Influence of particle shape on the statics and dynamics of granular materials

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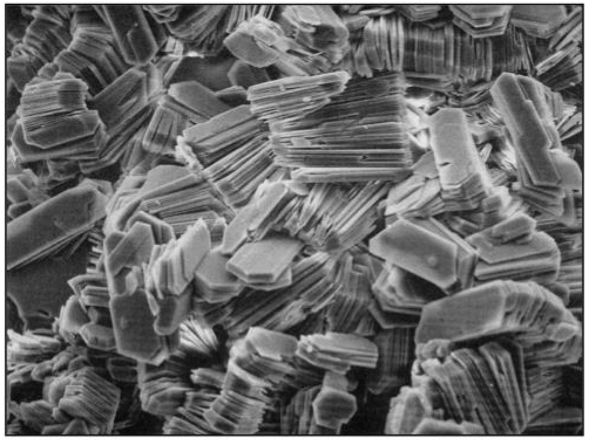
Introduction

Particle shape is a key element of the rich behavior of granular materials.



Particle shape strongly affects major properties of granular materials:

- Space-filling properties, shear strength, dilatancy, elastic moduli, cohesion, flow behavior…

Which particle-scale mechanisms are mostly involved by particle shape?

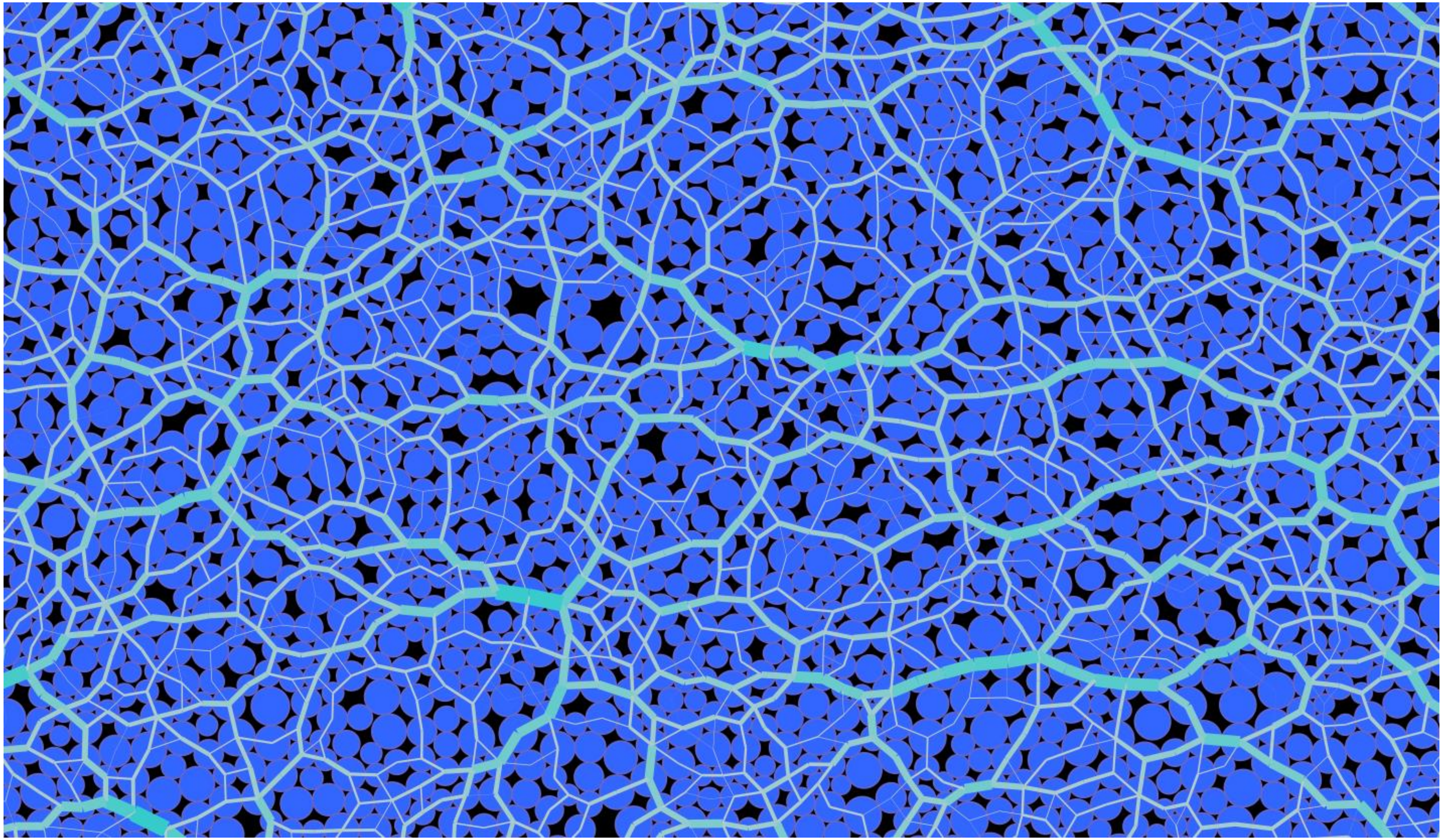
- Arching
- Contact and force networks
- Particle rotations
- Local ordering…

Related issues:

- What is the lowest-order shape parameter?
- Can the effect of particle shape be captured by particle interactions?
- What is the effect of shape polydispersity?
- How does particle shape impact energy dissipation?

Arching

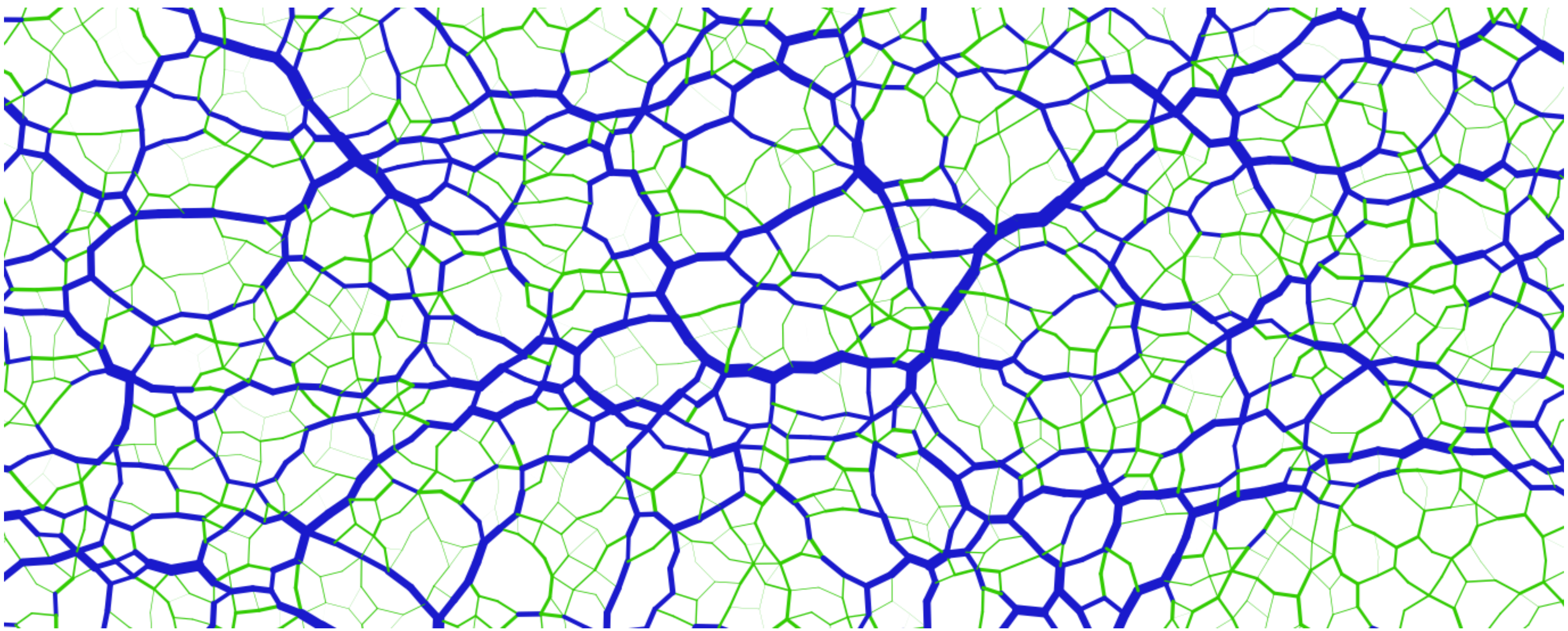
The arching effect refers to local force-bearing arrangements of particles in a granular packing.



Tensorial arching effect

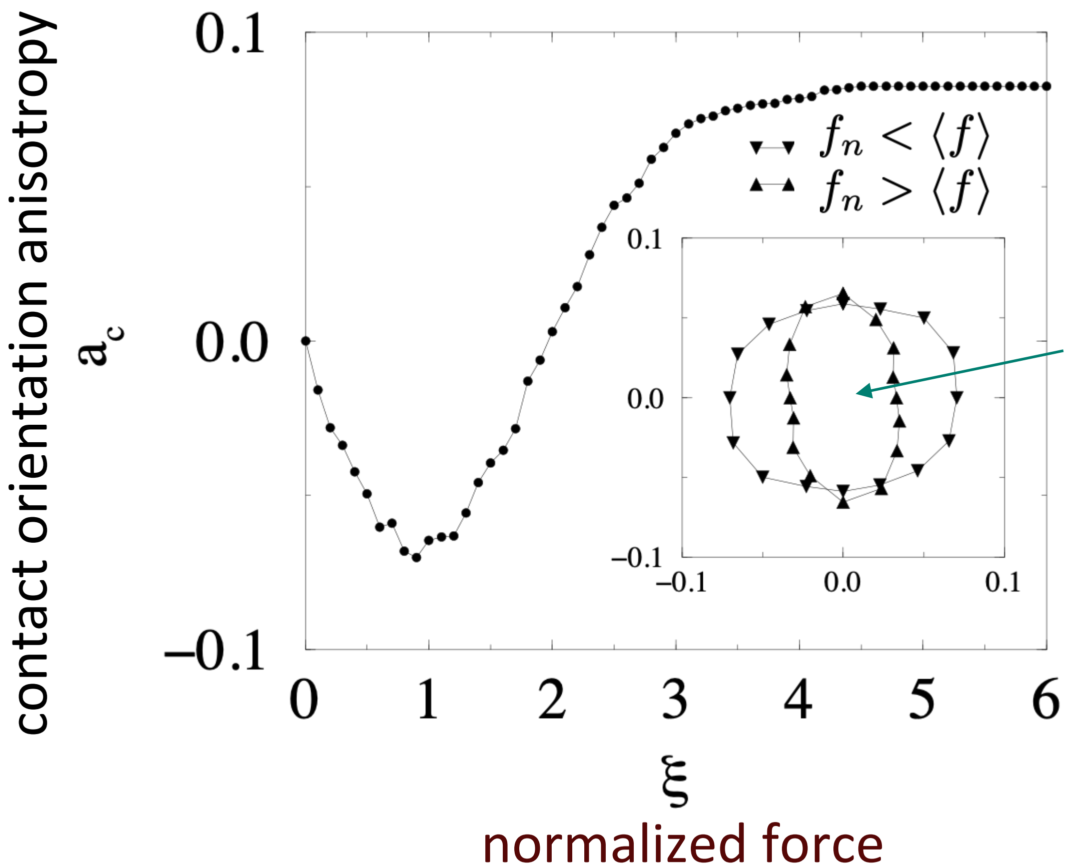
Spontaneous breakage of the symmetry of the force network and creation of two sub-networks.

Strong force chains are supported by weak contacts in analogy to flying buttresses supporting an arch.



Radjai, D. Wolf, M. Jean, J.J. Moreau, PRL 80 (1998)

Example: anisotropy of contact orientations

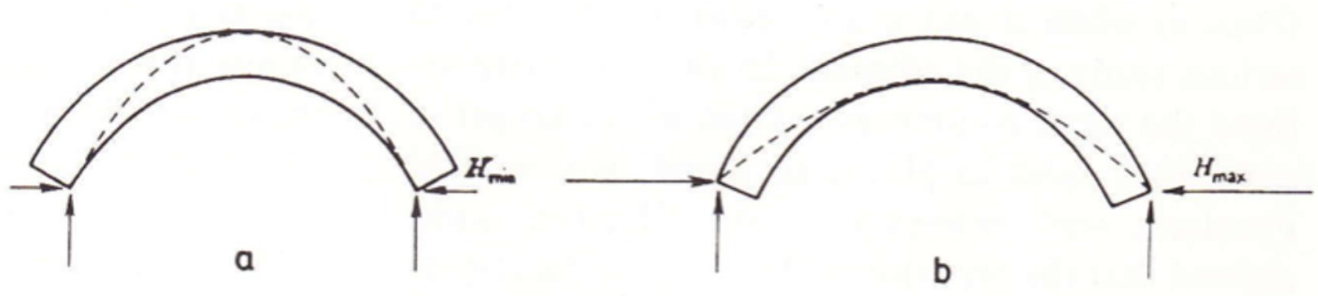


Polar diagram of the distributions of contact normals for forces below the mean force (weak) and above the mean force (strong)

Weak contacts tend to be along the minor principal stress direction.

$\Rightarrow$ Sequences of strong forces are propped by side-wise weak forces.

Analogy with structural arches



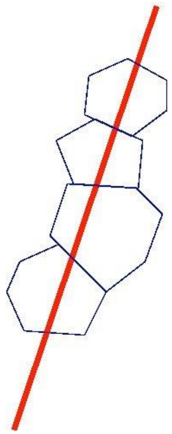
Thrust lines according to Coulomb in two limit states.

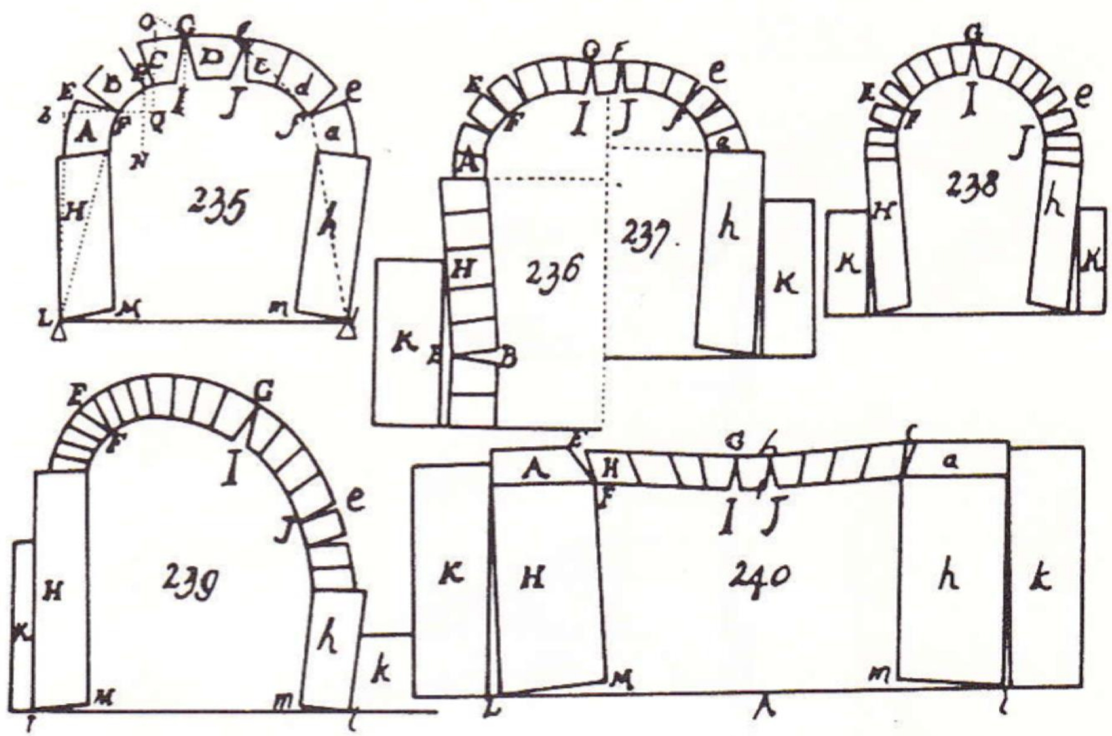
With spherical particles, the only possible arch is an inverse catenary shape. With blocks, there are more degrees of freedom.

