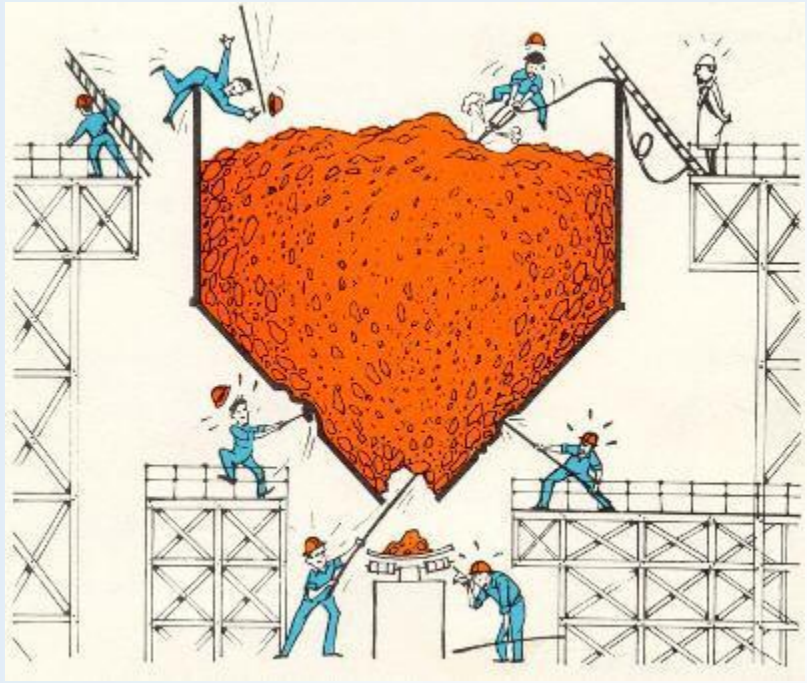
Research Gaps in Powder Flow: Purpose Driven Particle Scale Product Design to Solve Process Issues

By

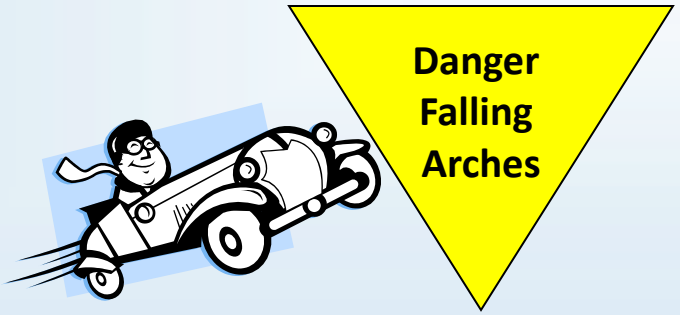
Dr. Kerry Johanson

Industrial Problem : Cohesive Hang-ups



Causes

- Poor process design
- Inaccurate property measurement
- Unknown design equations
- Poor scale-up models



Unscheduled down time costs billions in lost production

Relating Particle Scale Properties to Bulk Behavior and Then to Process Behavior

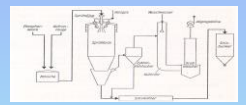
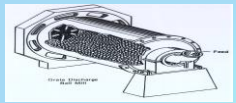
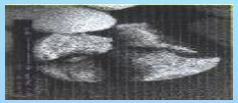
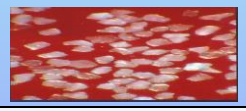
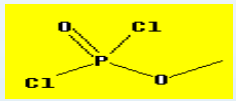
(or Bulletin 321: intelligent product design, a precursor of Bulletin 123)

Intriguing Question:

Can one use particle scale parameters and design a bulk material with a prescribed cohesive strength?

If so, what are the product design rules?

The Model and The Measurement

Length Scale		Model	Measurement
Environment		Chemistry	Satellite
Process		Population Balance	Size Distribution
Equipment		CFD Finite Element	Pilot Plant
Powder Slurry		Constitutive Equation	Shear Cells Viscometers
Particle		DEM	AFM,Indenter
Molecular		Molecular Simulation	Spectroscopy

What part of the sandbox do you want play in?

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Objective for Bulletin 321

- Measure inter-particle force laws
- Predict bulk behavior from interparticle force laws
- Optimize and extend bulk properties to processes

