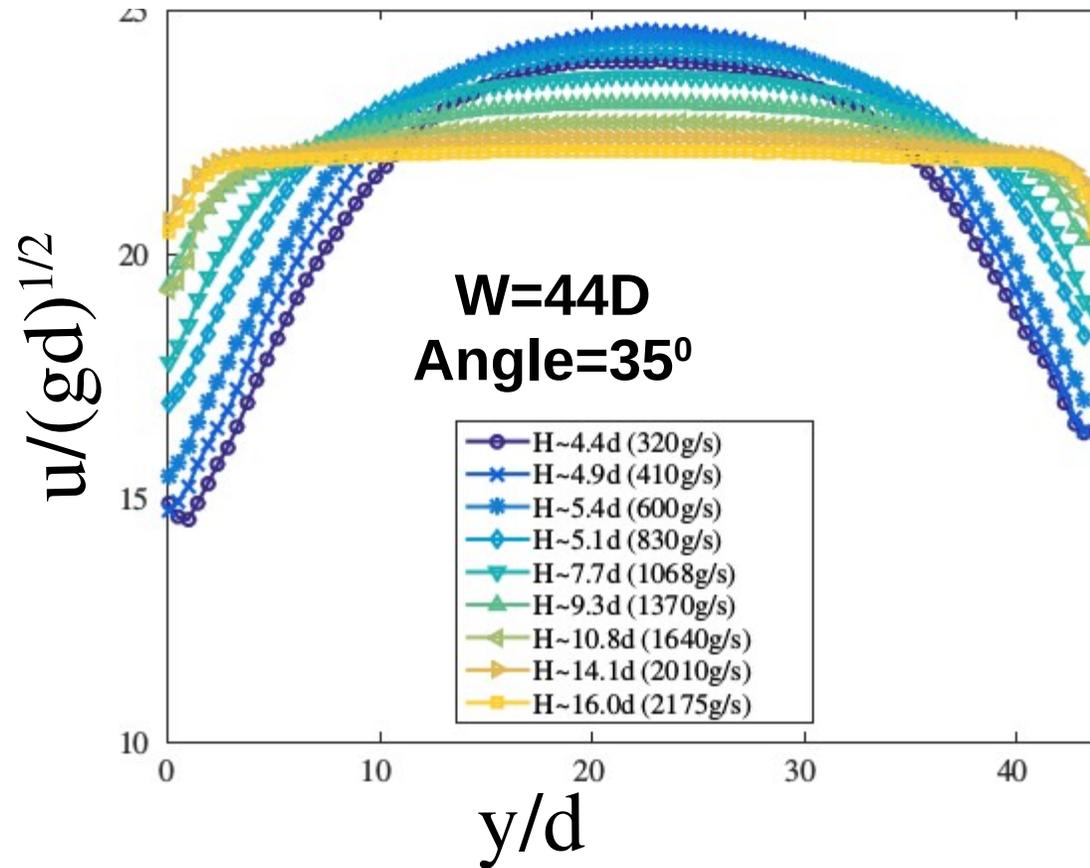
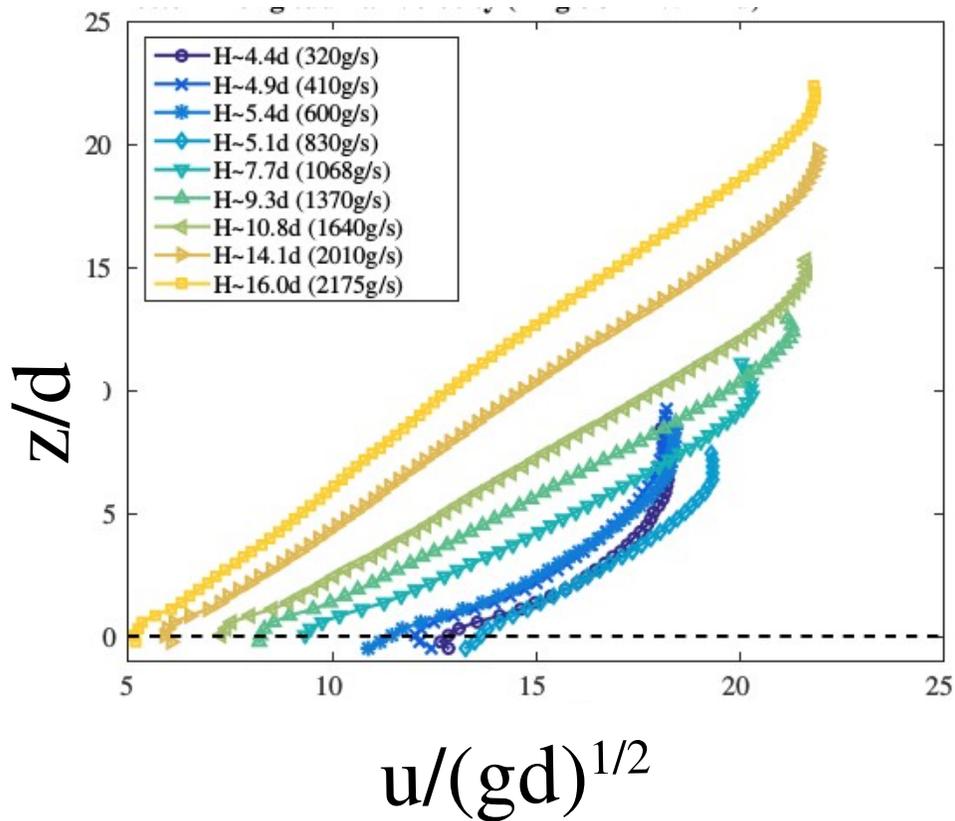