Catenary

arches made of blocks

Force lines

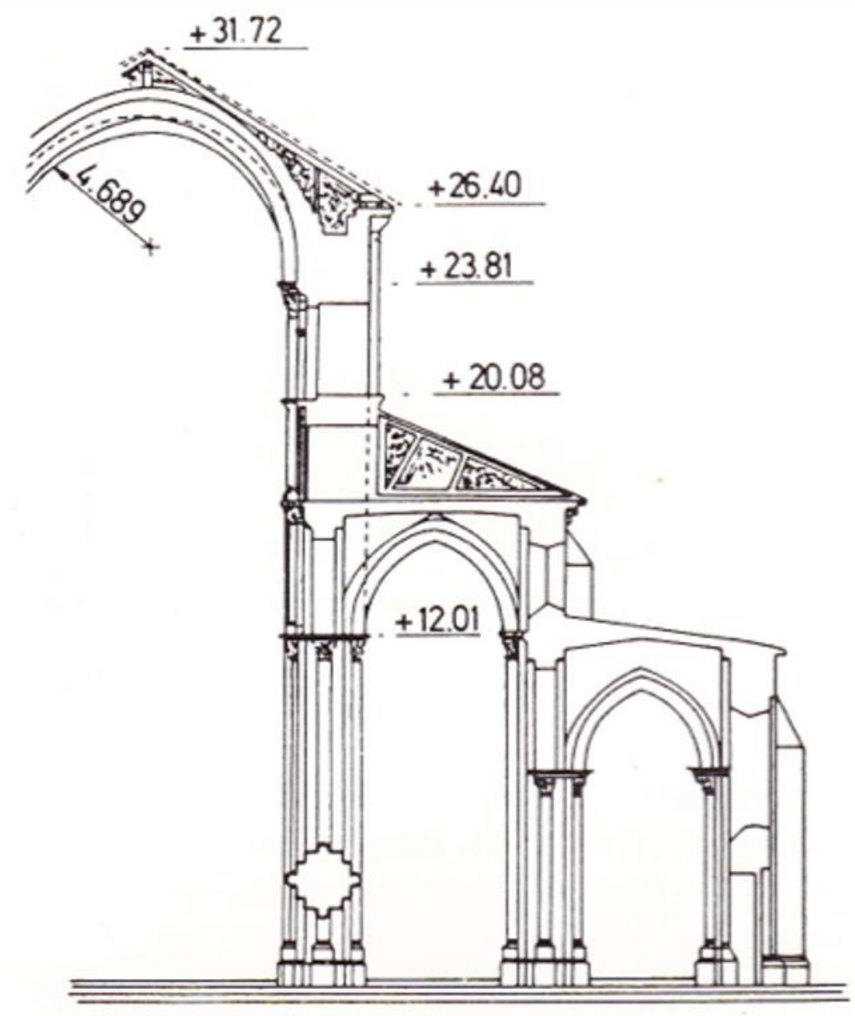




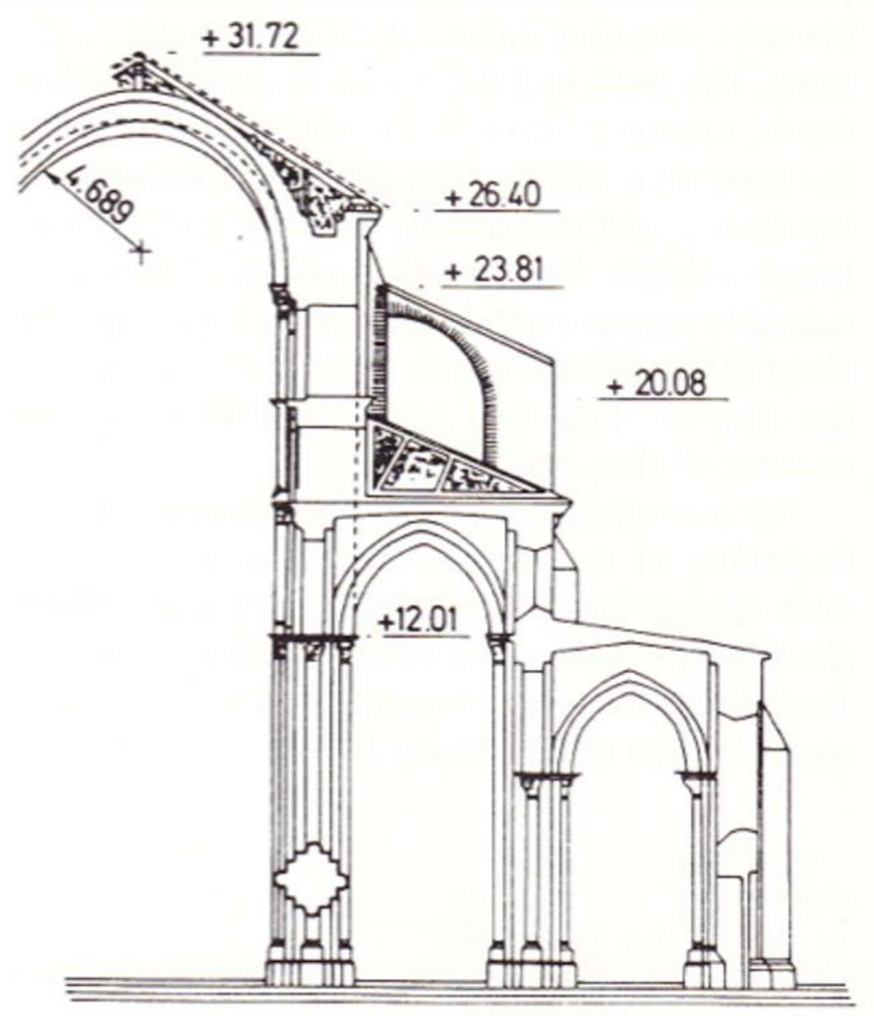
Experiments of Augustin Danyzy (1698-1777) presented in 1732 in Montpellier

Leonardo da Vinci: "A vault comprises two weaknesses, which both work for its collapse but which can be transformed into a strength."

"Down to Earth", Jean Kerisel, Balkema 1991



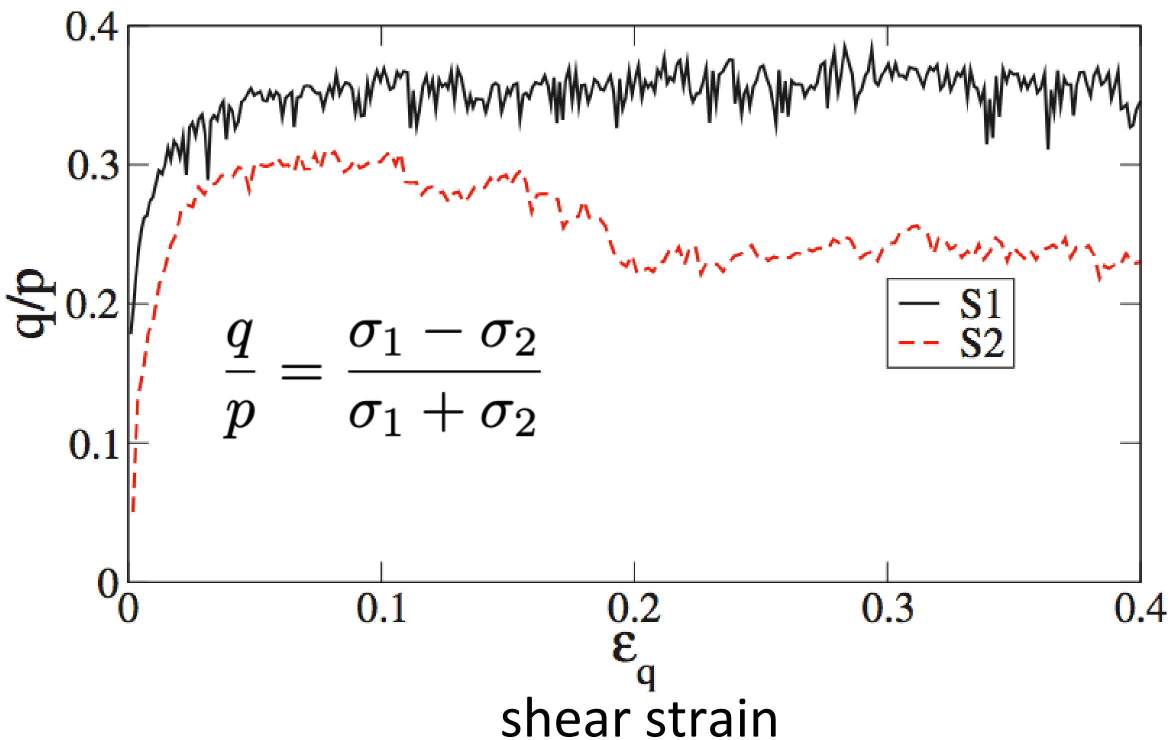
Cluny III before collapse in 1125



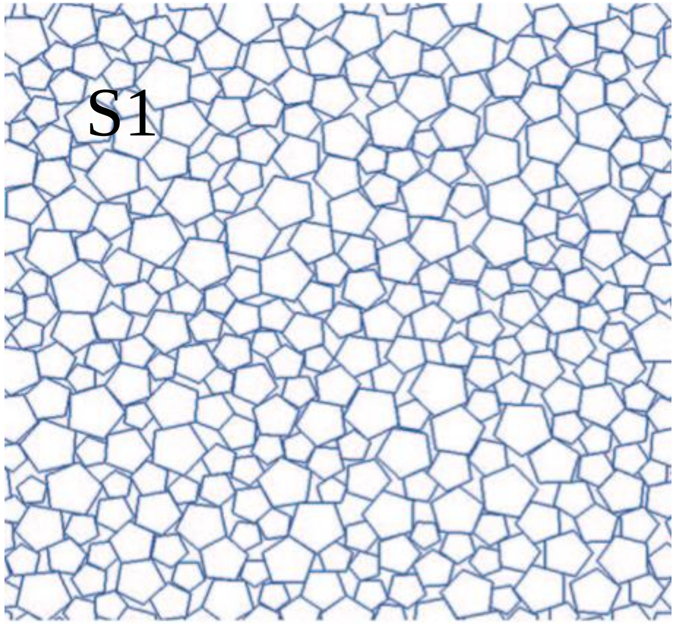
Cluny III after repairs in 1130

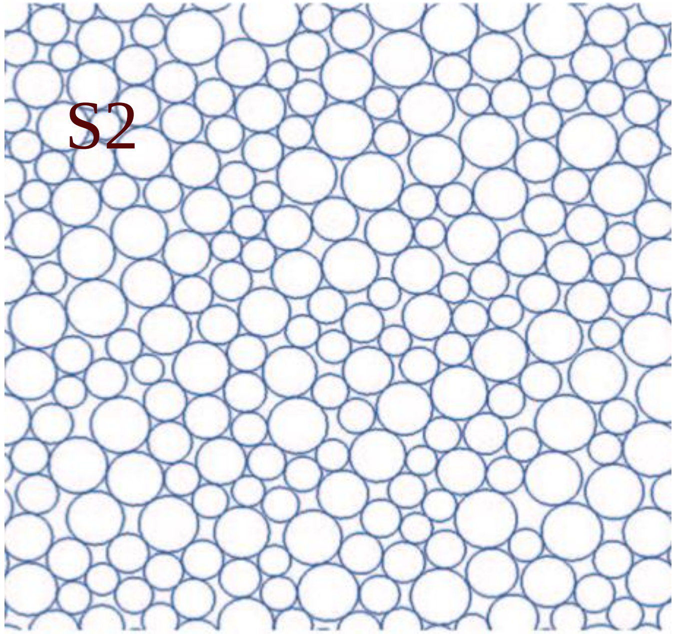
Effect of particle shape on arching

Polygon packing and disk packing simulated with exactly the same numbers of particles, particle size distributions and friction coefficient.

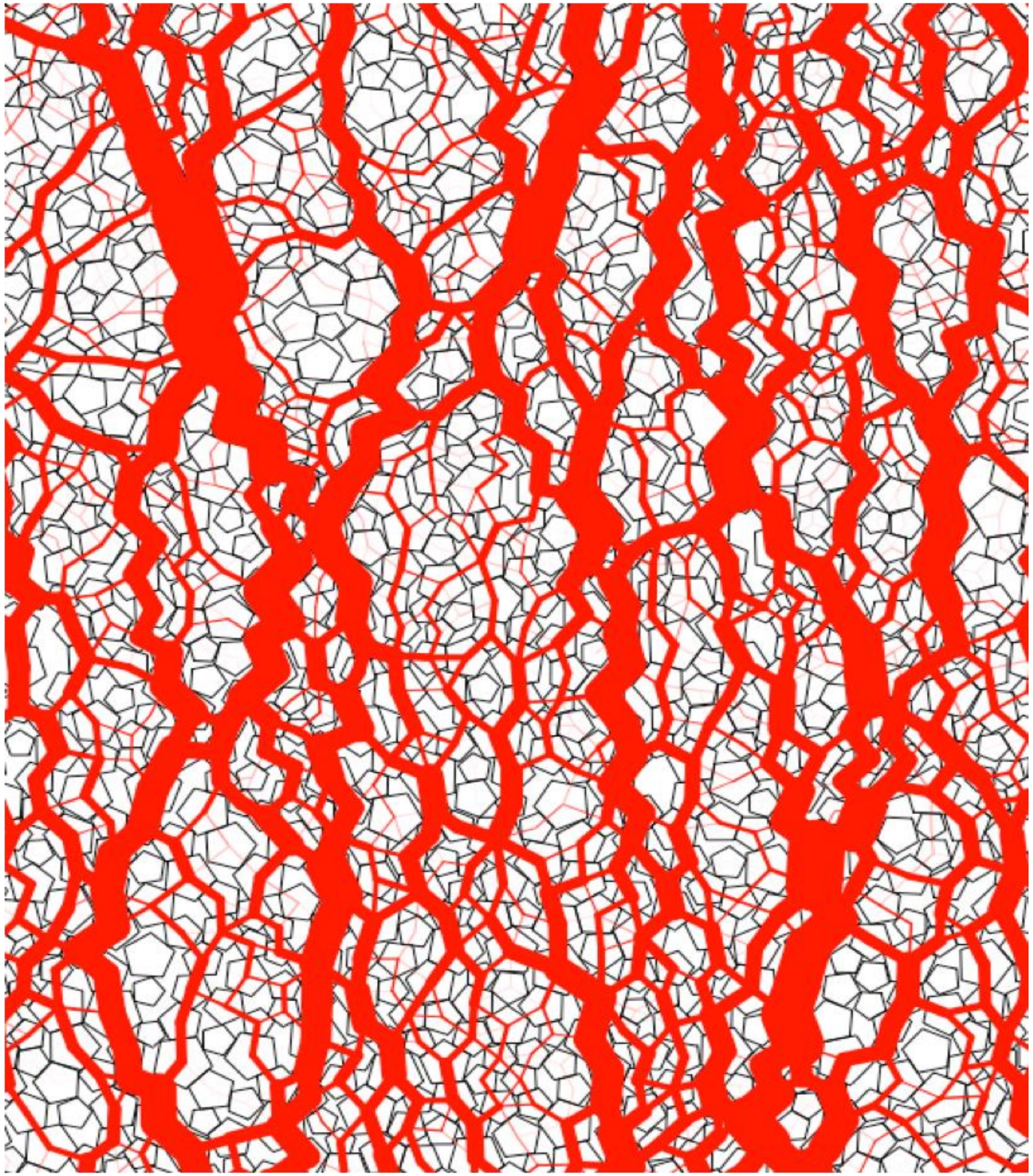


Azéma, Radjai, Peyroux, Saussine, PRE 76 (2007)