Increased Industry Productivity

Process Design

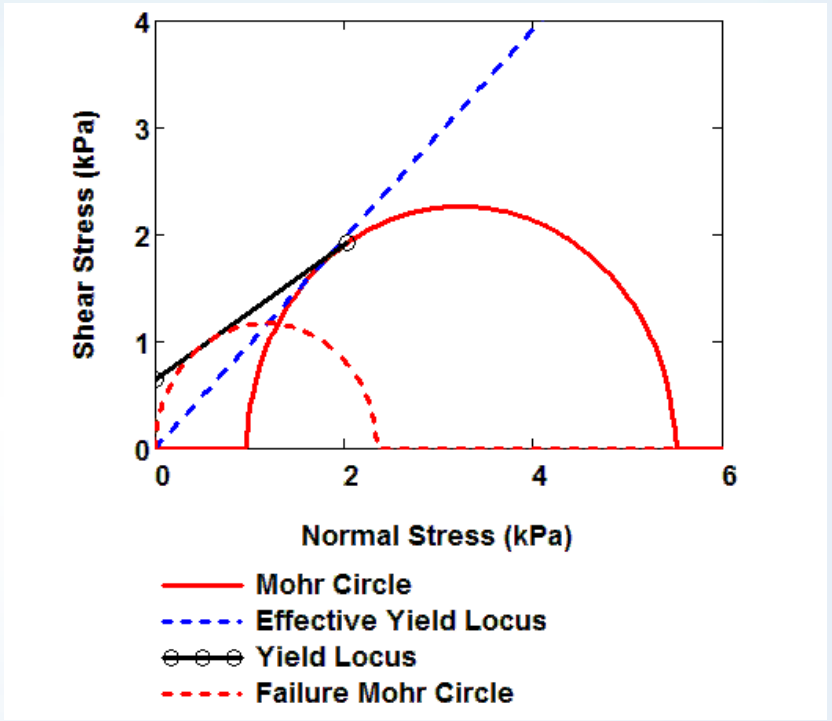
Bulk Behavior

Particle Forces

Chemistry

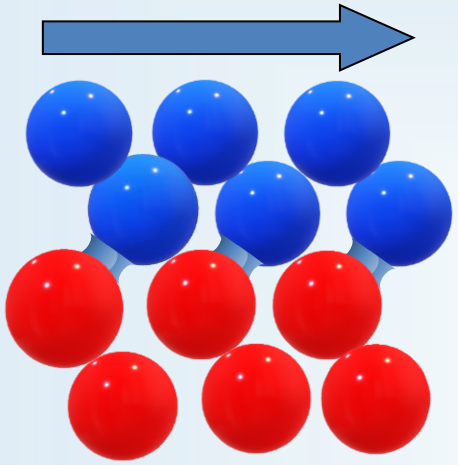
Understanding what influences bulk strength will help predict:

- Arching
- Rathole
- Blending
- Agglomeration
- Milling
- Segregation
- Fluidization
- Caking

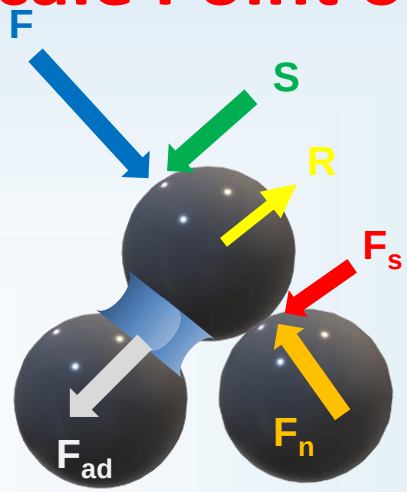


The yield locus is the envelope of stress states that will cause incipient failure of a bulk material subjected to the stress condition given by a particular Mohr circle shown here. There is a unique yield locus (black line) for each stress condition given by the largest Mohr circle.

Understanding Strength from a Particle Scale Point of View



Shear causes breakage of adhesion and friction bonds



$$E_{mech} = 2 \cdot \tau \cdot A_{sl} \cdot R$$

Mechanical work to move all particles past the next adjacent particle

$$W_{step} = W_1 \cdot N \cdot A_{sl} \cdot \frac{C_n}{2} \cdot n_{layer}$$

Work to separate the adhesive bridges between the particles

W_1	= work to separate one bridge
N	= number of particles in shear area
N_{1ayer}	= number of shearing layers
C_n	= coordination number

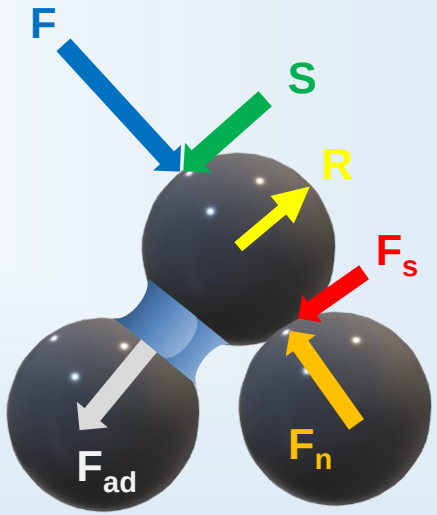
Incorporation of Friction Force in Bulk Strength Model