Side view →



# Profiles of the longitudinal velocity

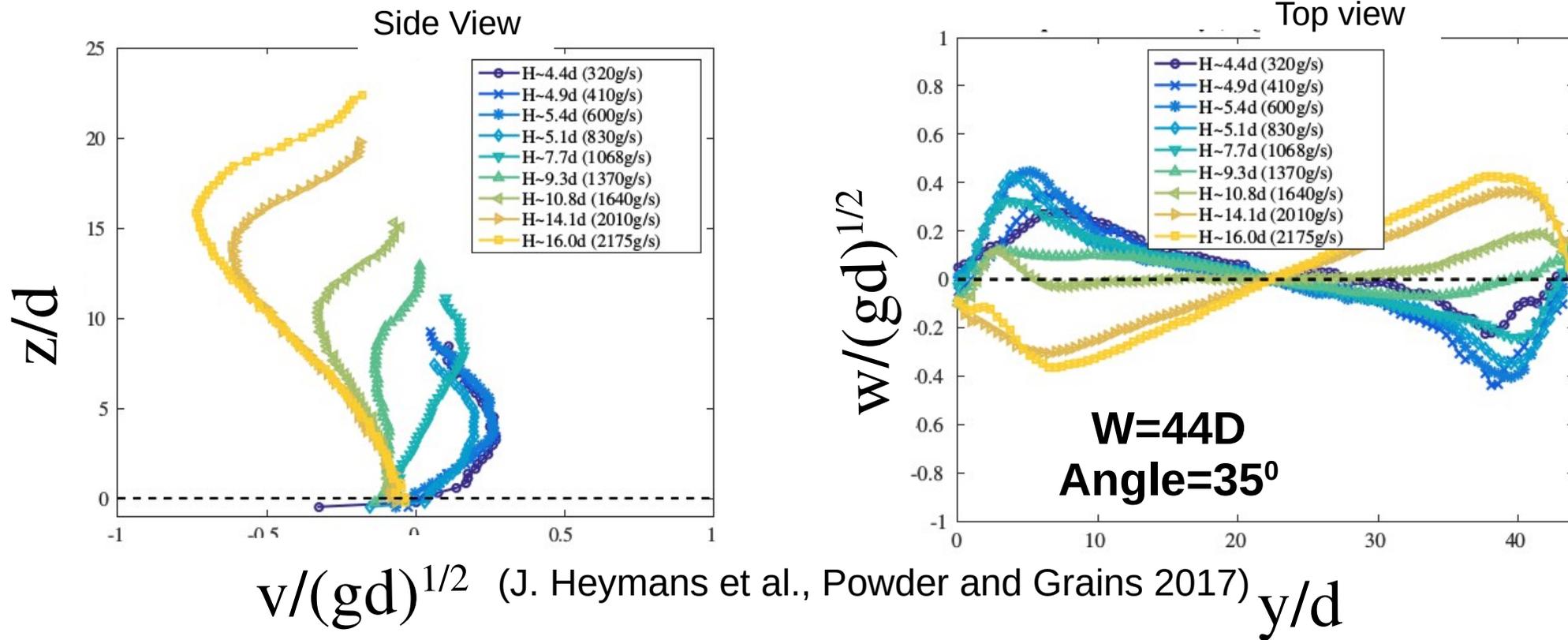
Top view →



← Side View

(J. Heymans et al., Powder and Grains 2017)

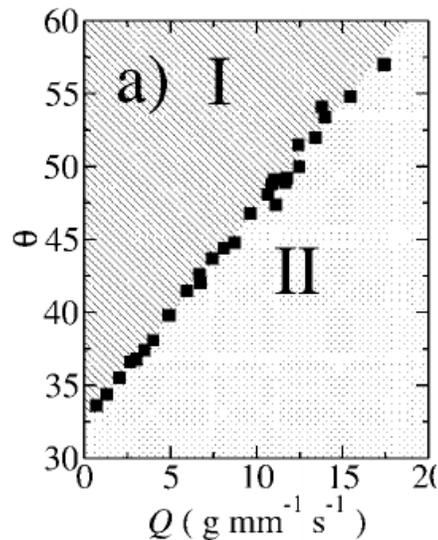
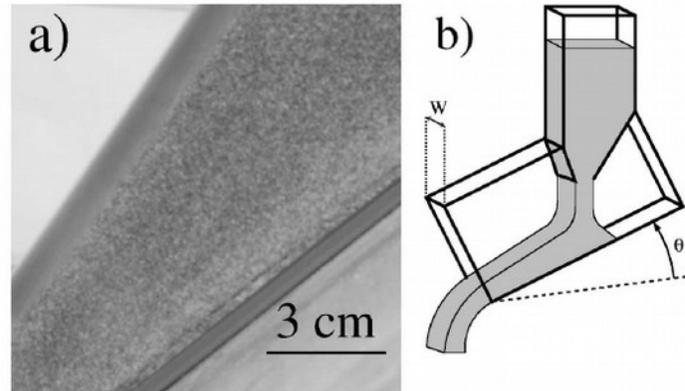
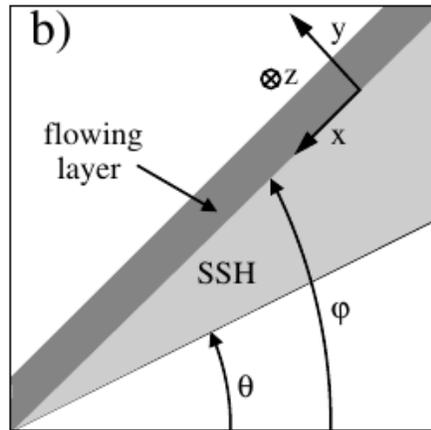
# Profiles of vertical velocities