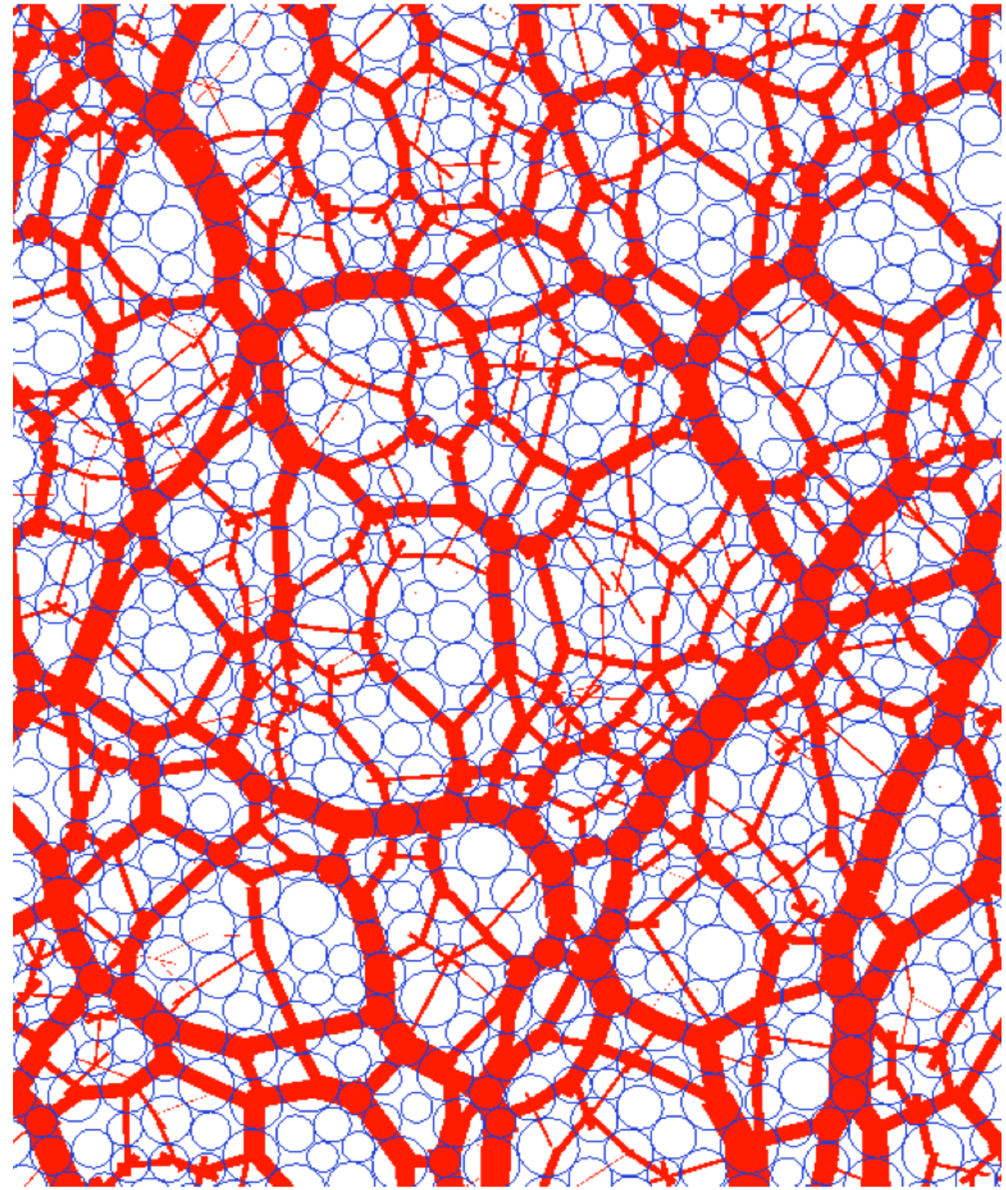




Polygons

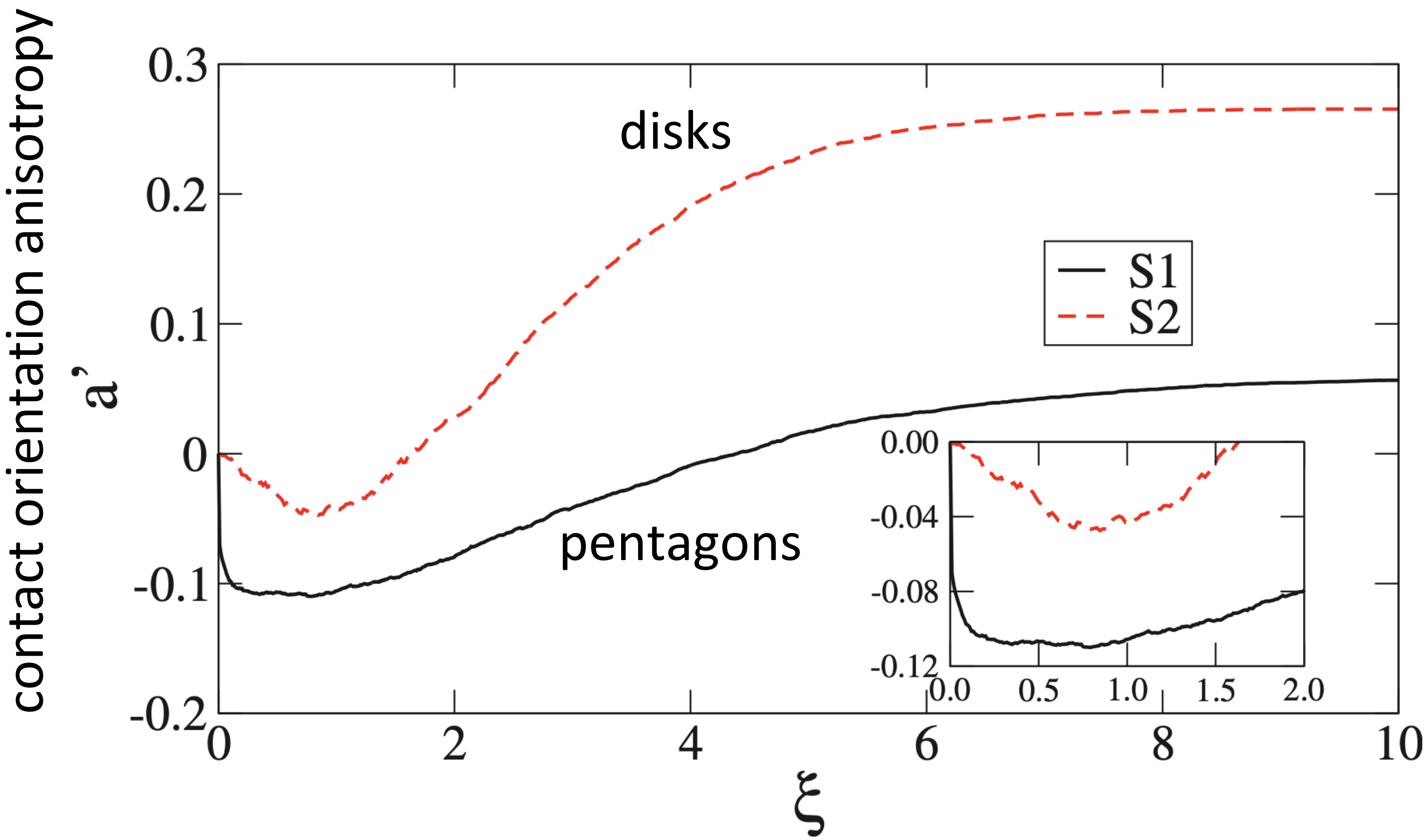


Disks



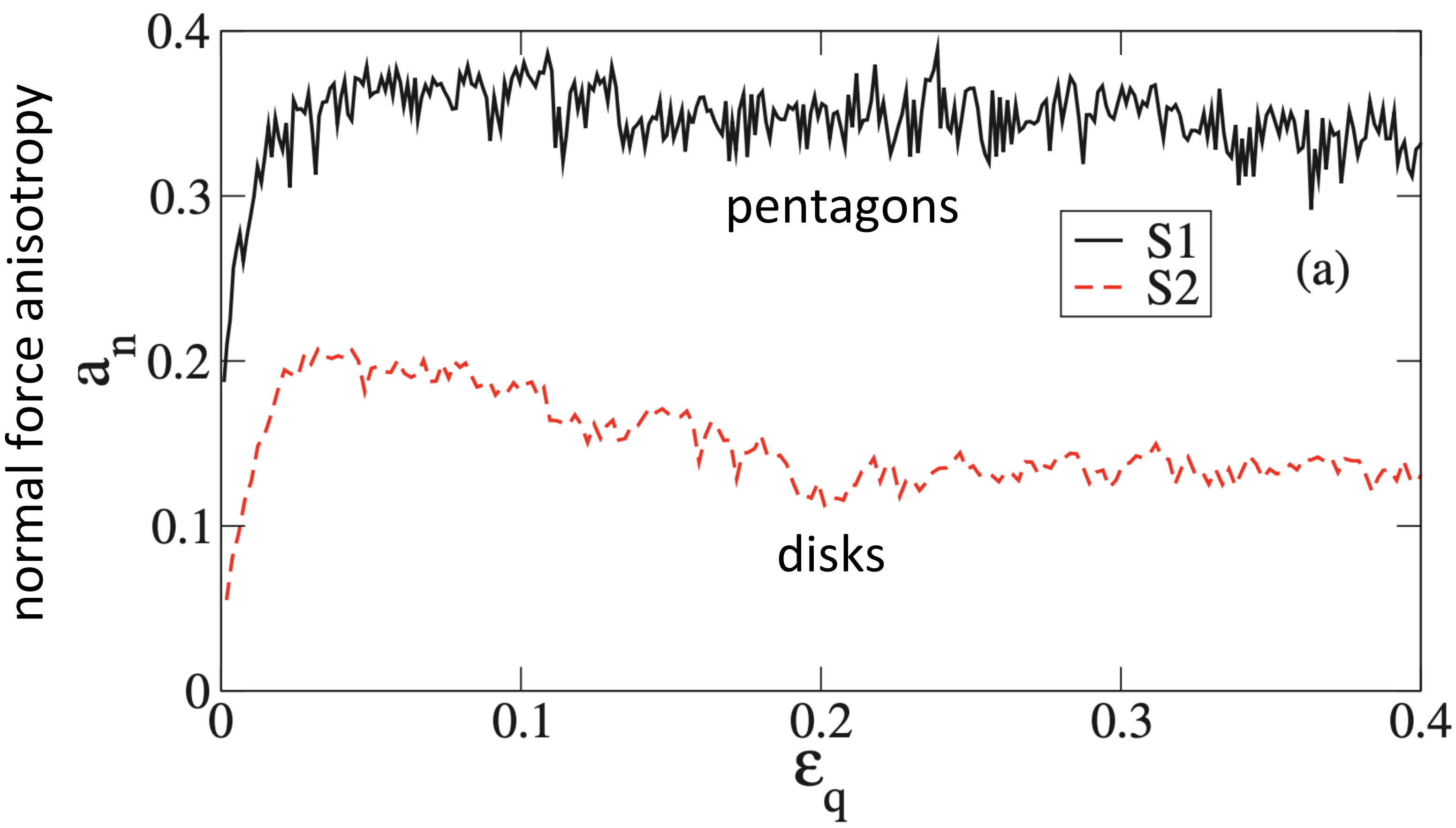
Compared to the disk packing, *pentagons develop a higher anisotropy in the weak network* and a lower anisotropy in the strong network!

We expected higher overall anisotropy of the contact network in the case of polygon packing.

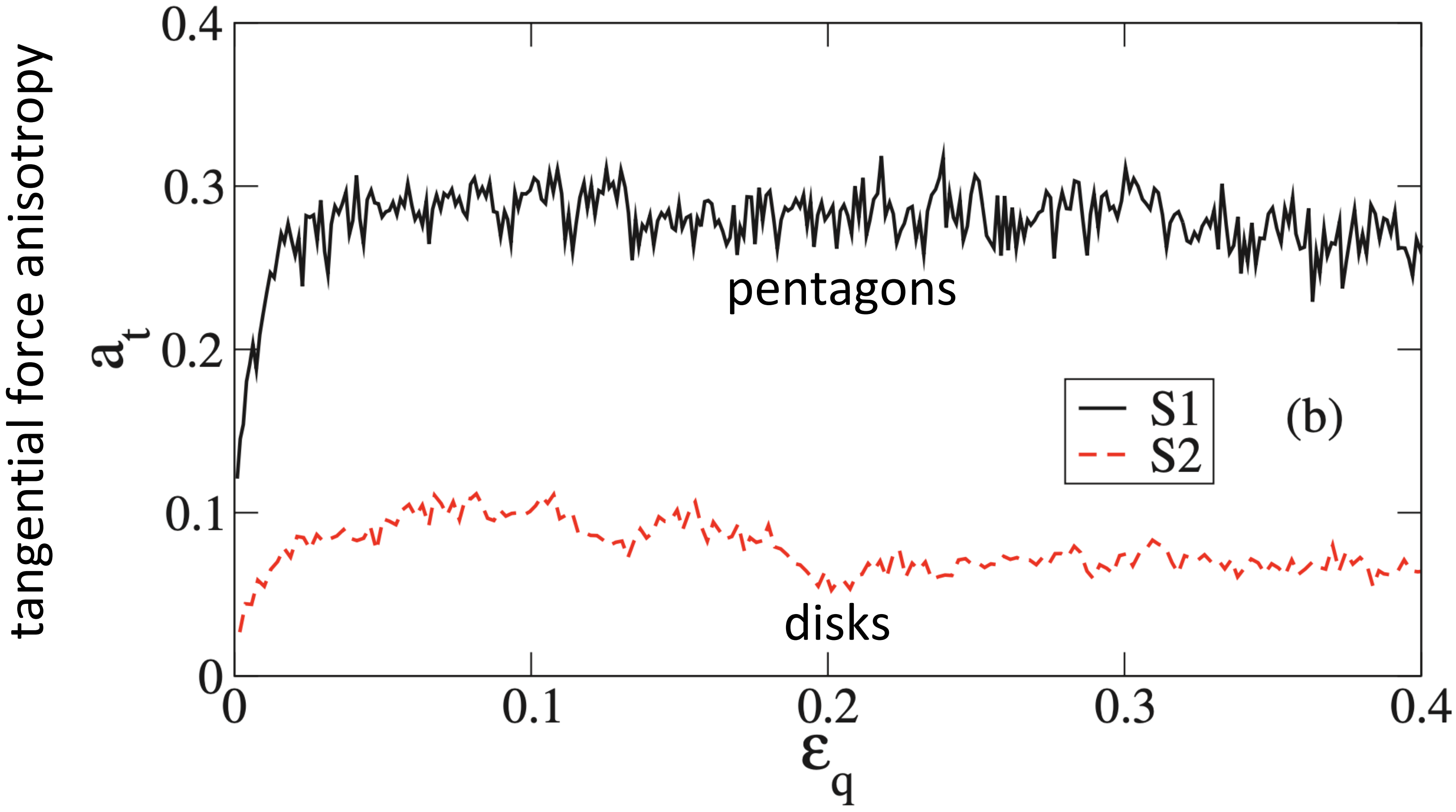


The higher contact anisotropy of the weak network leads to a higher force anisotropy of the strong network.

Normal force anisotropy reflects the higher value of the average force at contacts oriented along the major principal stress direction.



Tangential force anisotropy reflects the higher friction mobilization at contacts oriented at 45 degrees to the major principal stress direction.

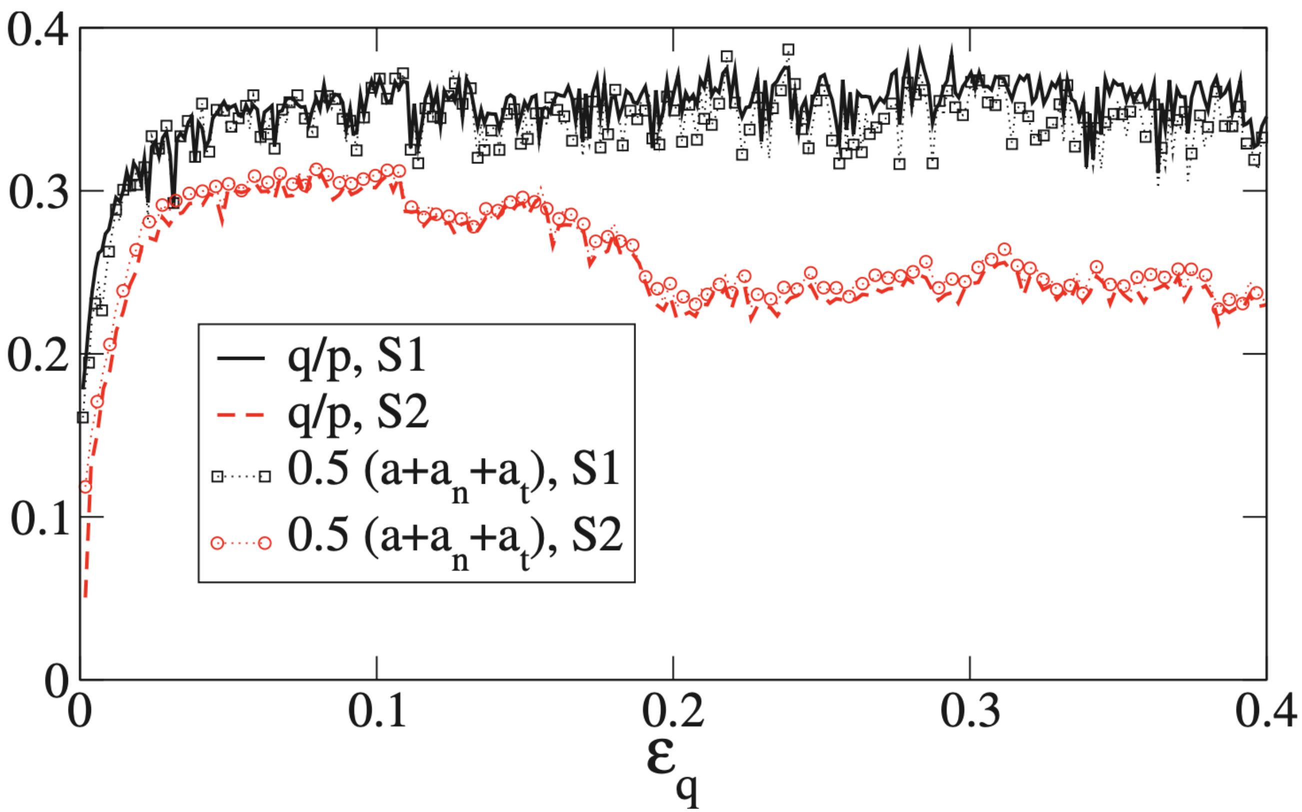


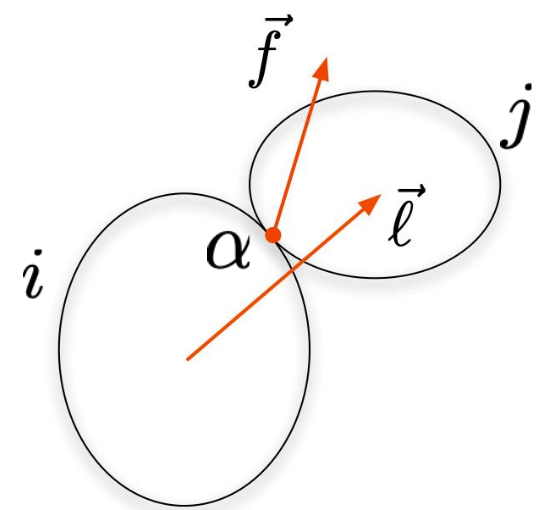
The net effect is the higher shear strength of the packing of pentagonal particles.

$$\sigma_{ij} = n_b \; \langle \ell_i^\alpha f_j^\alpha \rangle_\alpha$$ stress tensor

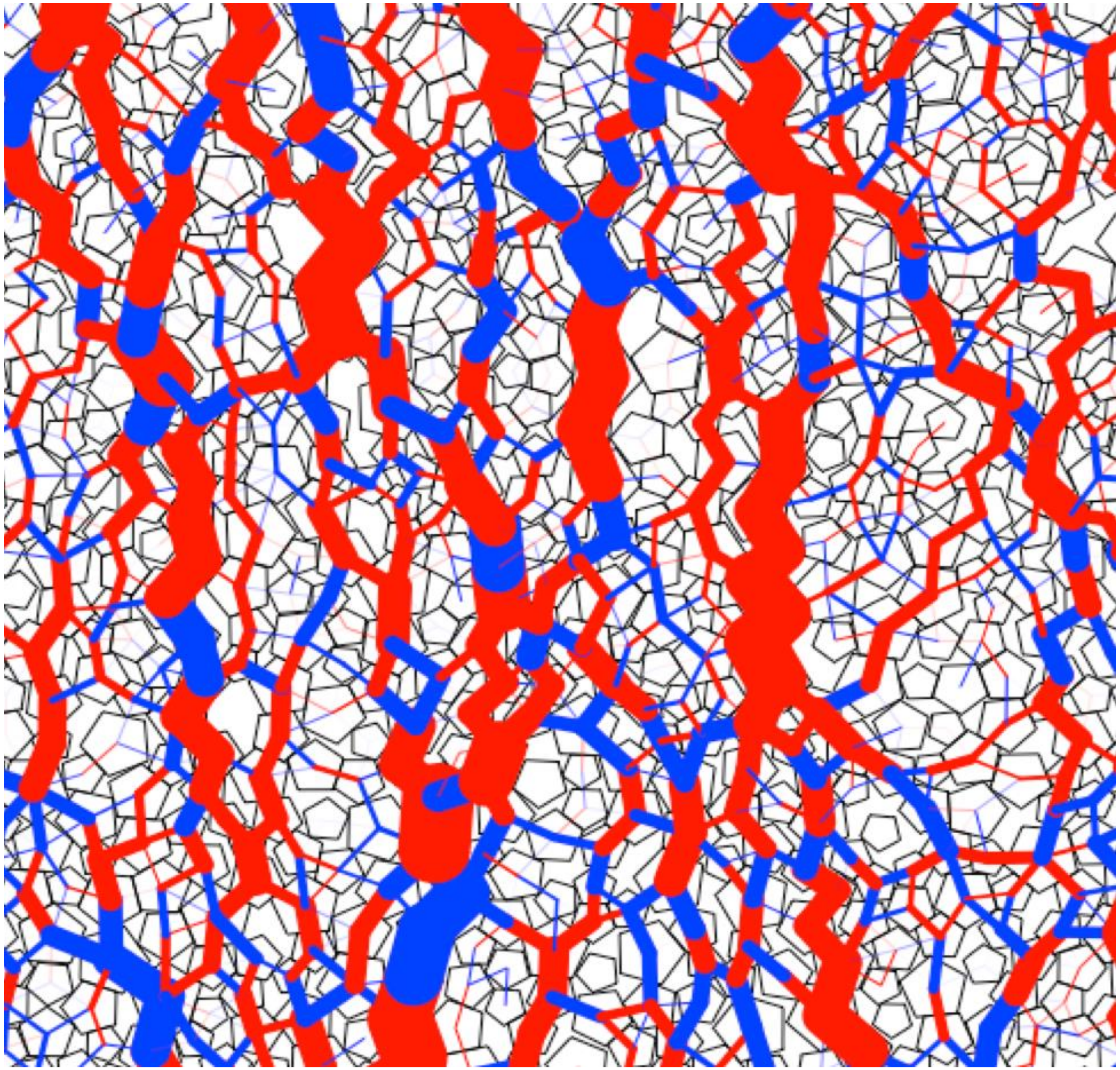
- n_b — number density of contacts
- $\vec{f}^\alpha$ — contact force
- $\vec{\ell}^\alpha$ — branch vector

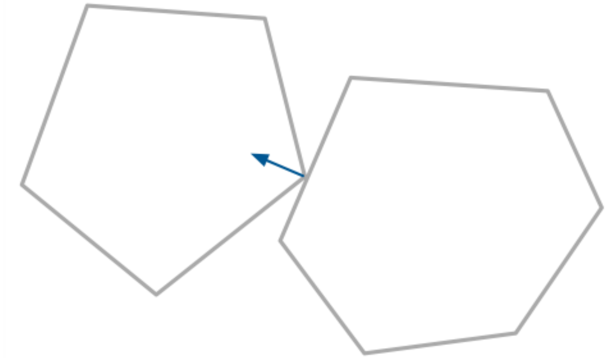
$$\Rightarrow \quad \frac{q}{p} = \frac{1}{2}(a + a_n + a_t)$$