$$W_{mech} = E_{br} + E_{fr}$$

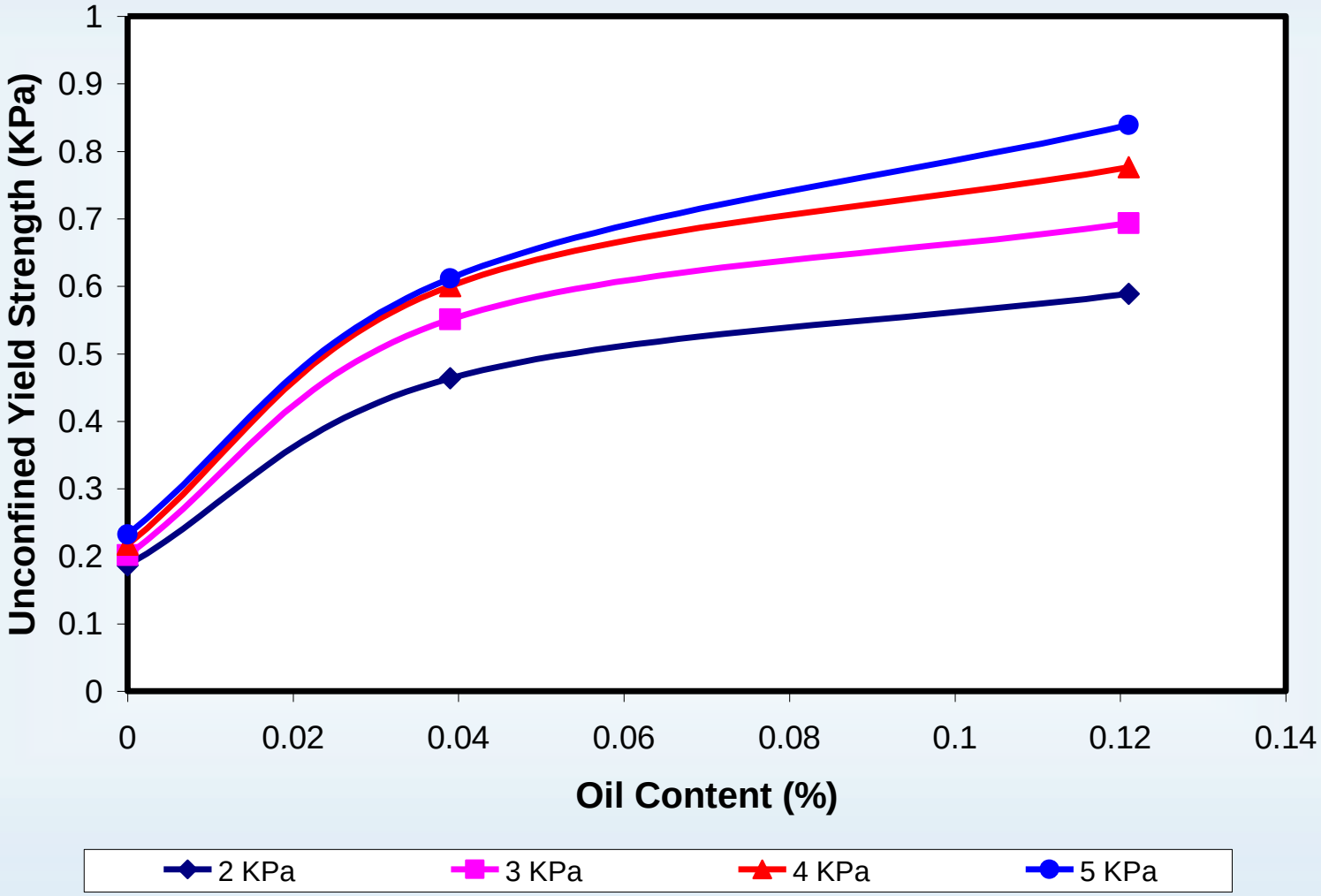
$$f_c = \frac{8 \cdot \gamma \cdot k_r \cdot n_l \cdot p \cdot \cos\theta}{\sqrt[3]{k_h} \cdot Dp \cdot \cos\phi} \sqrt{\frac{C \cdot n_c \cdot \rho_{sol}}{3 \cdot \rho_{liq}}} + \frac{\mu \cdot A_h \cdot n_c \cdot n_l \cdot p \cdot \sqrt[3]{k_r^{\,2}}}{6 \cdot \pi \cdot Dp \cdot H^2 \cdot rf \cdot \cos\phi}$$