Presence of a pair of longitudinal vortices:  
The direction of rotation is reversed at high flow rates

# Confined geometry: flow over an erodible bed

Transition at large mass flow rates : A Sidewall Stabilized Heap (SSH) forms as shown by Taberlet et al. experiments (PRL 2003)



For a given inclination angle the transition occurs above a critical flow  $Q_c(\theta)$ :

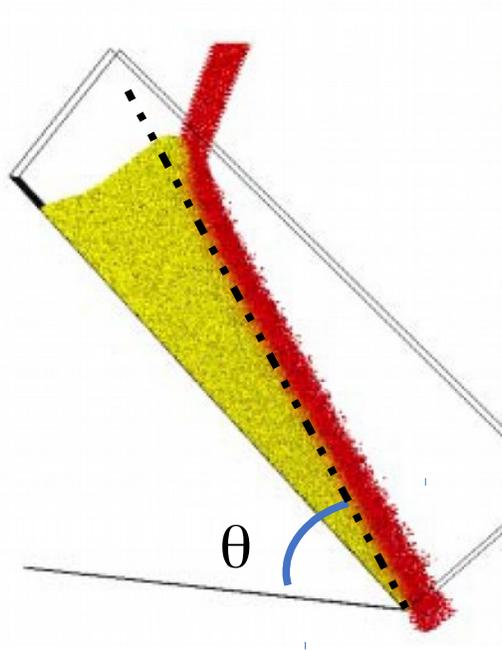
Regime I :  $Q < Q_c(\theta)$  (Flows over the rigid base)

Regime II:  $Q > Q_c(\theta)$  (Flows over a heap )

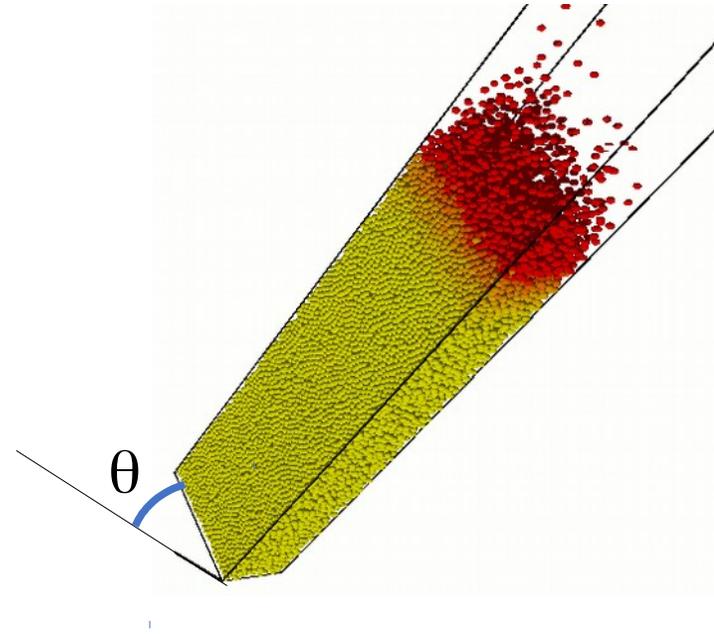
# Confined geometry: flows over an erodible bed

Simulations are able to reproduce what is seen in the experiments:

An open cell with a prescribed mass flow  $Q$  rate as in the experiments:  
The angle  $\Theta$  is self-adjusted by the system



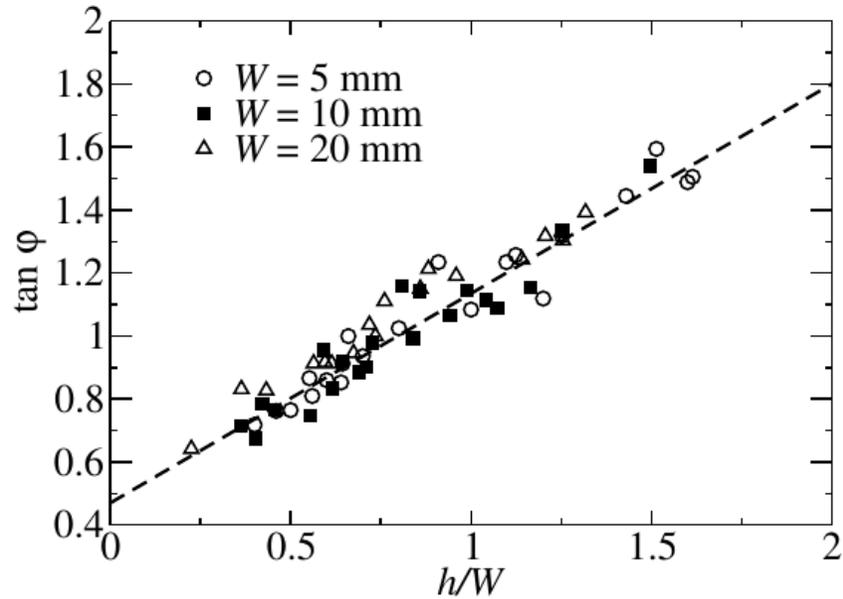
A cell with periodic boundary in the stream-wise direction and a prescribed inclination angle  $\Theta$ :  
 $Q$  is self-adjusted by the system