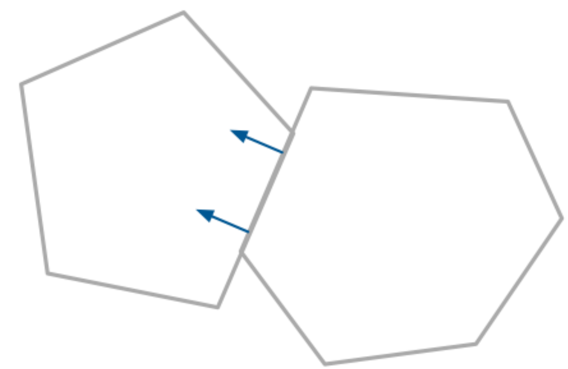
Role of side-side contacts





side-vertex contact
= simple contact

1 geometrical constraint



side-side contact
= double contact

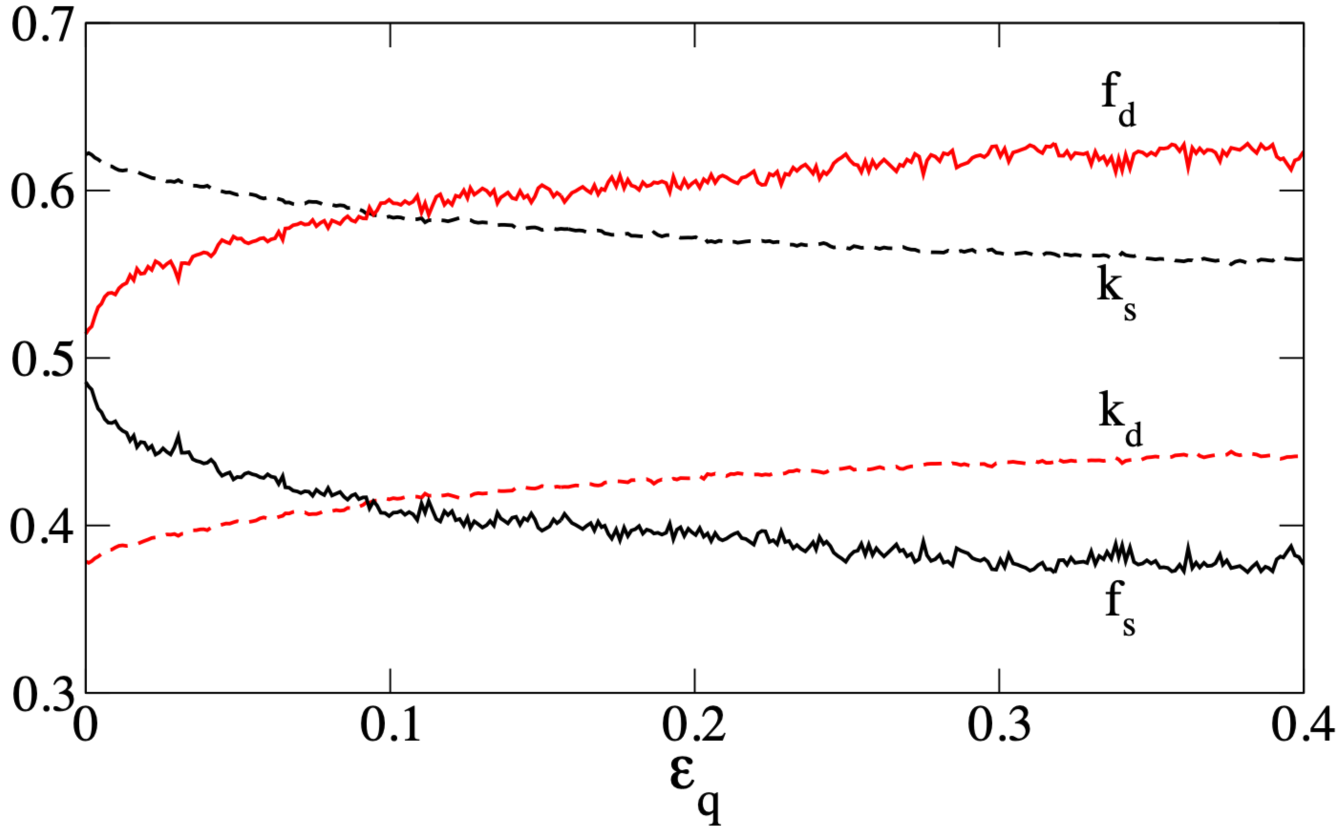
2 geometrical constraints

k_s proportion of simple contacts

k_d proportion of double contacts

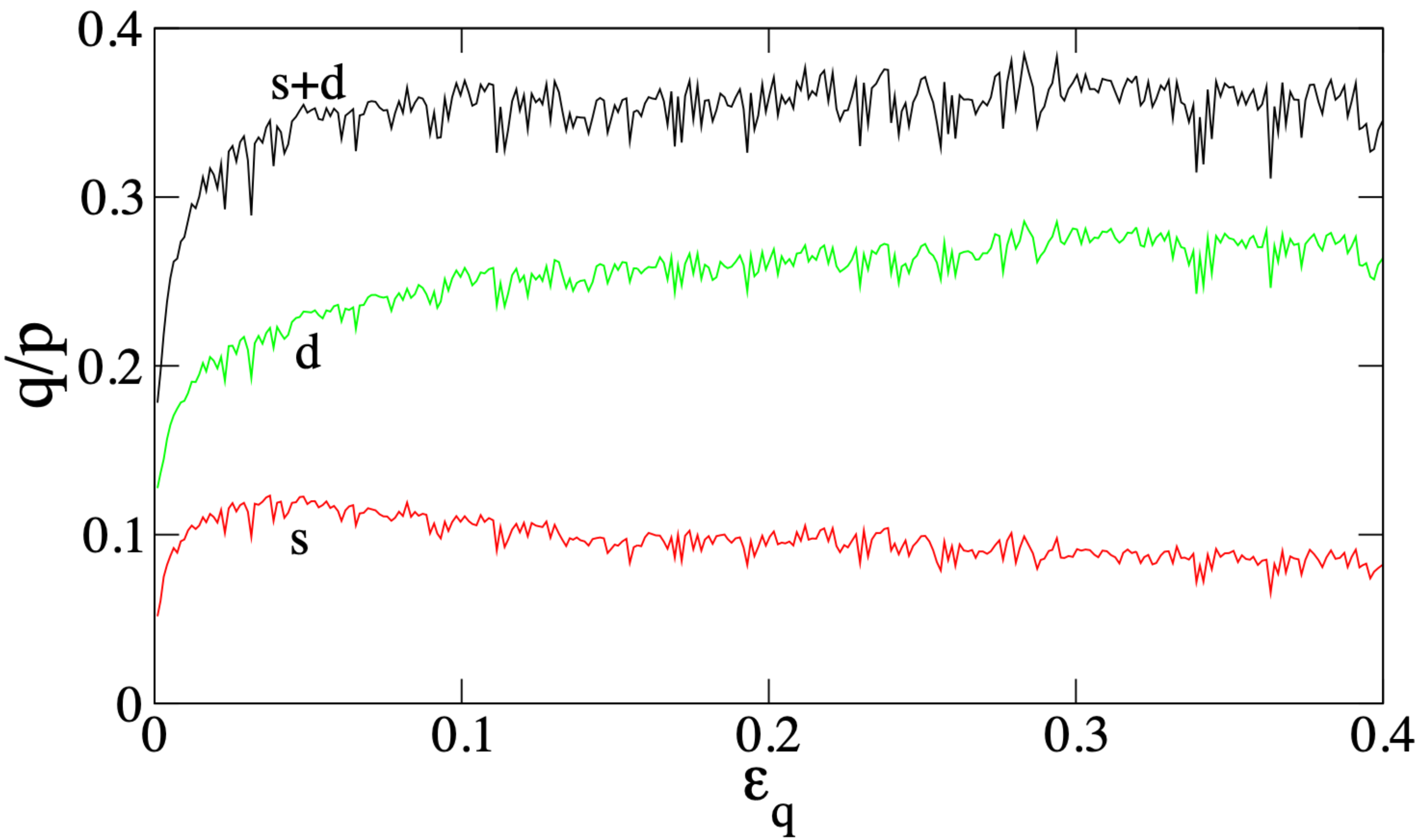
f_s average force of simple contacts

f_d average force of double contacts



Double contacts are less in number, but they capture strong forces.

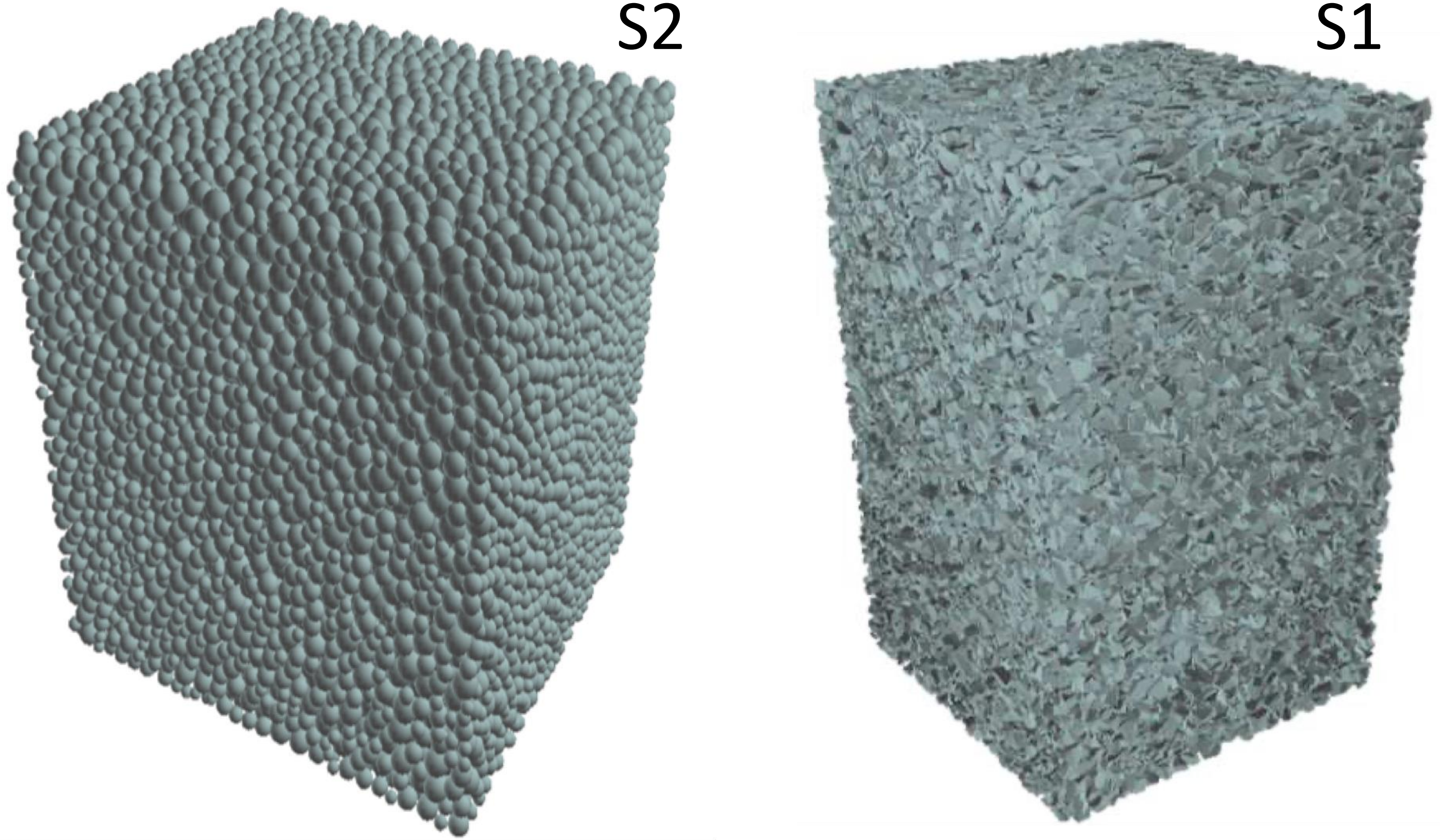
Partition of shear stress



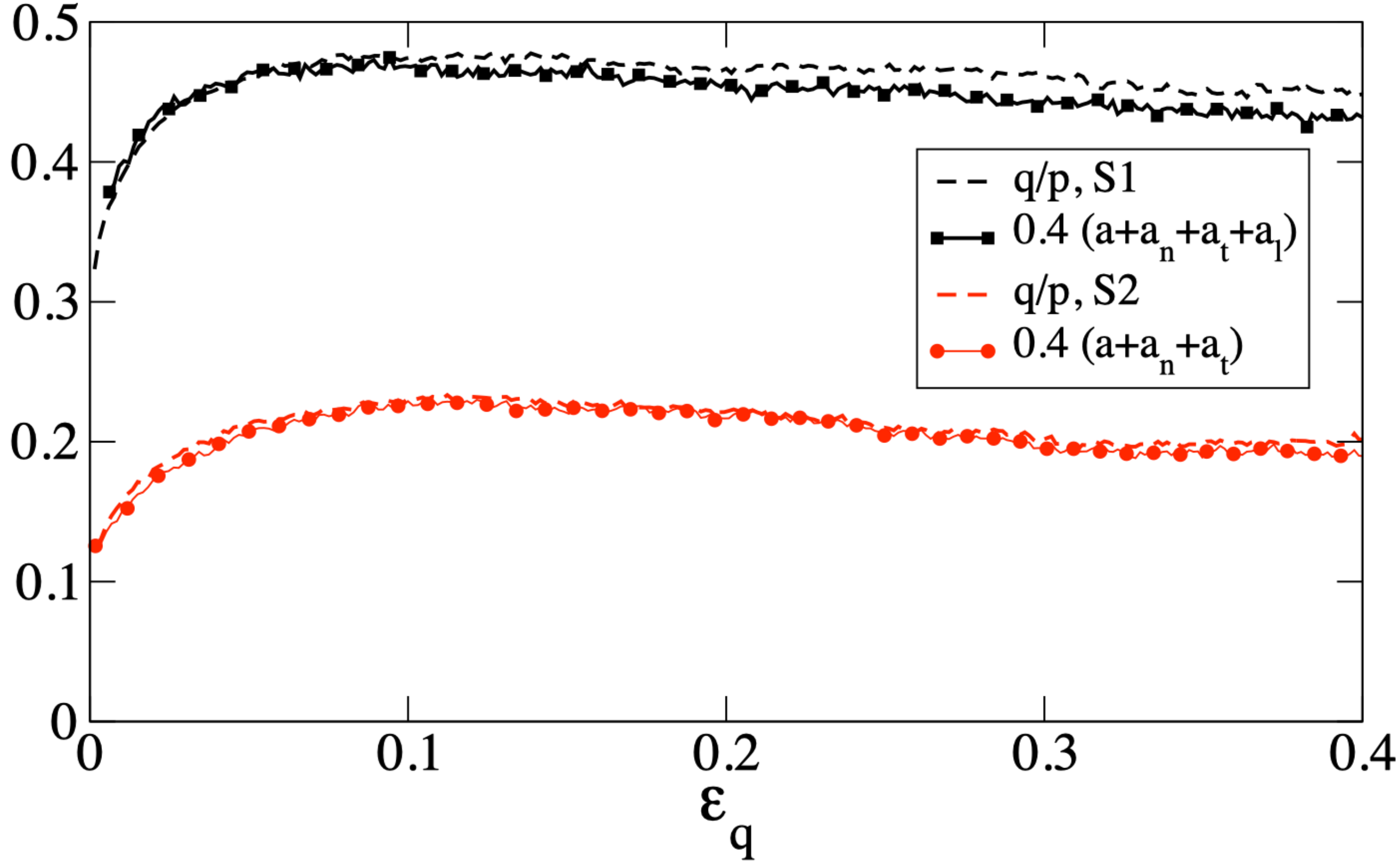
Double contacts carry most of the shear stress.

Polyhedra vs. spheres

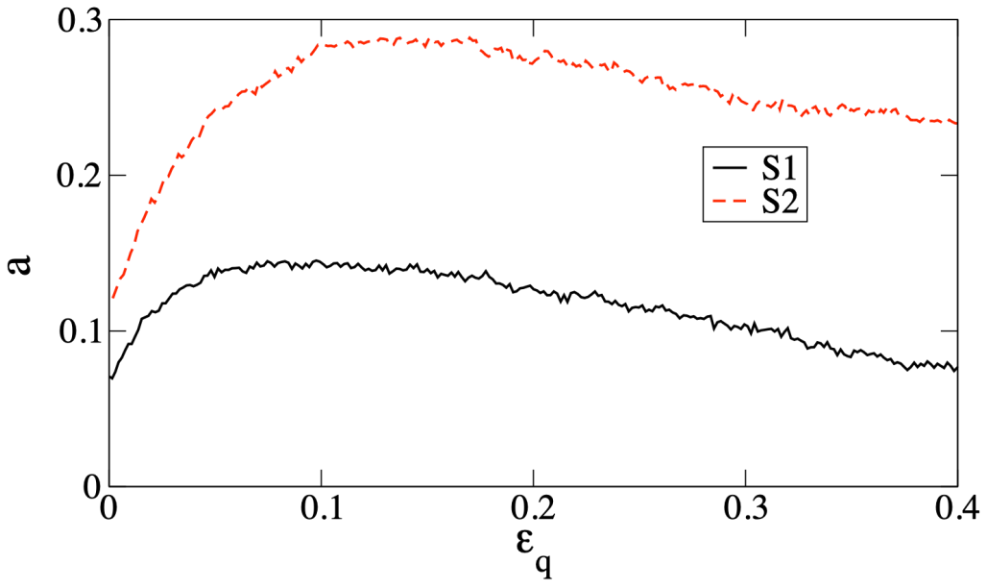
Similar analysis and similar results

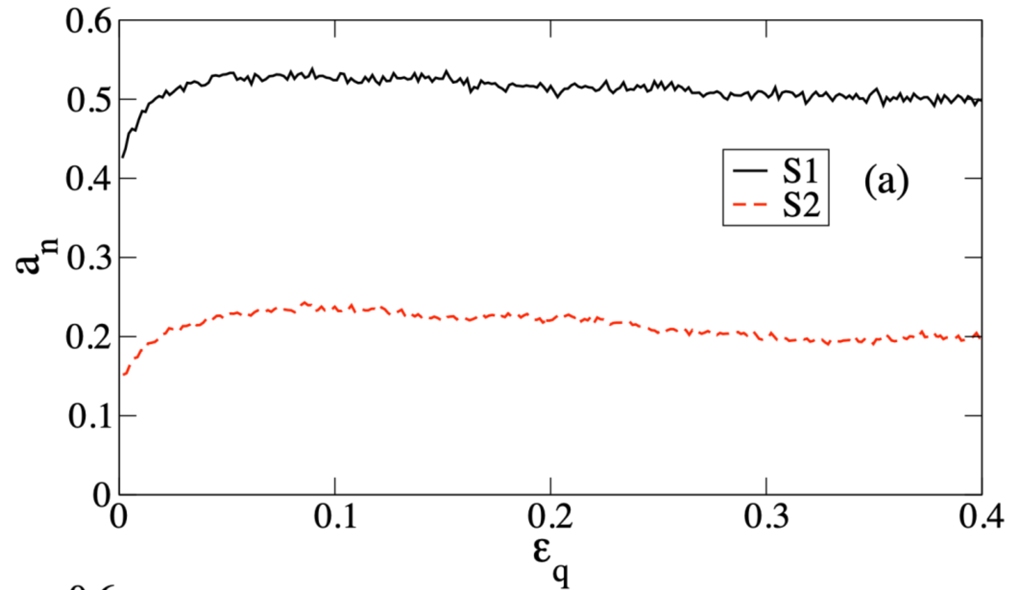


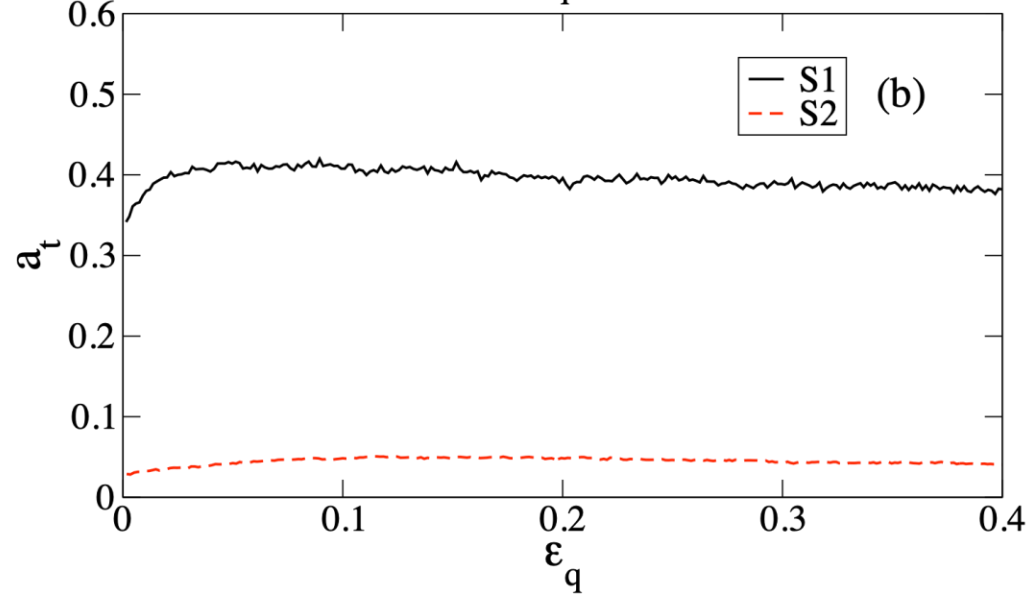
E. Azéma, F. Radjai, G. Saussine, Mech. Mat. 41, 2009