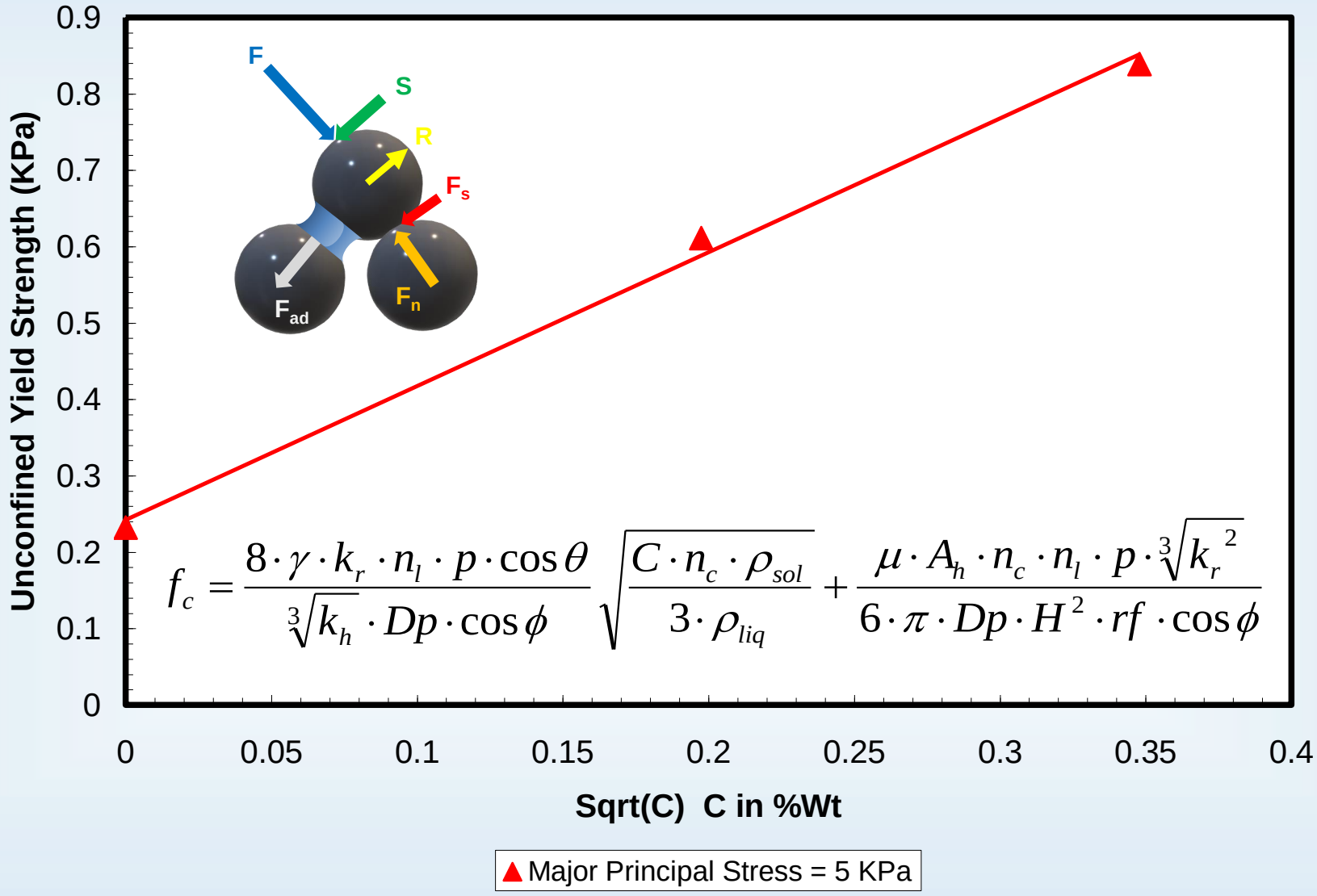
γ – surface tension
ρ_{liq} – density of binder
n_c – coordination number
p – probability of rupture
k_h – hexagonal packing factor
rf – roughness factor
ϕ – angle of internal friction

θ – contact angle
μ – friction coefficient
n_l – number of shear layers
k_r – random packing factor
H – separation distance
ρ_{sol} – density of particles
A_h – Hamaker constant



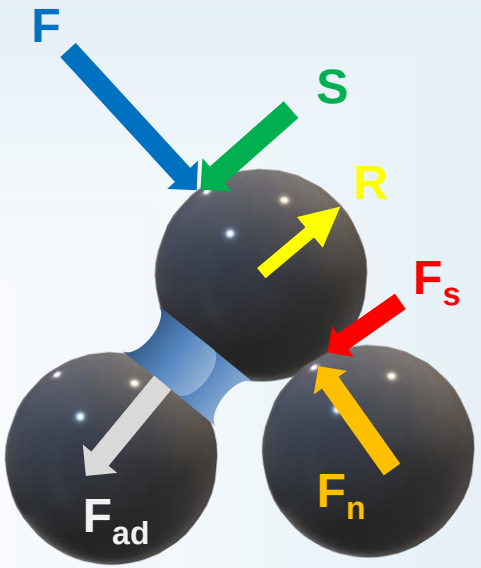
Glass Bead Strength

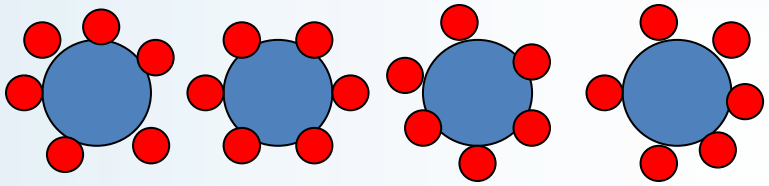



Unconfined Yield Strength (KPa)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Sqrt(C) C in %Wt
F
S
R
Fs
Fad
Fn
$f_c = \frac{8 \cdot \gamma \cdot k_r \cdot n_l \cdot p \cdot \cos\theta}{\sqrt[3]{k_h} \cdot Dp \cdot \cos\phi} \sqrt{\frac{C \cdot n_c \cdot \rho_{sol}}{3 \cdot \rho_{liq}}} + \frac{\mu \cdot A_h \cdot n_c \cdot n_l \cdot p \cdot \sqrt[3]{k_r^{\,2}}}{6 \cdot \pi \cdot Dp \cdot H^2 \cdot rf \cdot \cos\phi}$
Major Principal Stress = 5 KPa