# SSH flows: Experiments vs Simulations

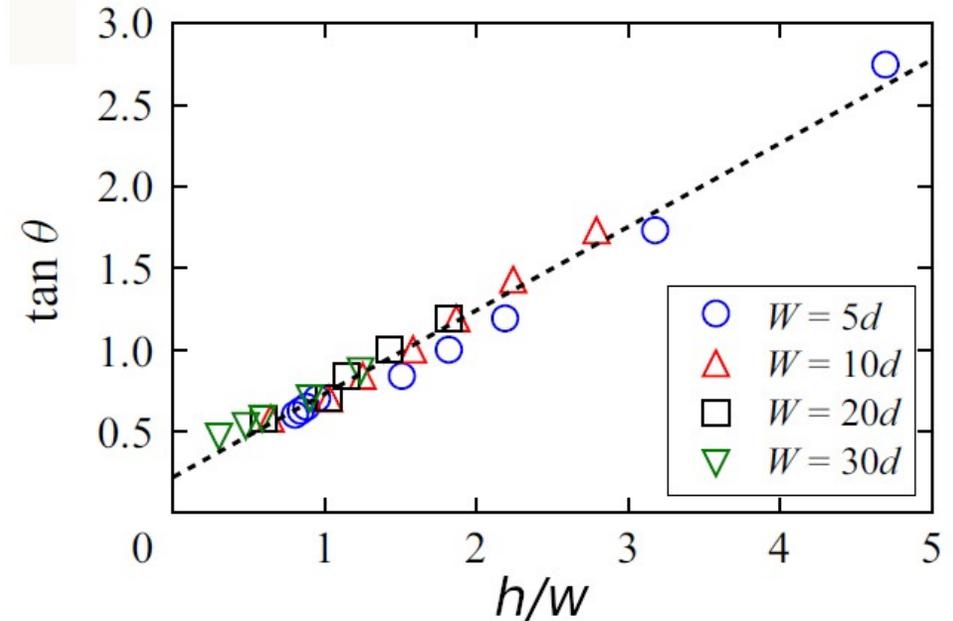
Experiments:

(N. Taberlet et al., PRL 2003)



Simulations

(P. Richard et al., JFM 2022)