0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
ε_q
q/p, S1
0.4 $(a+a_n+a_t+a_l)$
q/p, S2
0.4 $(a+a_n+a_t)$







a contact orientation anisotropy

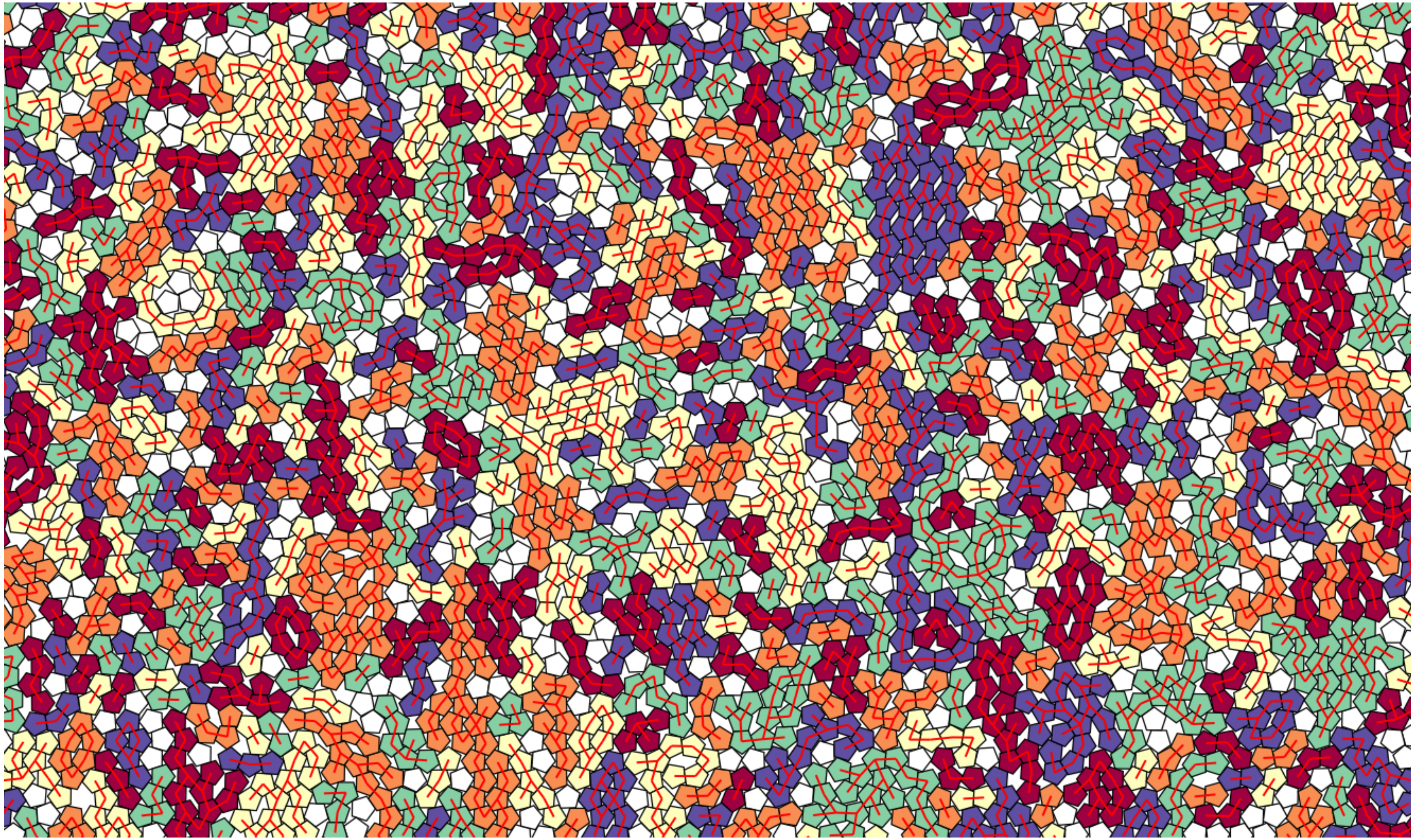
a_n contact normal anisotropy

a_t friction mobilization

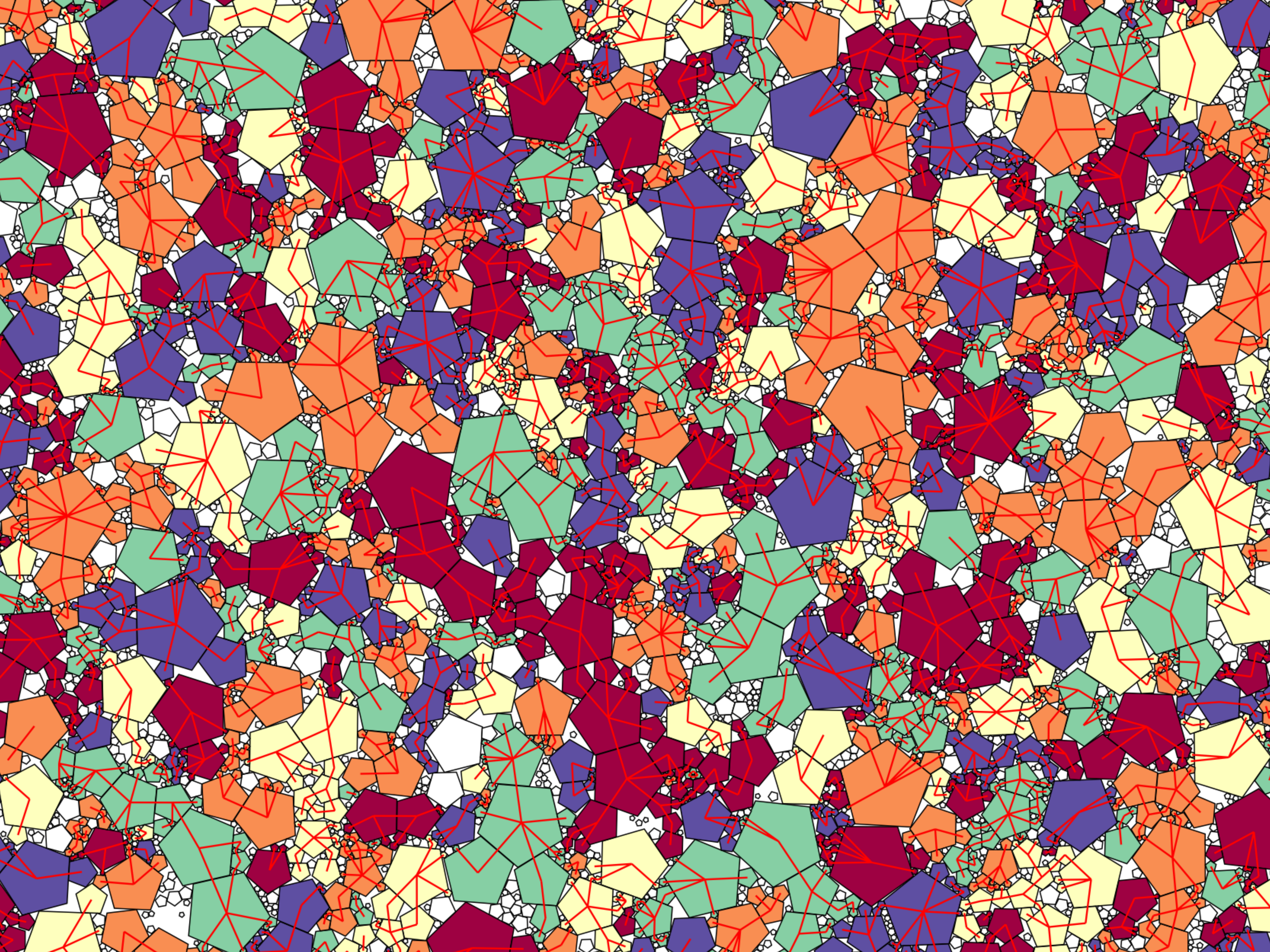
Local ordering

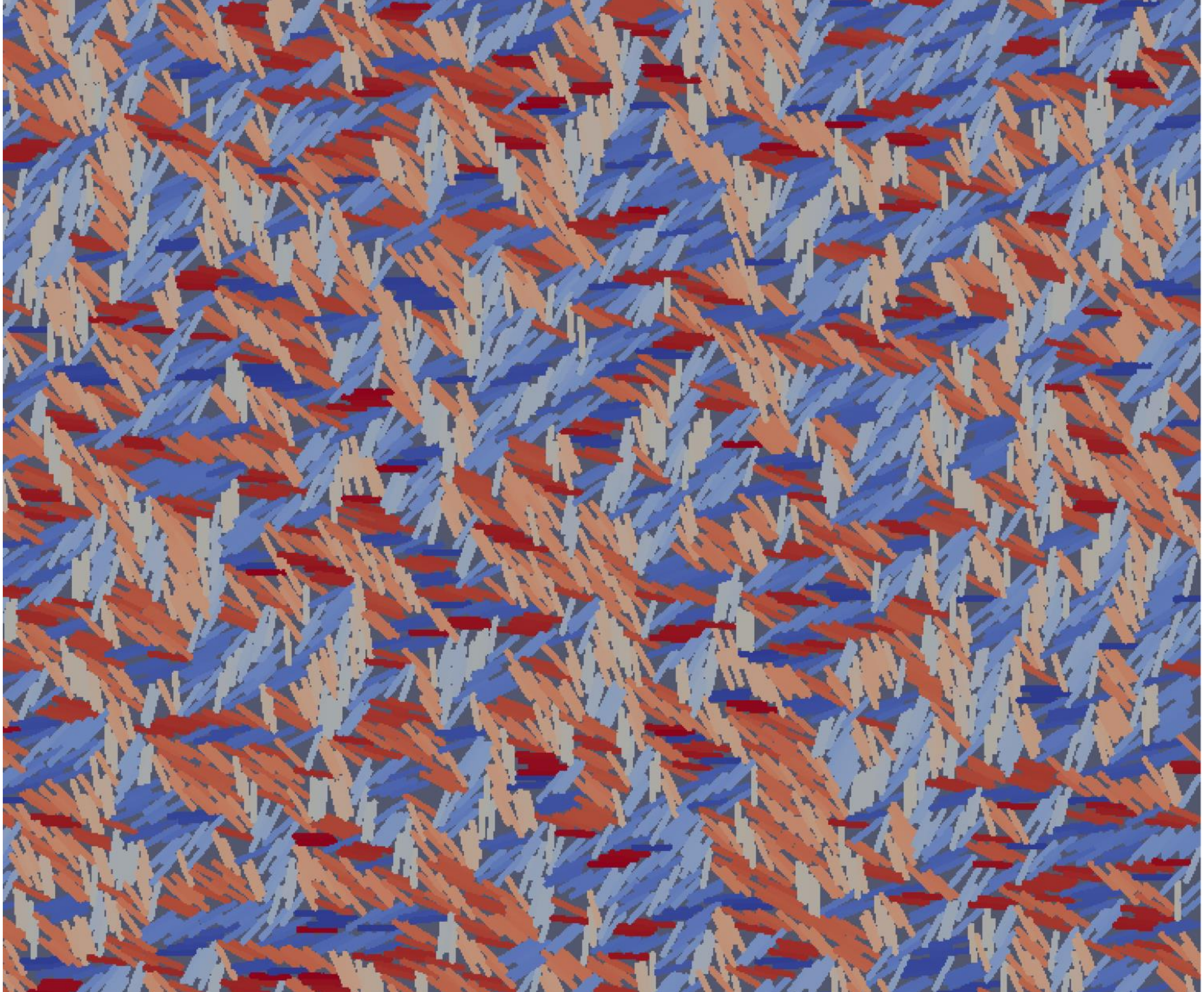
Shape parameters induce particle clustering.

Example: clustering of particles according to double contacts.



DH Nguyen, E. Azéma, F. Radjai, P. Sornay, PRE 90 (2014)



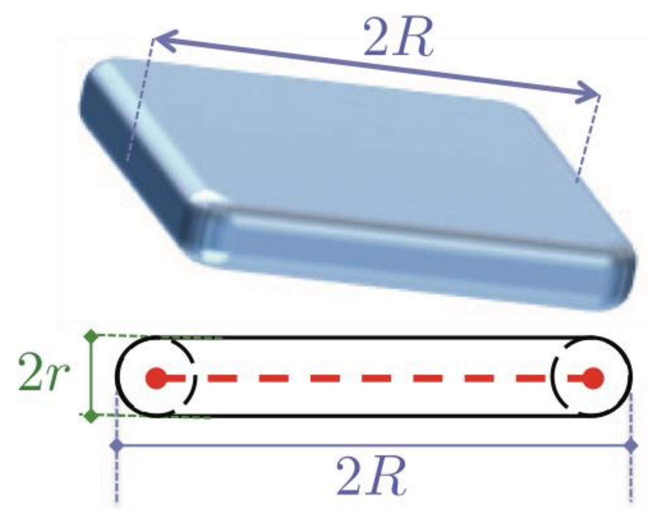


Shape parameters induce local ordering in competition with short-range order induced by volume exclusions between particles.

Sphero-plates

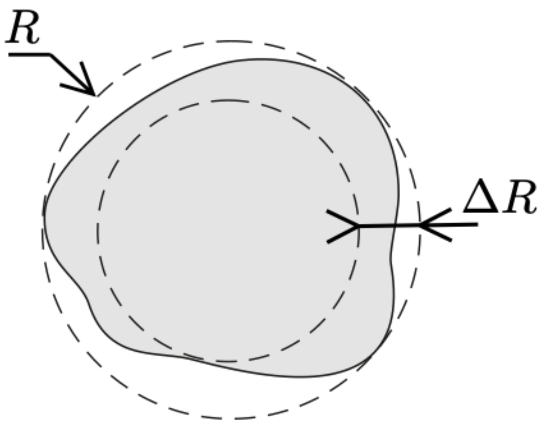
Shape parameter: $$\eta = \frac{R - r}{R}$$

M. Boton, E. Azéma, N. estrada, F. Radjai, A. Lizcano, PRE 87 (2013)

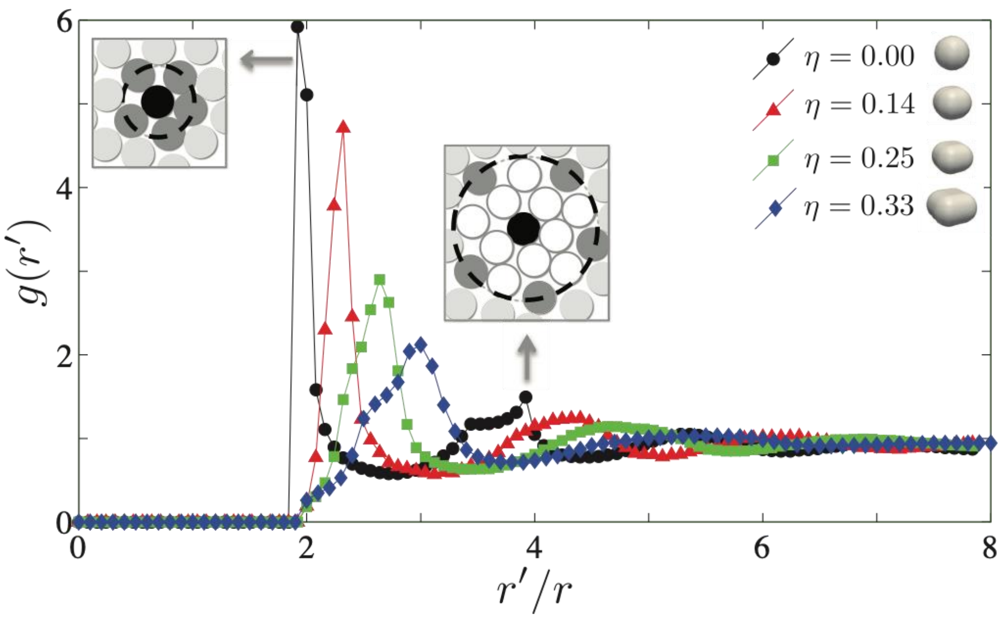


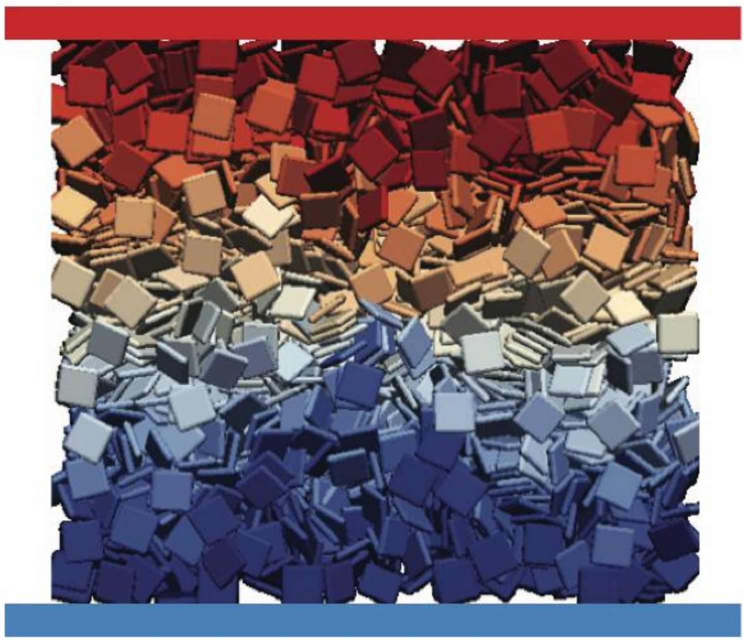
This is a general shape parameter, which measures deviation from spherical shape.