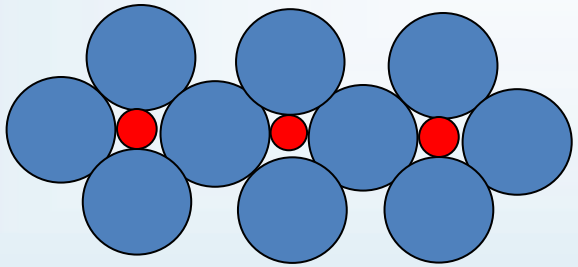
Factors Affecting Cohesive Strength

- van der Waals bonds
- Moisture (capillary forces)
- Mechanical interlocking of particles
- Chemical reactions

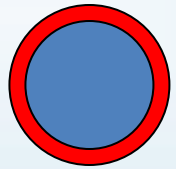




Coating of coarse with fines



Surrounding fines with coarse



Surface coat (modification)

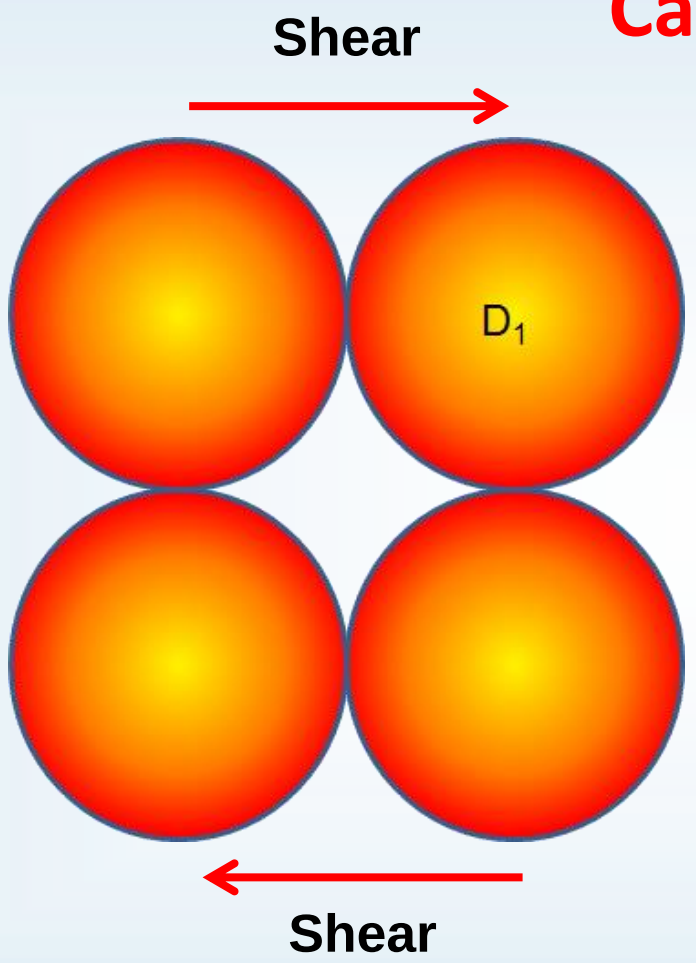
Strength Models for Mono Dispersed Systems

Model	Mechanism	Source
$fc = \frac{K_1}{Dp^2}$	Van der Waals forces	Mollerus [4]
$fc = \frac{K_2 \cdot \sqrt{C}}{Dp}$	Capillary forces	Rabinovich [2]
$fc = \frac{K_3}{Dp^{1/2}}$	Elastic fracture	Rumpf [1]
$fc = \frac{K_4}{Dp^n}$	Plastic-elastic fracture	Spetch [3]

$$fc = \frac{K_1 \cdot \sqrt{C}}{D_p} \cdot X_{capillary} + \frac{K_2}{D_p^{\,2}} \cdot X_{van} + \frac{K_3}{D_p^{\,n}} \cdot X_{fracture}$$