SSH Equation

$$\tan \theta = \mu_{b,h} + \mu_{w,h} \frac{h}{W}$$

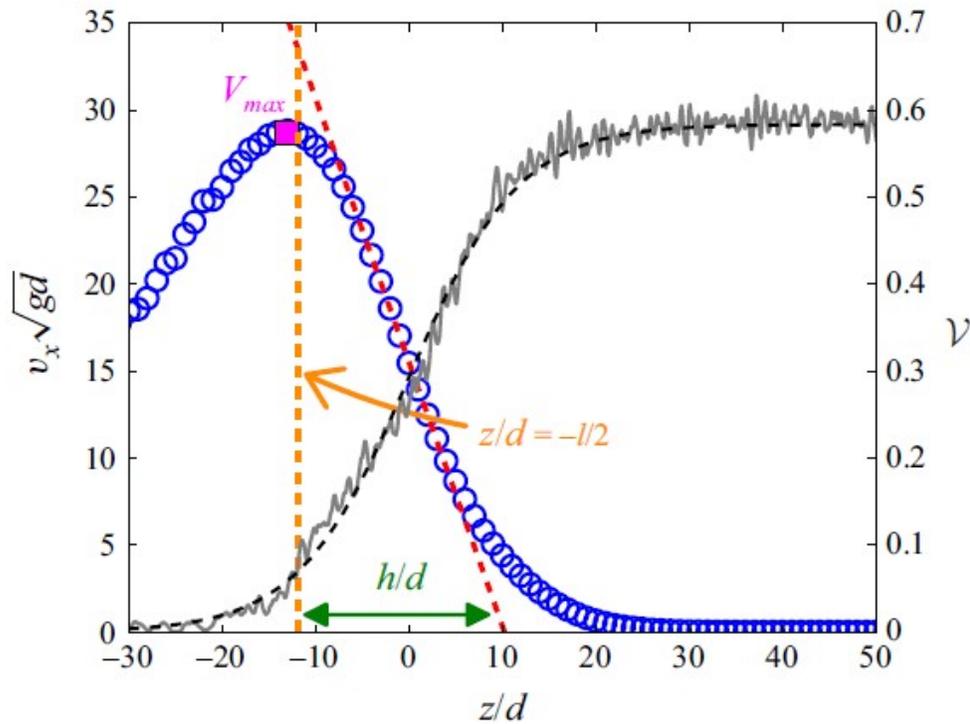
basal friction Sidewall friction

$h$ : flowing layer height

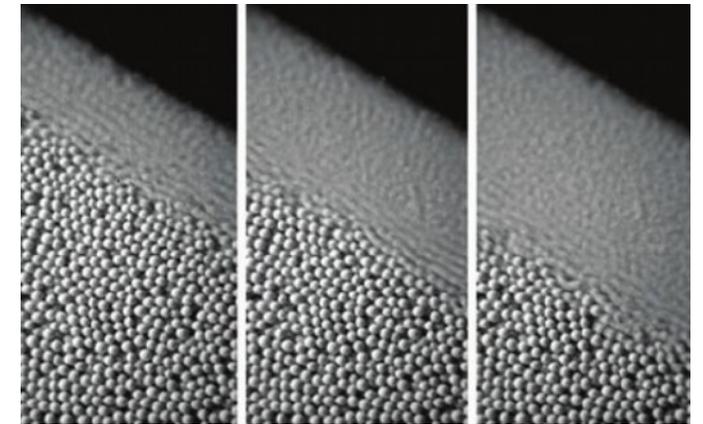
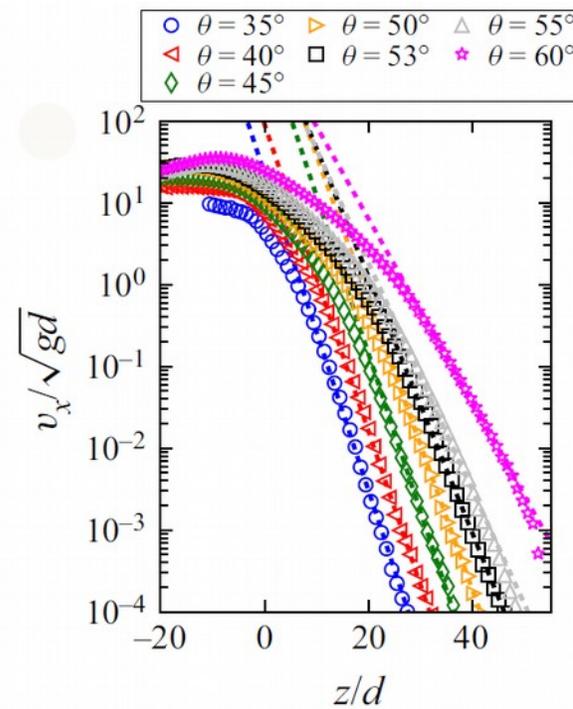
$W$ : flow width