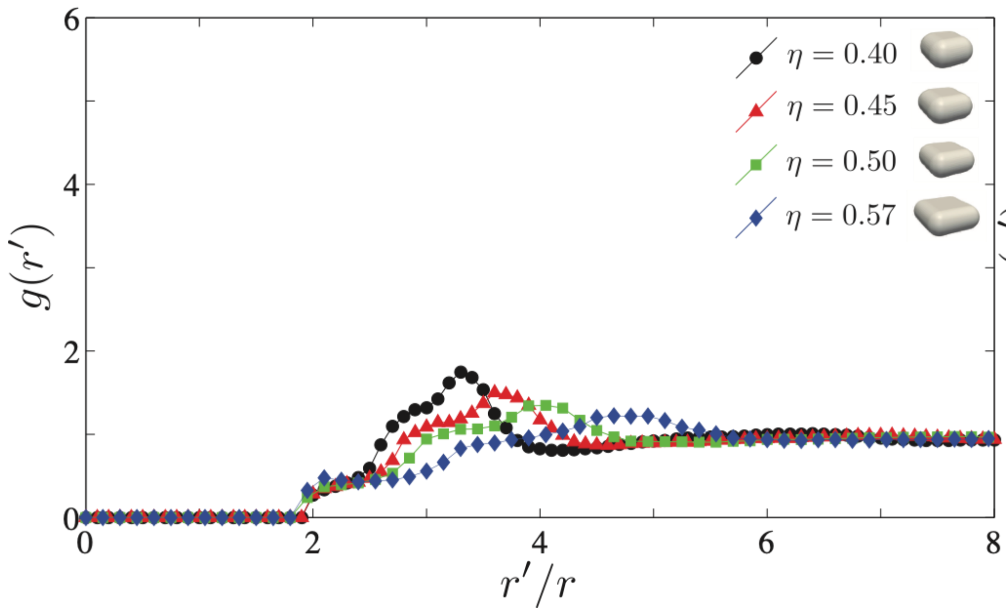
$$\eta = \frac{\Delta R}{R}$$

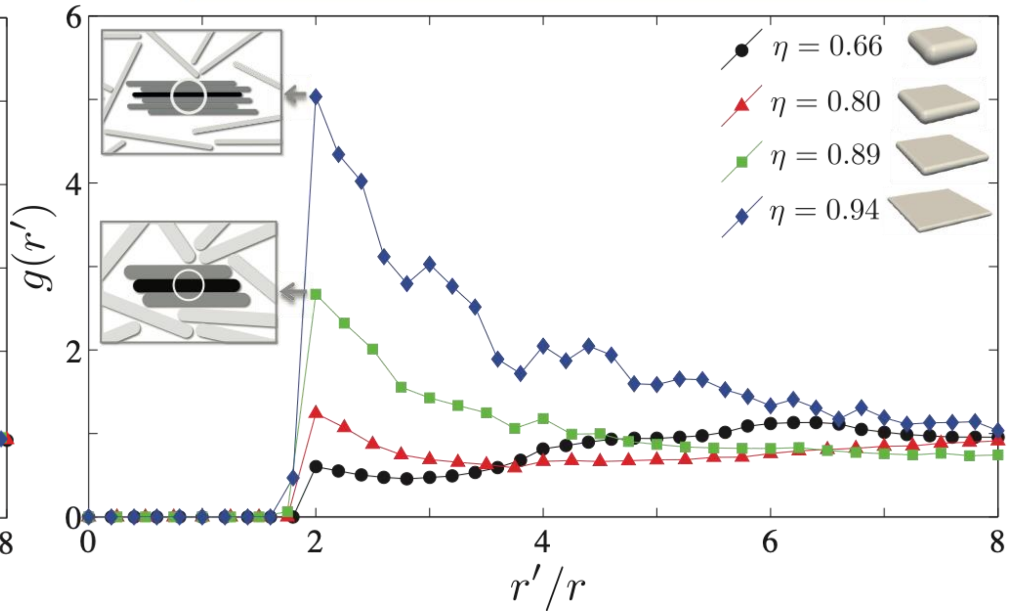


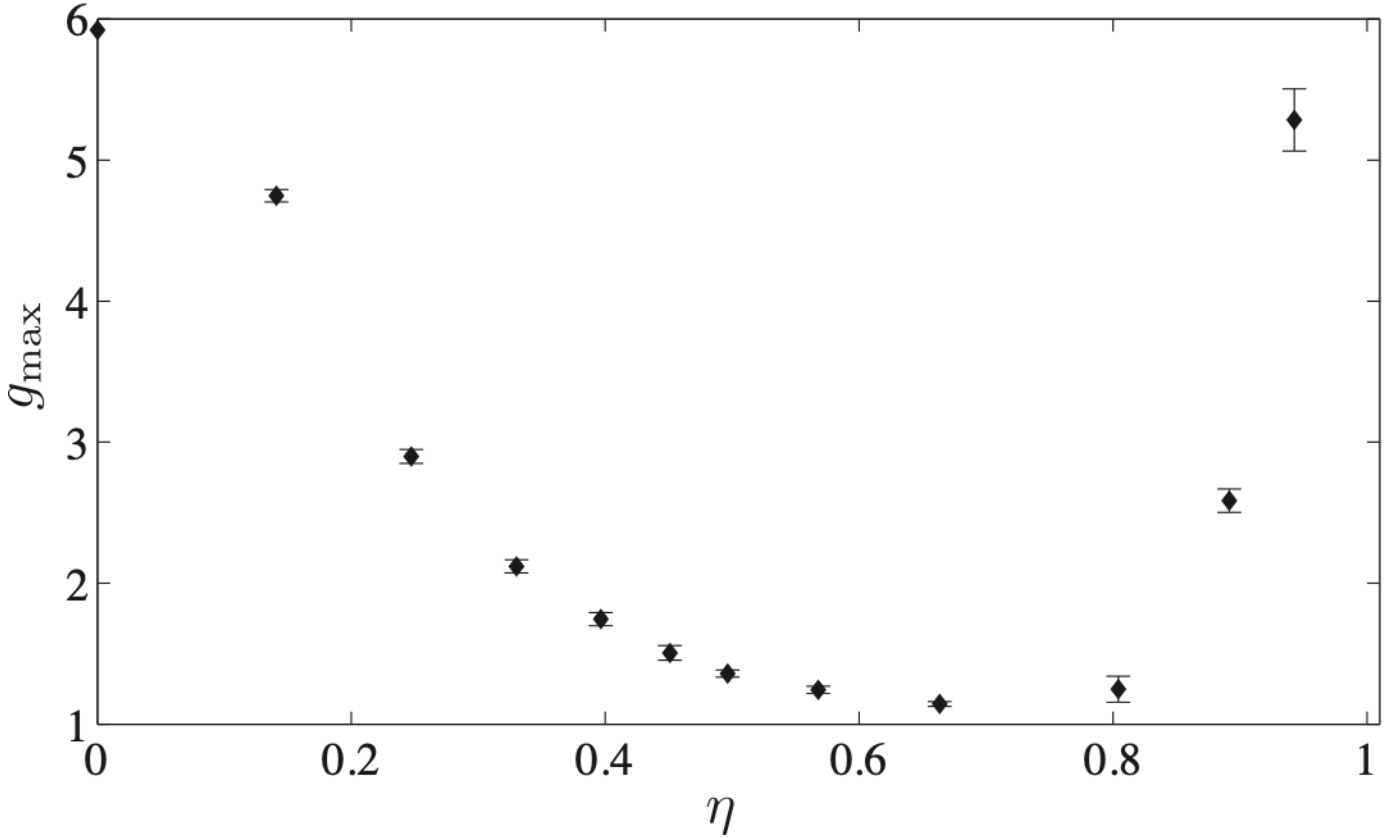
Pair correlation function



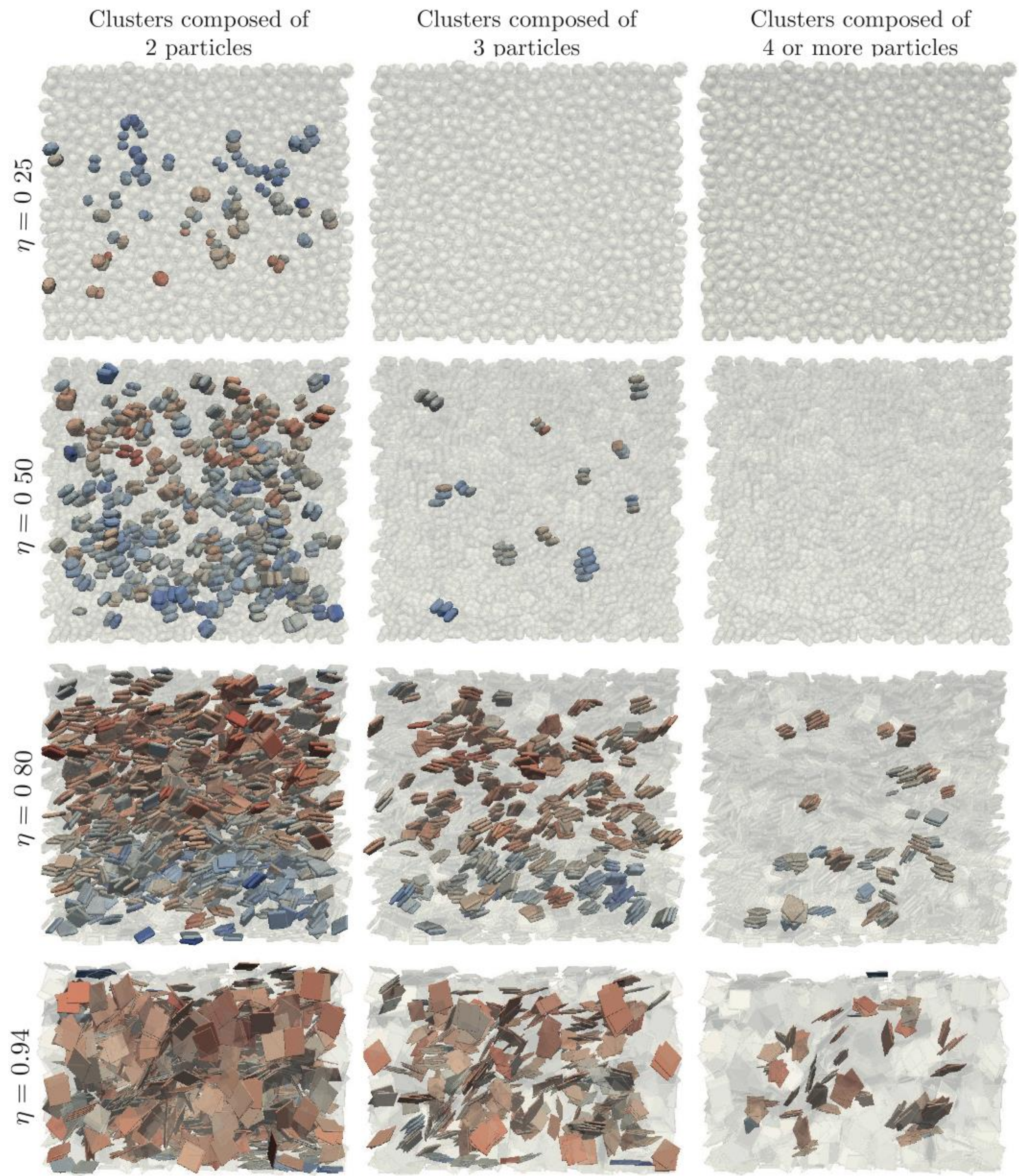






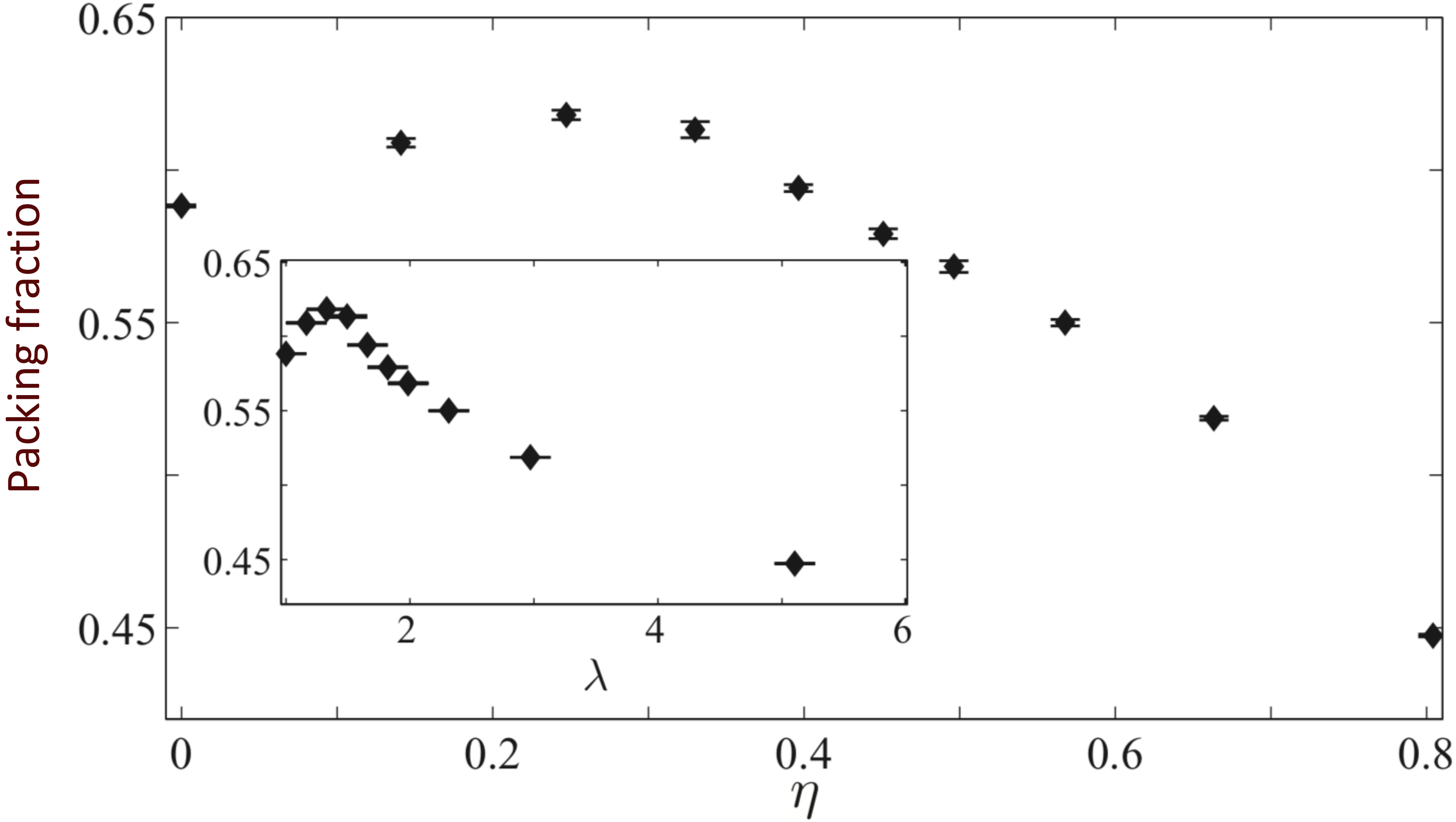


Transition from local ordering regime governed by steric exclusions to local ordering induced by face-face contacts.

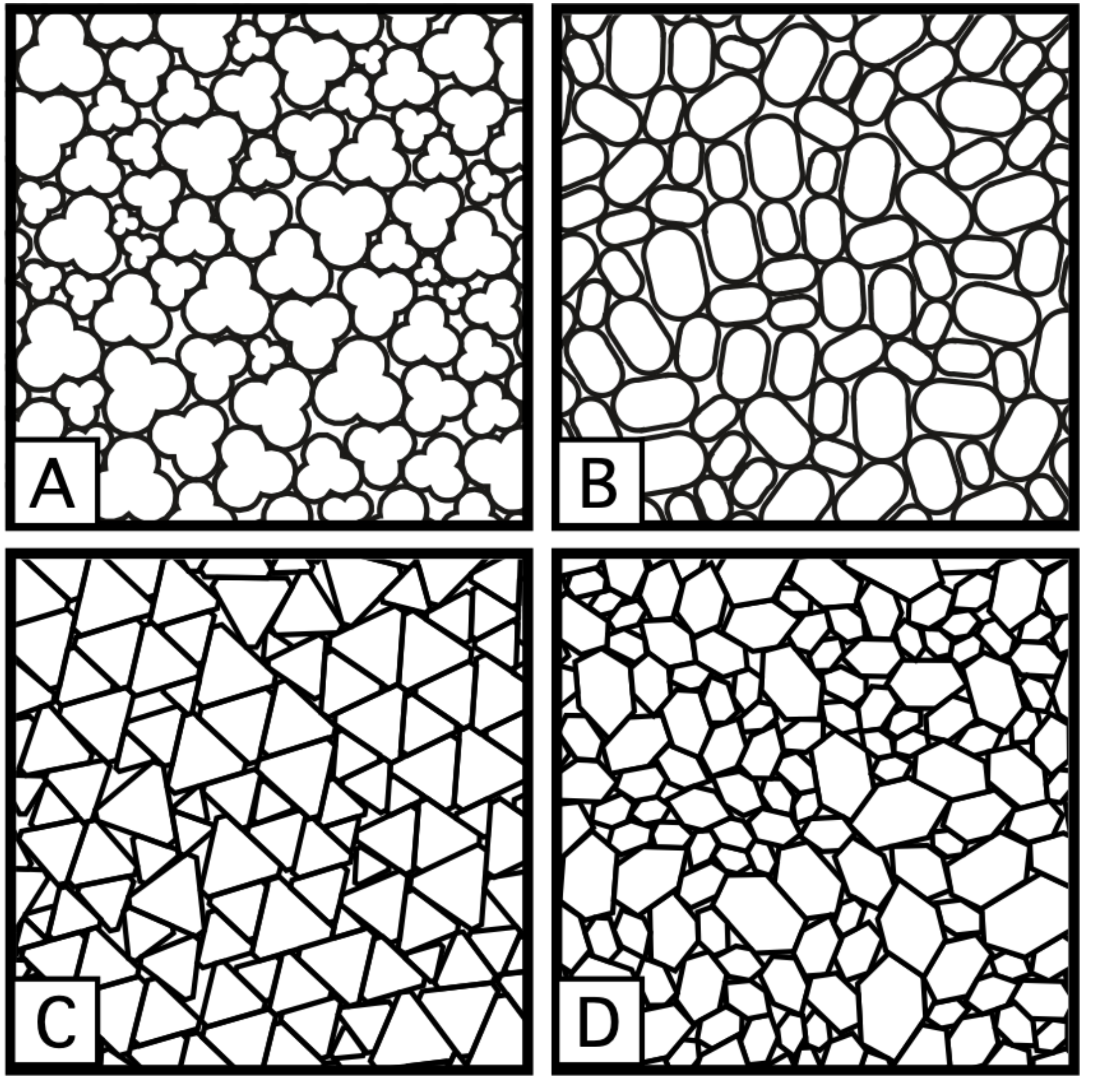

Clusters composed of 2 particles
Clusters composed of 3 particles
Clusters composed of 4 or more particles
$\eta = 0.25$
$\eta = 0.50$
$\eta = 0.80$
$\eta = 0.94$