Poly-Disperse Material

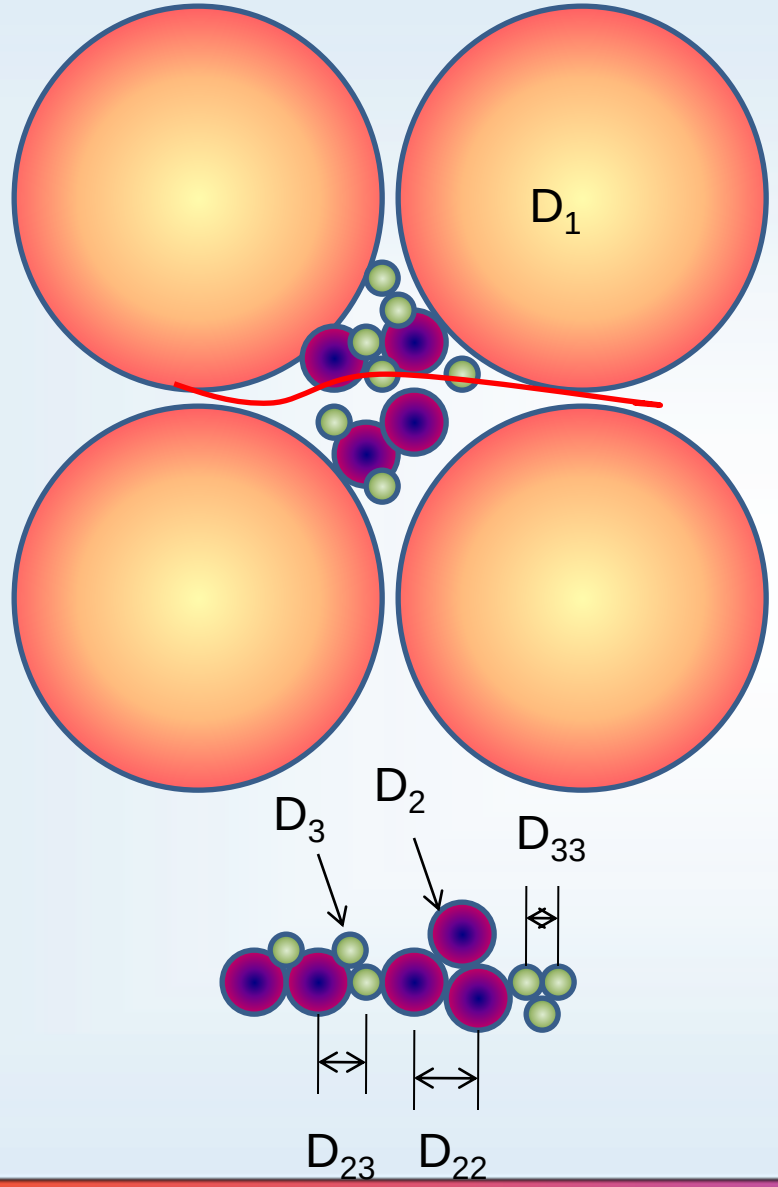
Case 1



There are no small particles in the system. Shear takes place between adjacent large particles. The strength of this structure is found from the strength values for mono-dispersed systems for large particles.

$$fc = \sum_{A} K_1 \cdot \frac{Dp_{50}}{Dp_A} + K_2 \cdot \left(\frac{Dp_{50}}{Dp_A} \right)^2$$

Where Dp_A is determined by the size of the larger particles in the bulk material

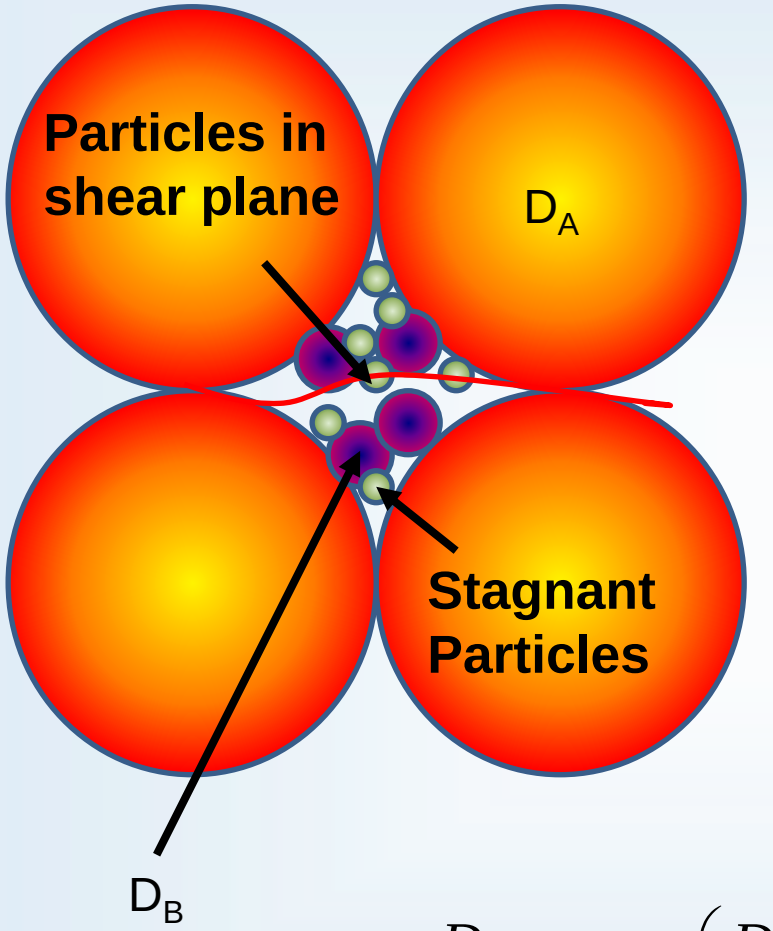


Case 2

There are an excess of small particles in the system. Shear takes place between adjacent small particles. The strength of this structure is found from the strength values for mono-dispersed systems for small particles where Dp_B is the average particle size in the void structure.

$$fc = \left[\left(K_1 \cdot \frac{Dp_{50}}{Dp_B} + K_2 \cdot \left(\frac{Dp_{50}}{Dp_B} \right)^2 \right) \right]$$