# SSH flows: Simulations

Large variation of the volume fraction



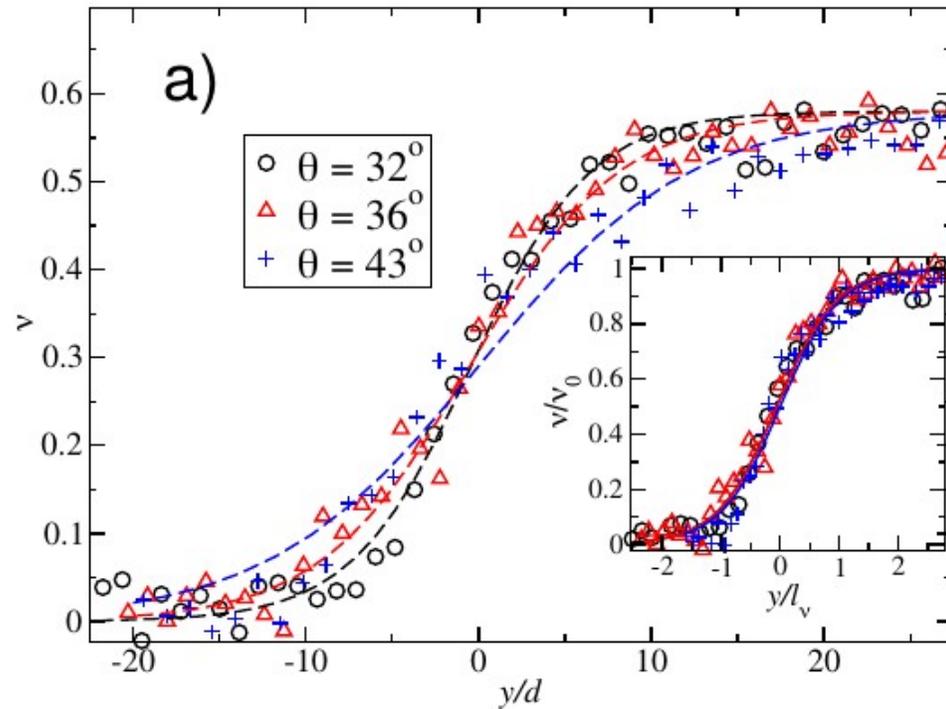
Creeping below the flowing layer



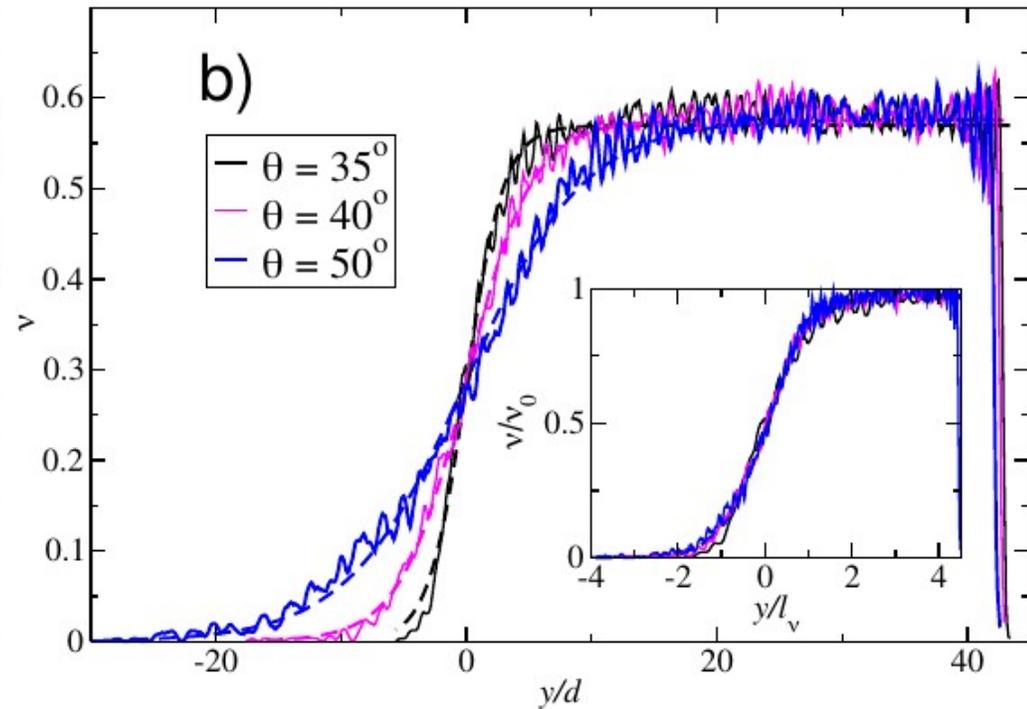
Komatsu et al. PRL 2001

# SSH flows: Experiments vs Simulations

Experiments:  $W=9d$



Simulations:  $W=10d$

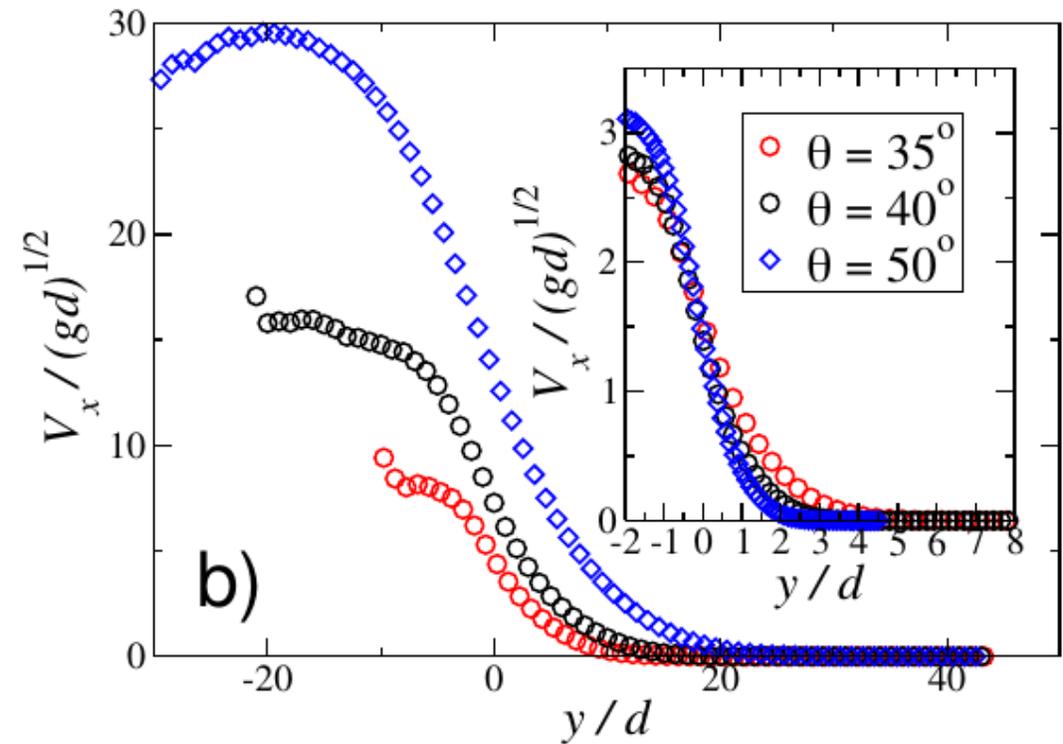
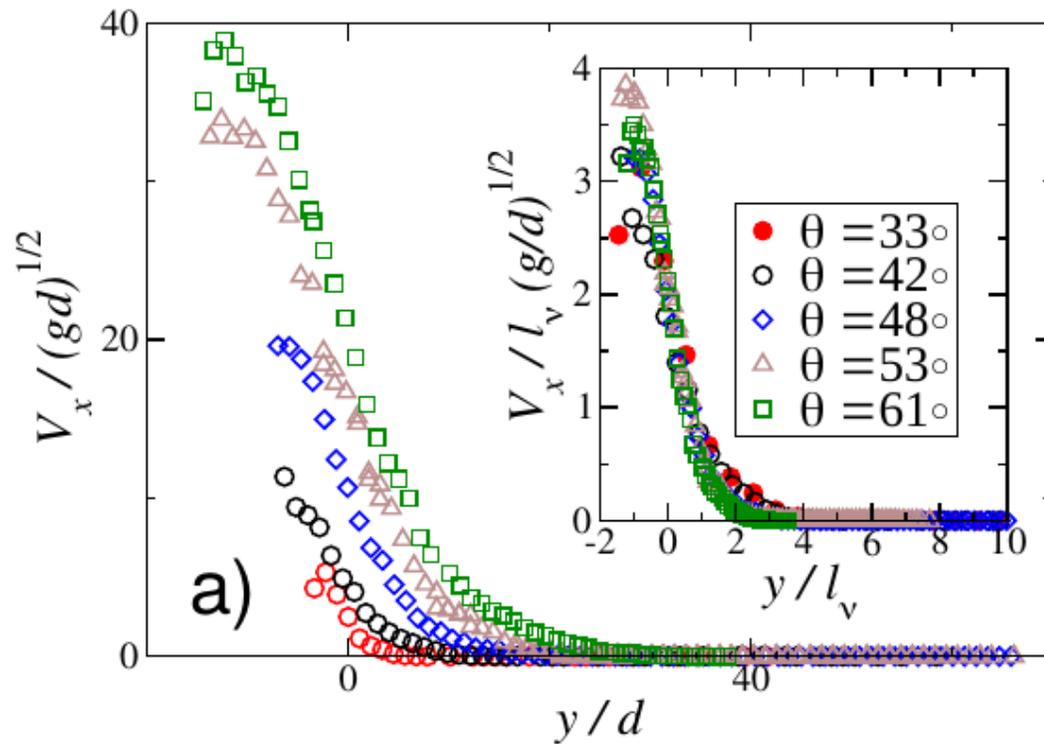