Packing fraction

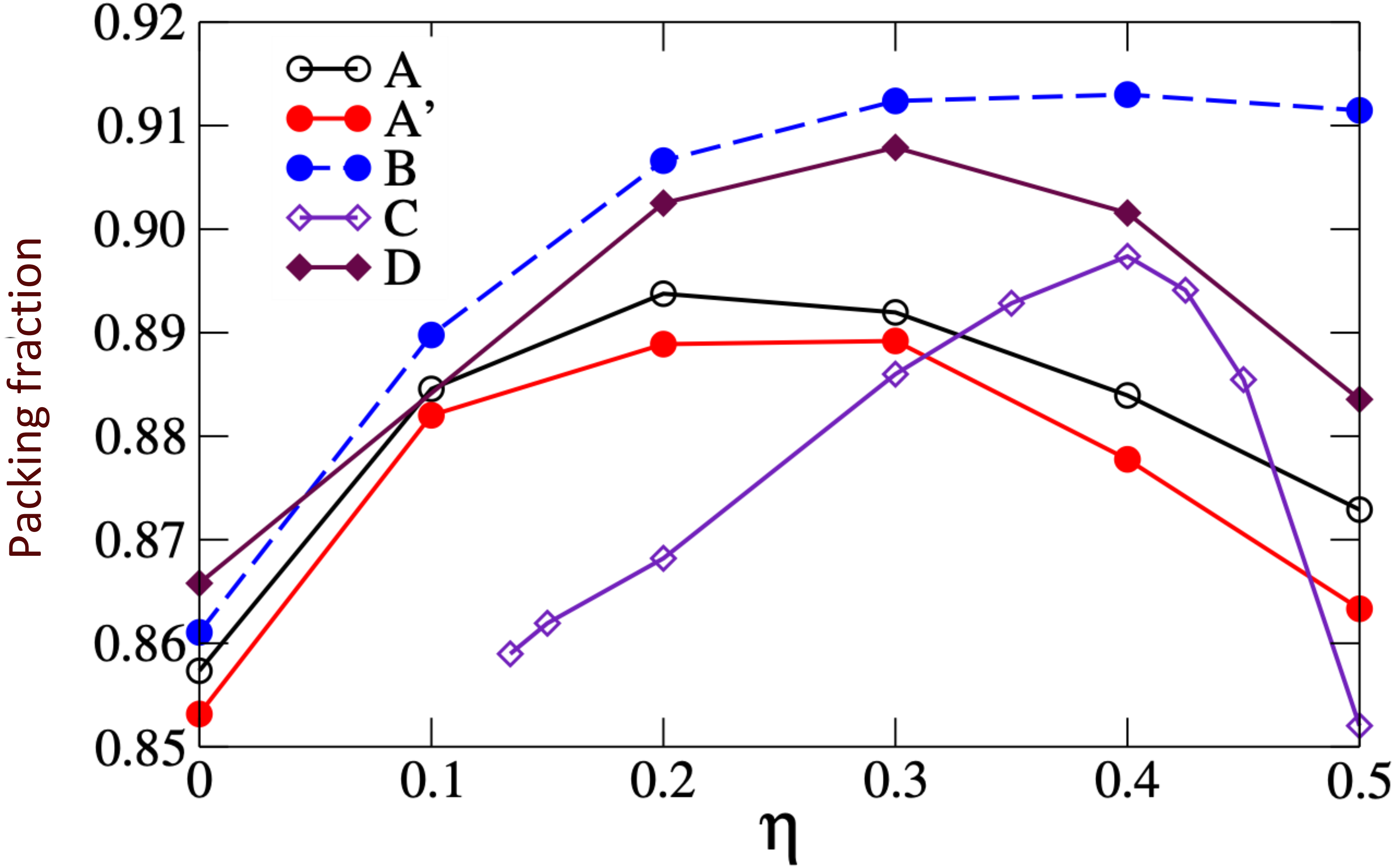
The packing fraction is an unmonotonic function of η



The same unmonotonic dependence is observed for all shapes.

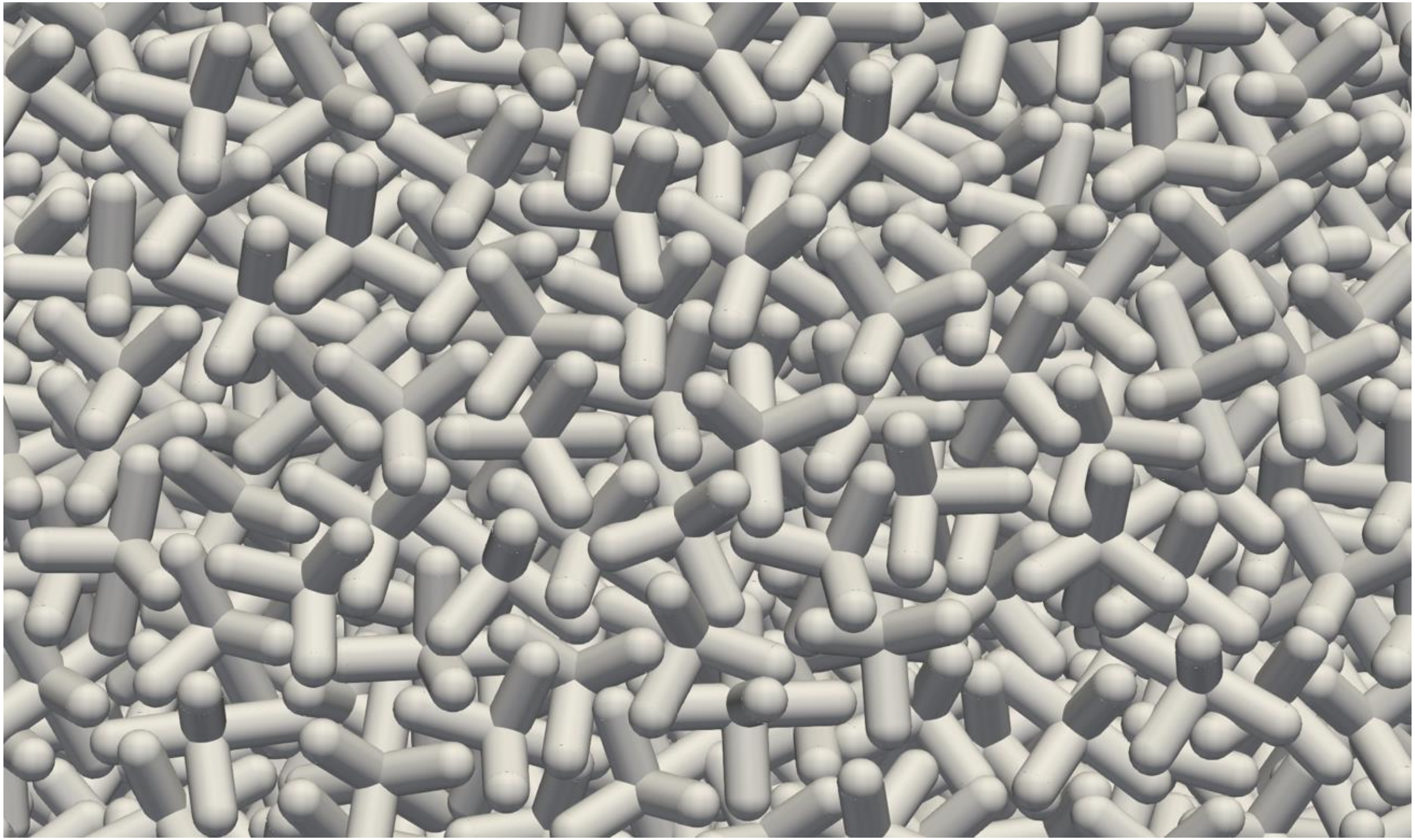


CEGEO et al., EPL 98 (2012)

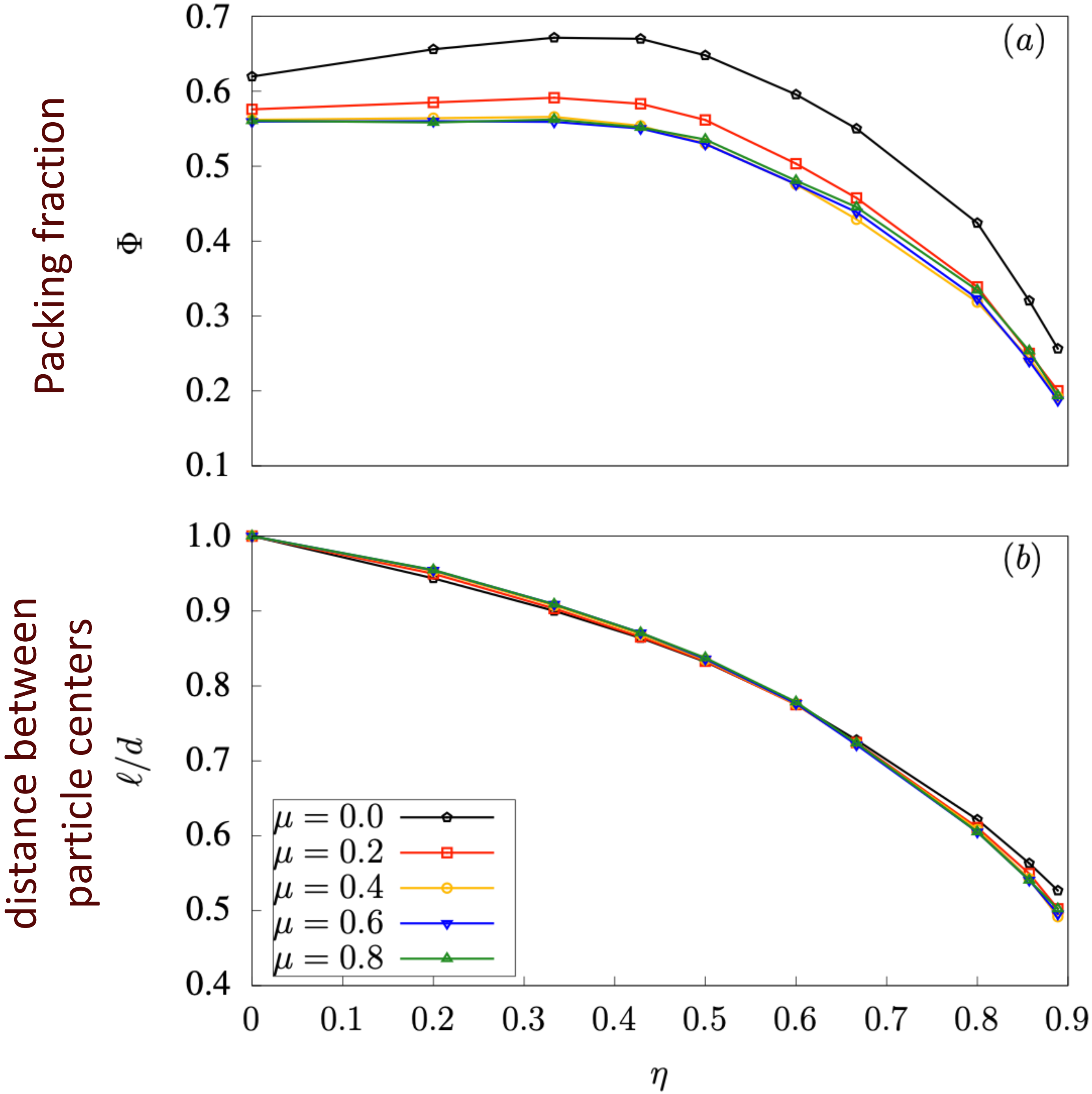


0.92
0.91
0.90
0.89
0.88
0.87
0.86
0.85
Packing fraction
A
A'
B
C
D
0
0.1
0.2
0.3
0.4
0.5
η

hexapods

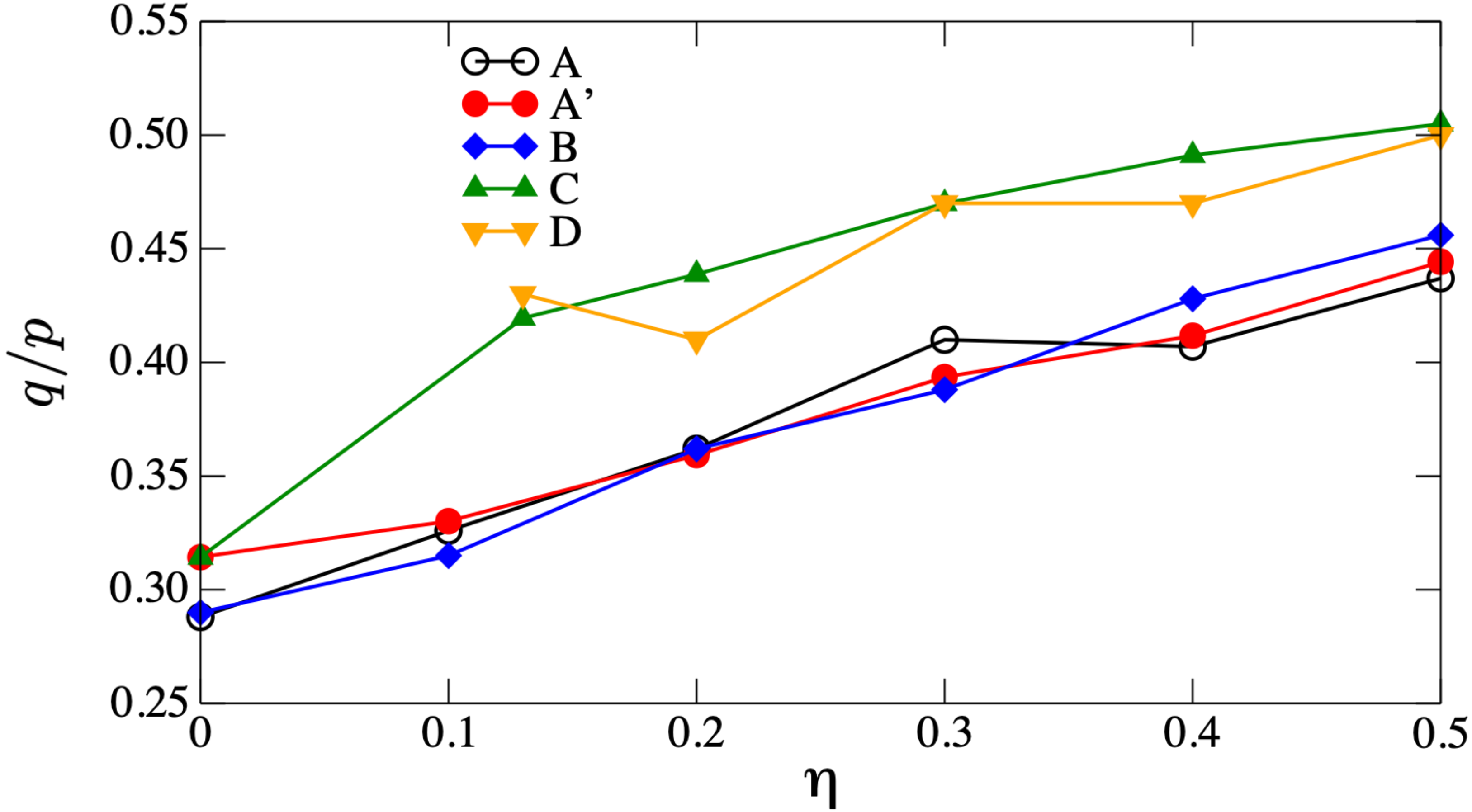


TD Tran et al. to be published (2023)



Packing fraction
Φ
(a)
distance between particle centers
ℓ/d
(b)
μ = 0.0
μ = 0.2
μ = 0.4
μ = 0.6
μ = 0.8
η

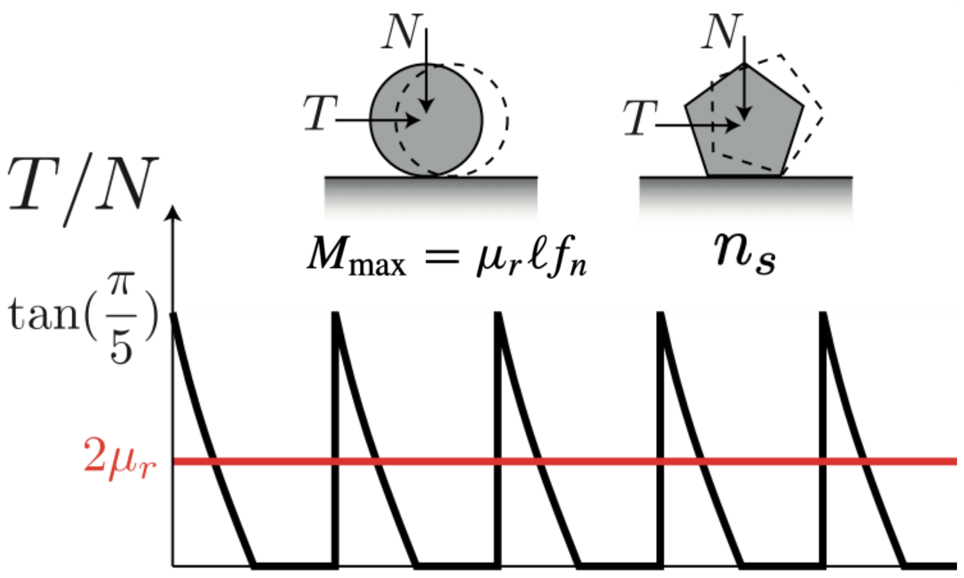
η is the also a generic parameter for shear strength.



Particle rotations

Rotations of aspherical particles imply local dilation and therefore higher energy cost.

Test: compare polygons with decreasing number of sides with disks with increasing rolling friction.