(P. Richard et al., AIP Conference Proceedings 2010)

# SSH flows: Experiments vs Simulations

Experiments:  $W=9d$

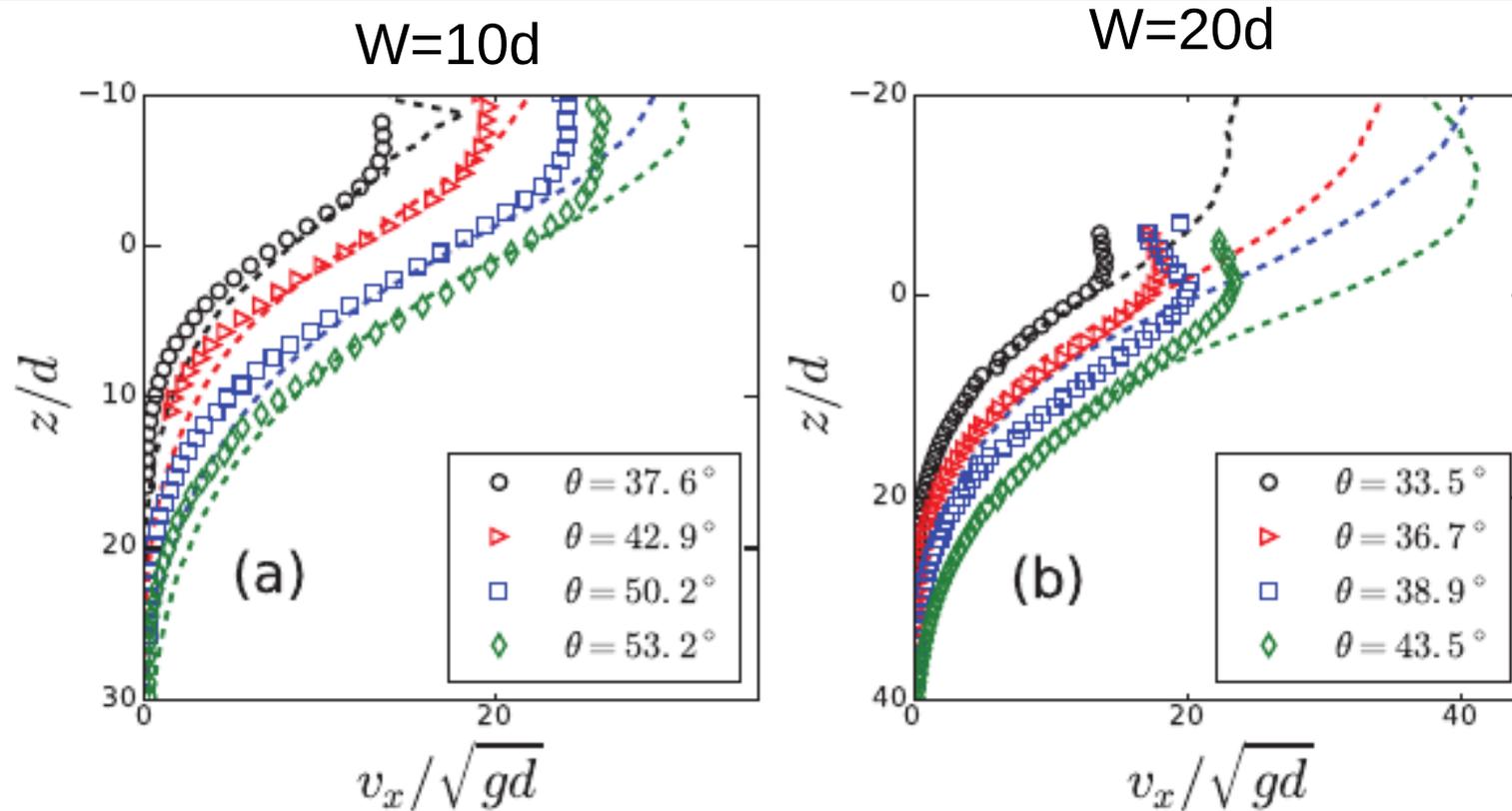
Simulations:  $W=10d$



(P. Richard et al, AIP Conference Proceedings 2010)

# SSH flows: Experiments vs Simulations

Quantitative agreement can be not achieved for any gap width  $W$  with the same set of the micro-mechanical parameters of the simulations



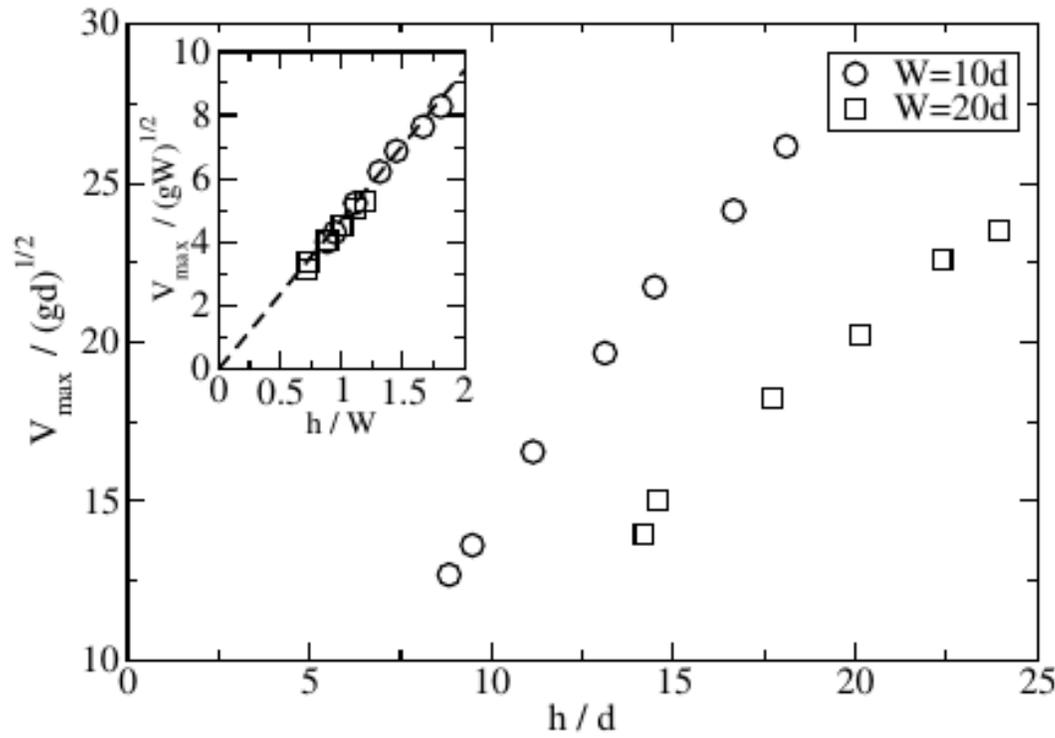
Experiments: symbols  
Simulations: dash lines

(P. Richard et al., Powder and Grains 2017)

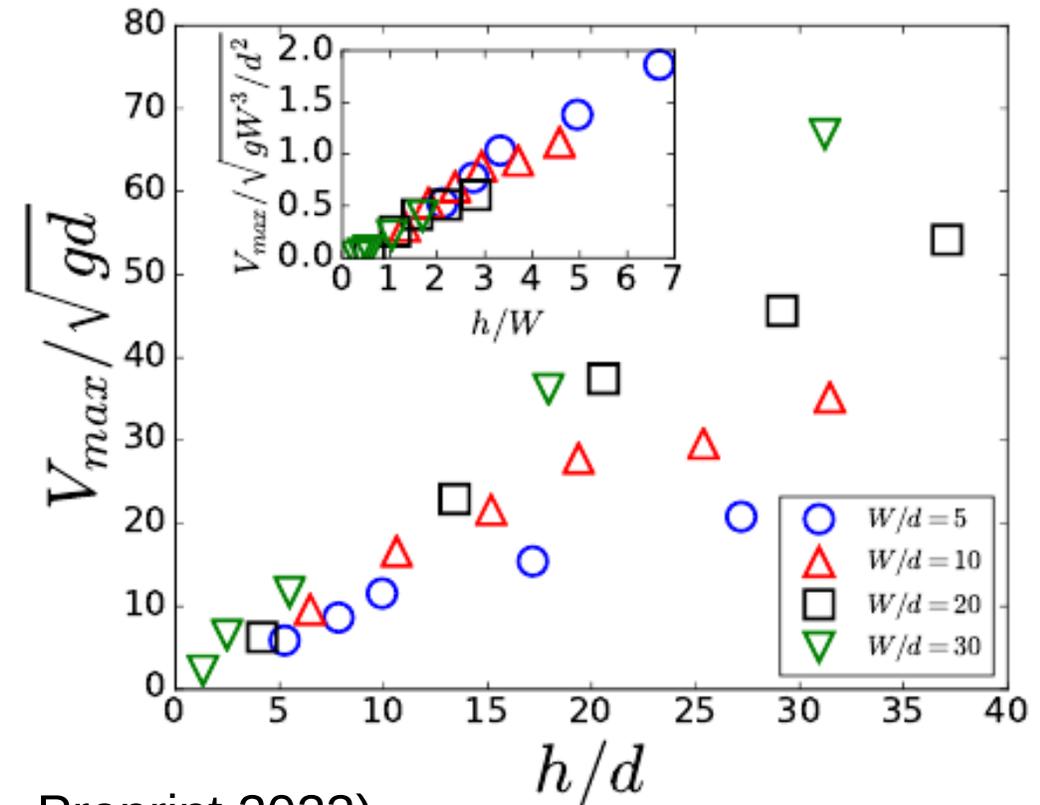
# SSH flows: Experiments vs Simulations

There is a surprising discrepancy between experiments and simulations concerning the velocity scaling with the gap width  $W$

Experiments:  $V_{\max} \approx (\tan \theta - \mu_b) (W/d)^{1/2}$



Simulations:  $V_{\max} \approx (\tan \theta - \mu_b) (W/d)^{3/2}$



(P. Richard et al., Preprint 2023)

# Conclusion

- Laterally bounded granular flows exhibit a rich variety of flow patterns in particular at high flow rate and on steep slopes
- The role of the boundaries and their influence on the flow patterns are not completely understood
- Numerical simulations based on discrete element methods are a good tool to predict the qualitative features of granular flows
- However, numerical simulations can dramatically fail for quantitative predictions