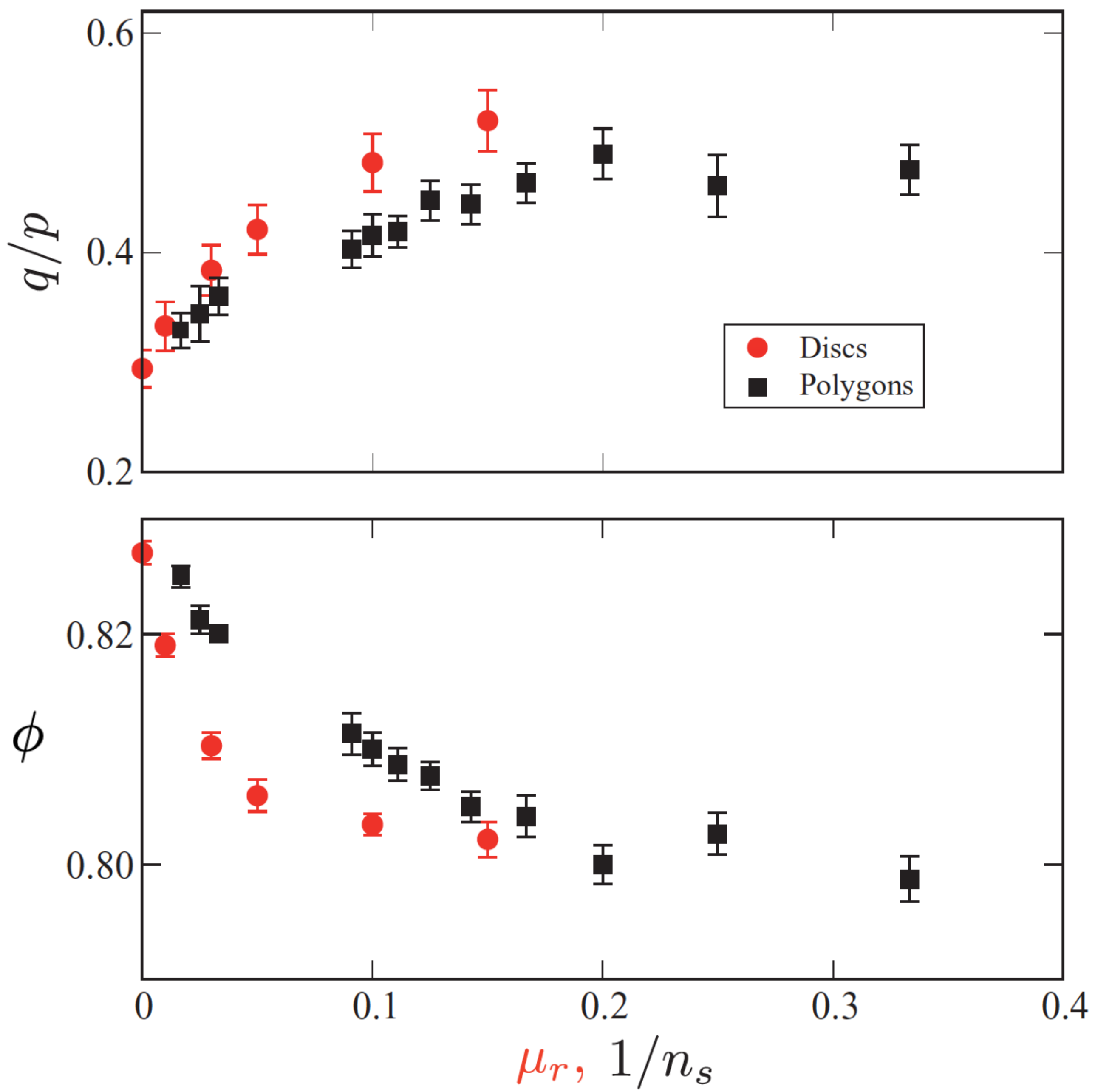


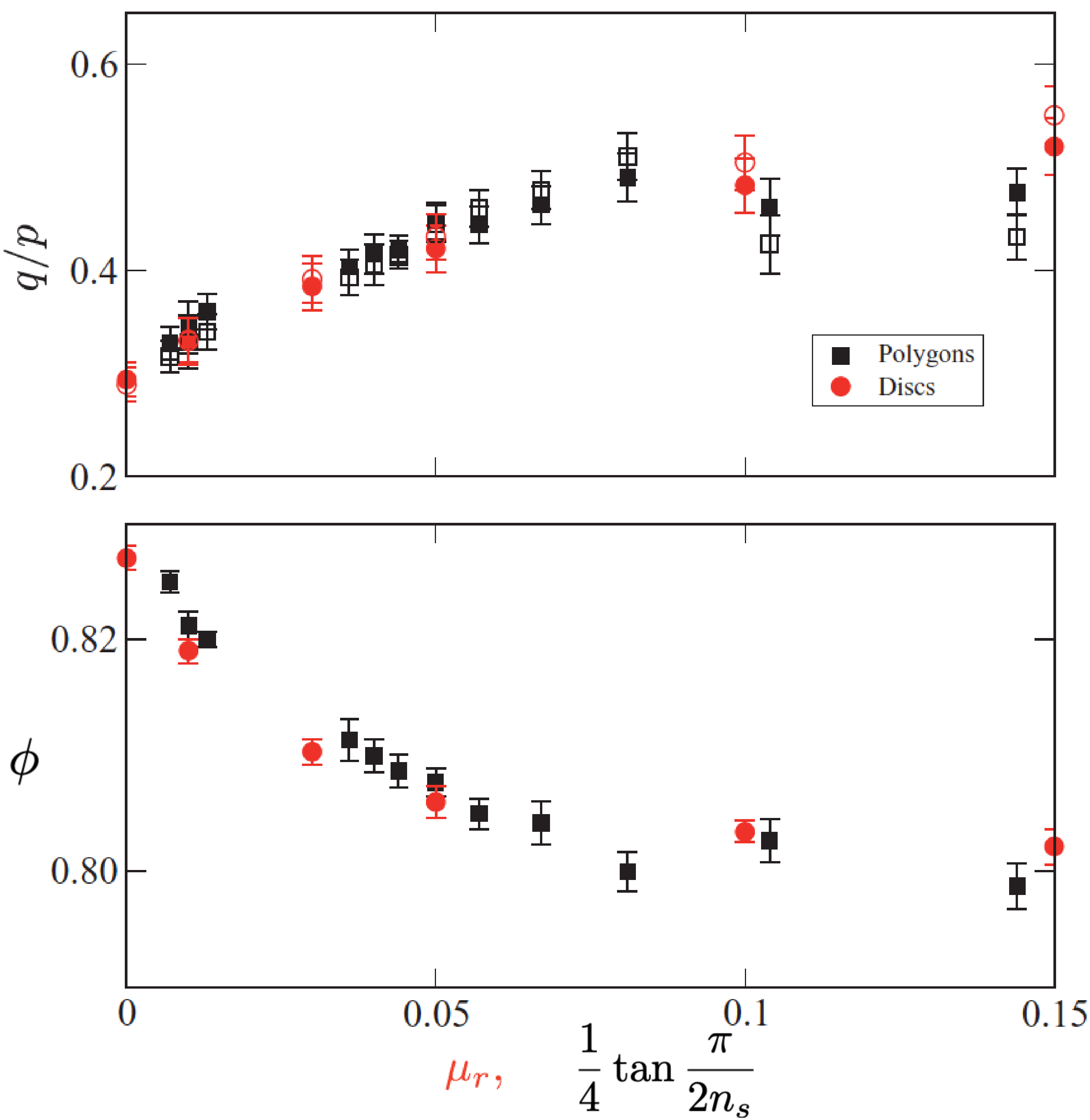
μ_r rolling friction coefficient

n_s number of sides

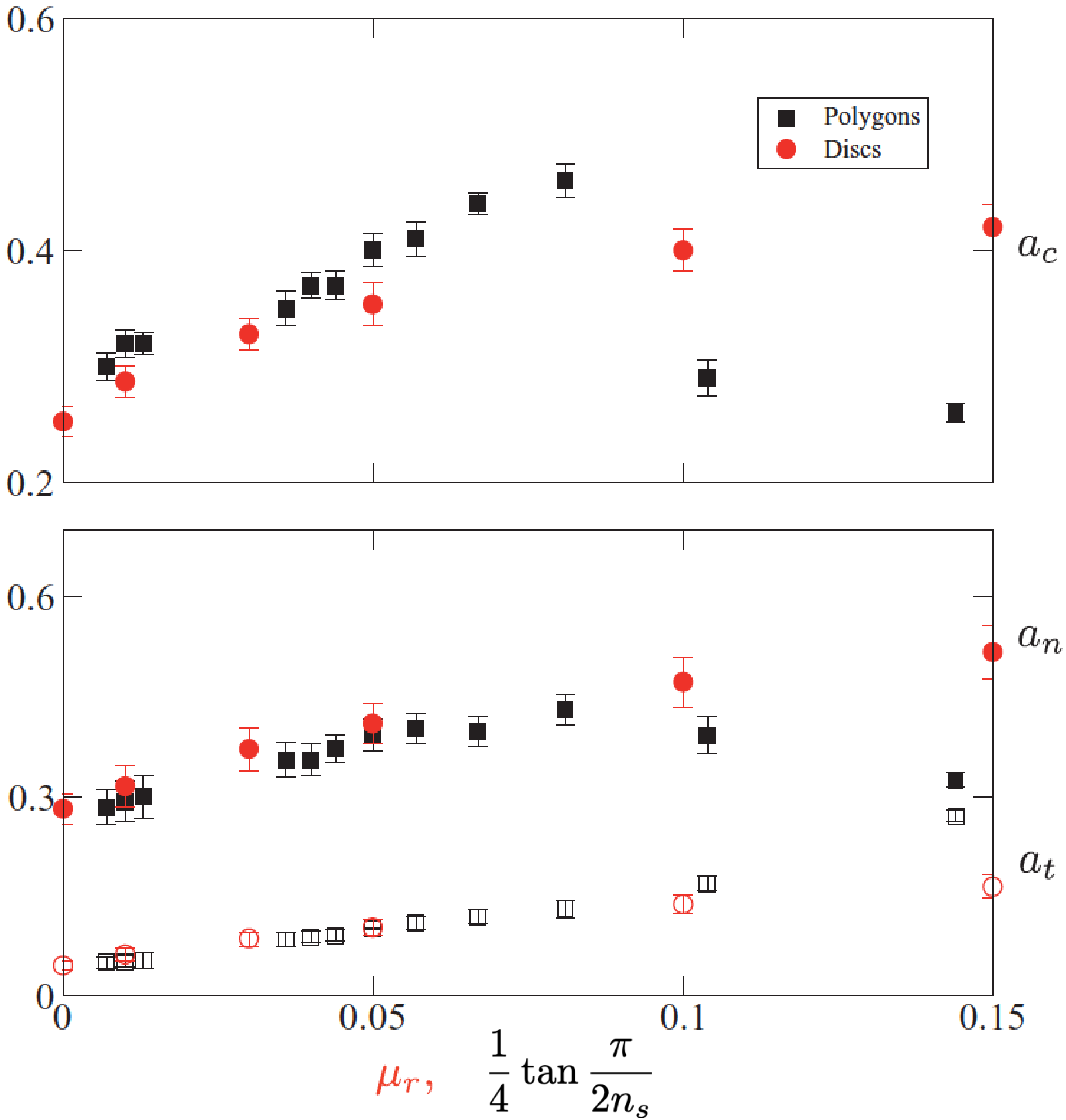
Equality of energy dissipation in one full rotation $\Longrightarrow$ $\mu_r = \frac{1}{4} \tan \frac{\pi}{2 n_s}$

N. Estrada, E. Azéma, F. Radjai, A. Taboada, PRE 84 (2011)

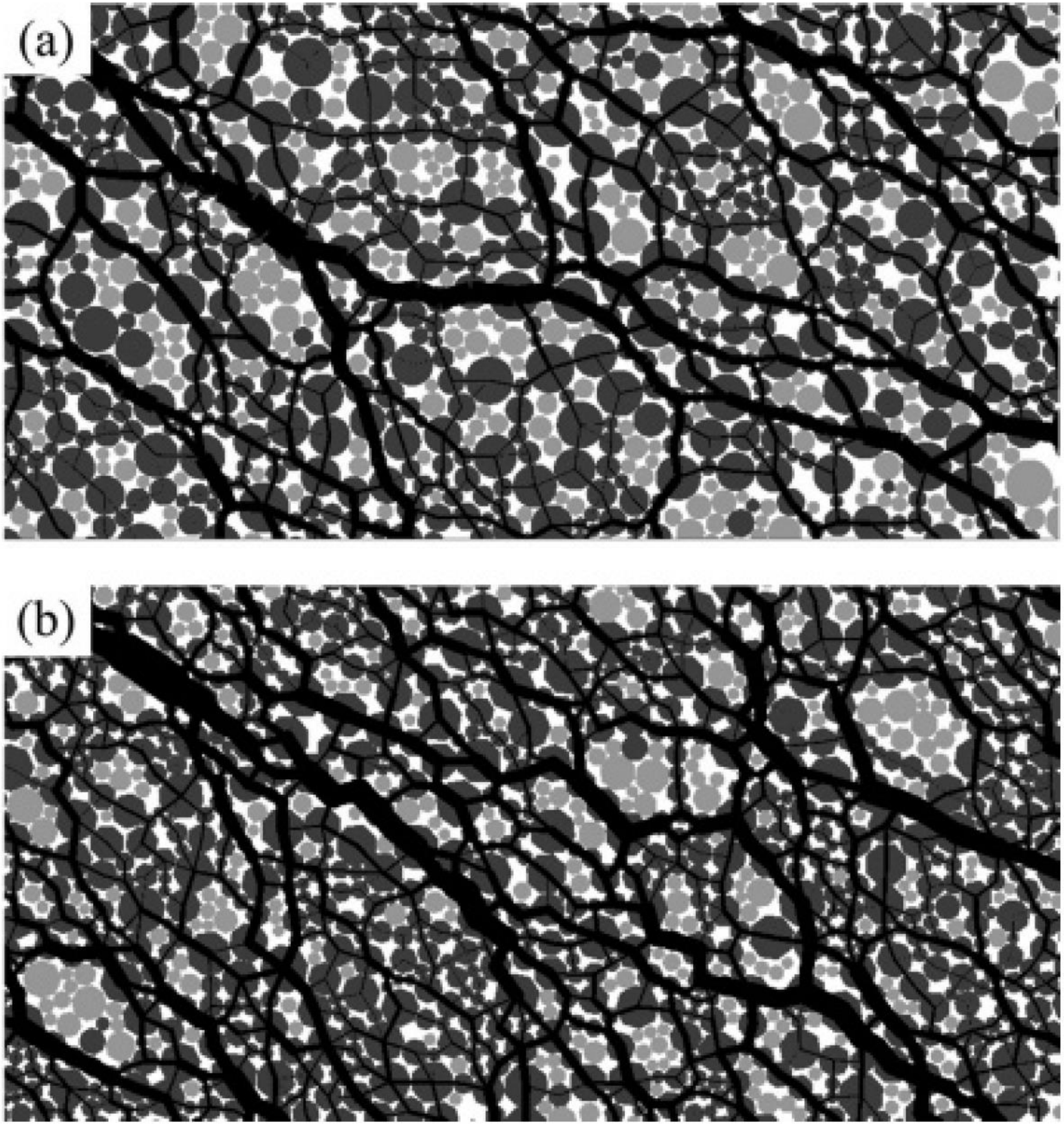

0.6
0.4
0.2
q/p
Discs
Polygons
0.82
0.80
φ
0
0.1
0.2
0.3
0.4
μ_r, 1/n_s

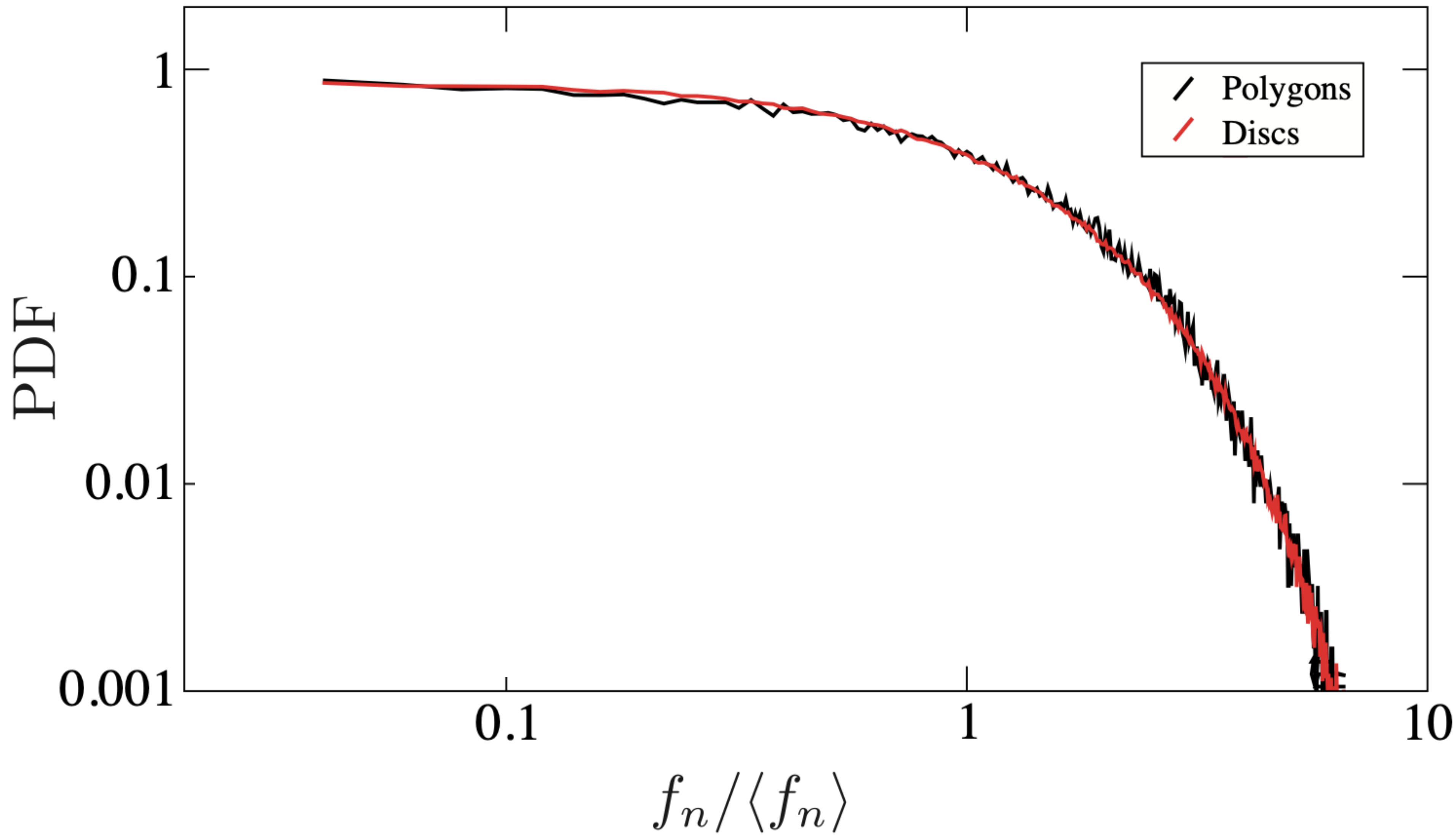


0.6
0.4
0.2
q/p
Polygons
Discs
0.82
0.80
ϕ
0
0.05
0.1
0.15
$\mu_r, \quad \frac{1}{4}\tan\frac{\pi}{2n_s}$



Polygons
Discs
0.6
0.4
0.2
0.3
0
0.05
0.1
0.15
a_c
a_n
a_t
μ_r, $\frac{1}{4}\tan\frac{\pi}{2n_s}$


(a)
(b)



PDF
1
0.1
0.01
0.001
0.1
1
10
$f_n/\langle f_n \rangle$
Polygons
Discs

Conclusions

We examined three particle-scale mechanisms that are influenced by particle shape: arching, local ordering, and particle rotation.

1) Particle shape affects contact orientation anisotropies in the weak and strong networks of a granular packing, allowing for higher force anisotropy in the strong network and therefore higher shear strength.

2) Face-face contacts lead to lower contact orientation anisotropy but larger normal force anisotropy (force chains) and tangential force anisotropy (friction mobilization): enhanced arching effect and stability.

3) Shape parameters induce local ordering in competition with short-range ordering induced by volume exclusions and unmonotonic dependence of packing fraction on the shape parameter.

4) A low-order shape parameter describes most of the trends: η

5) Aspherical particle shape leads to enhanced dissipation (compared to spheres), which is well captured by rolling friction.