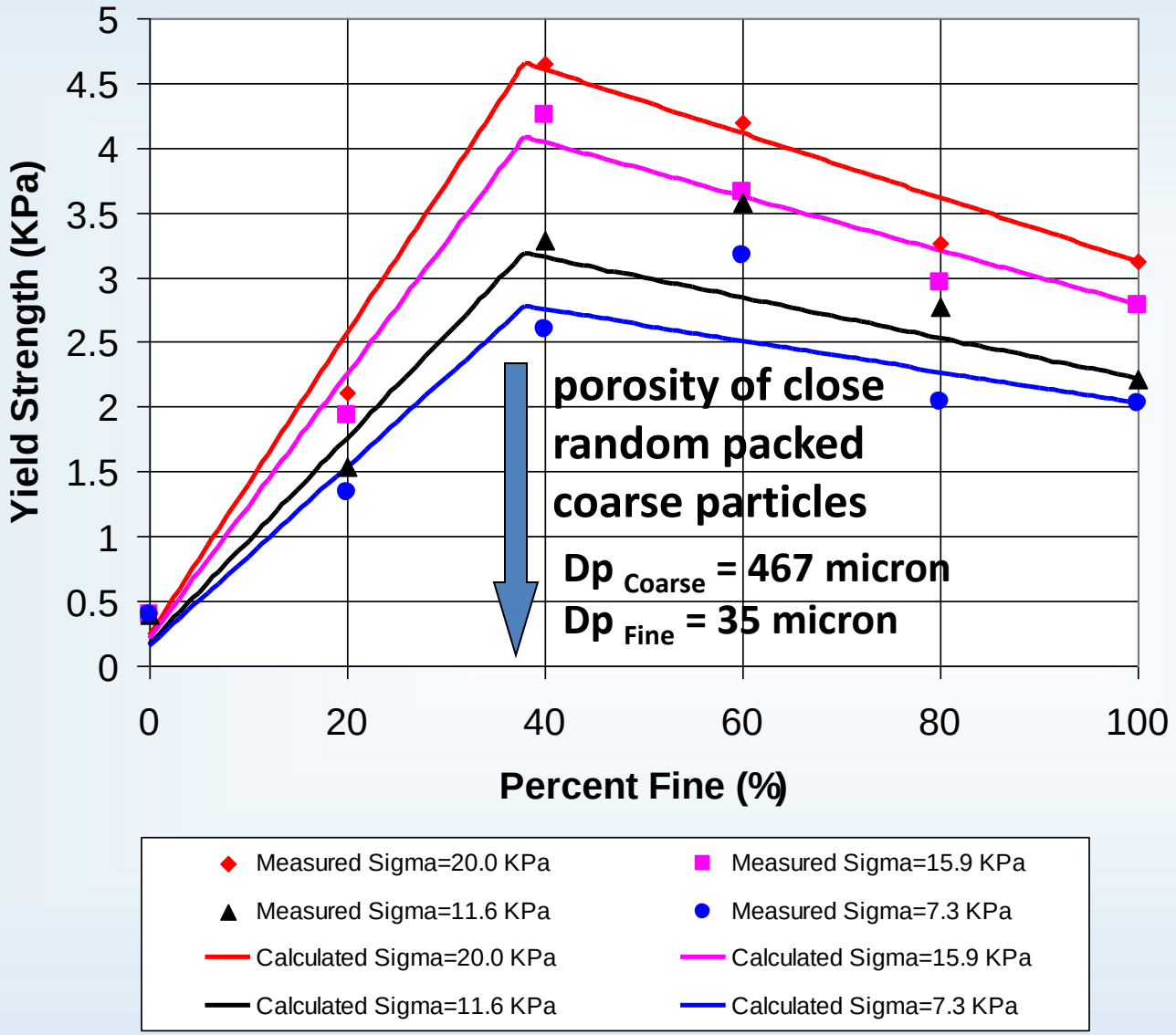
Case 3

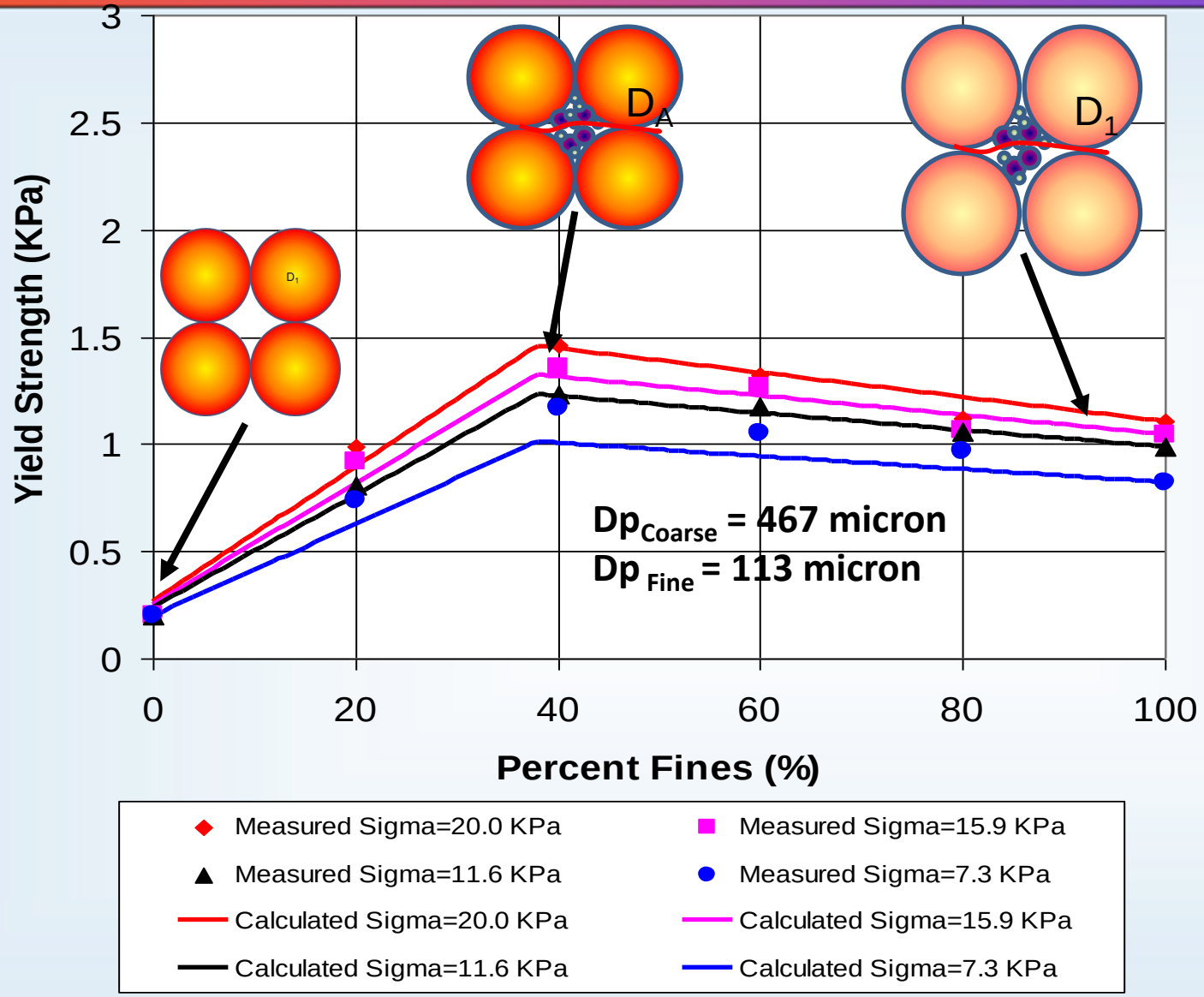


Dp_A is the particle size of the large particles
Dp_B is the particle size of the small particles
n_{shear} is the number of particles in the shear layer between larger particles

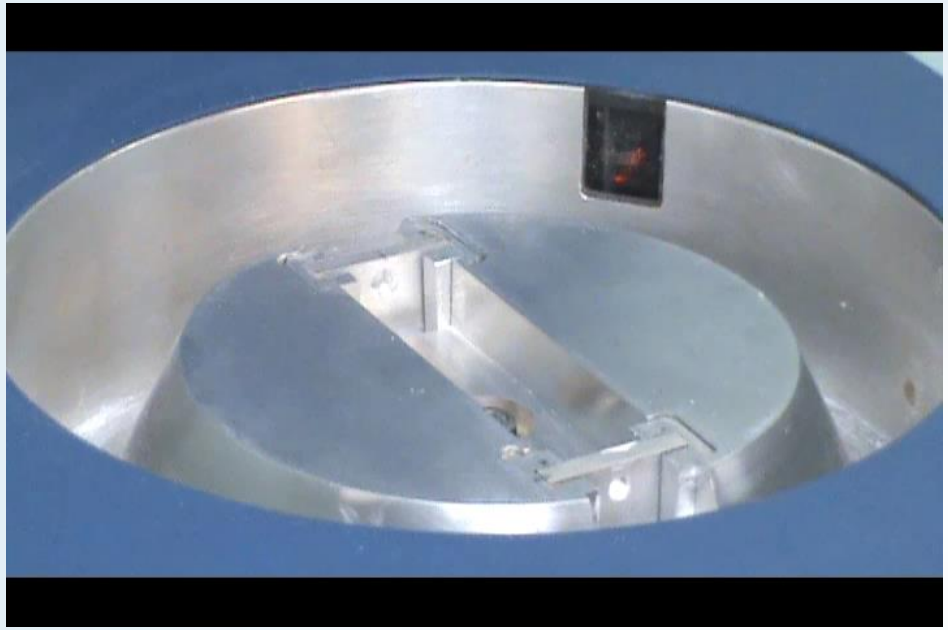
Strength includes the effect of the forces between the larger particles and the smaller particles in the mode of shearing between the larger particles. Note some particles in the void are carried with the larger particle and do not undergo shear. These particles are not counted when computing the strength.

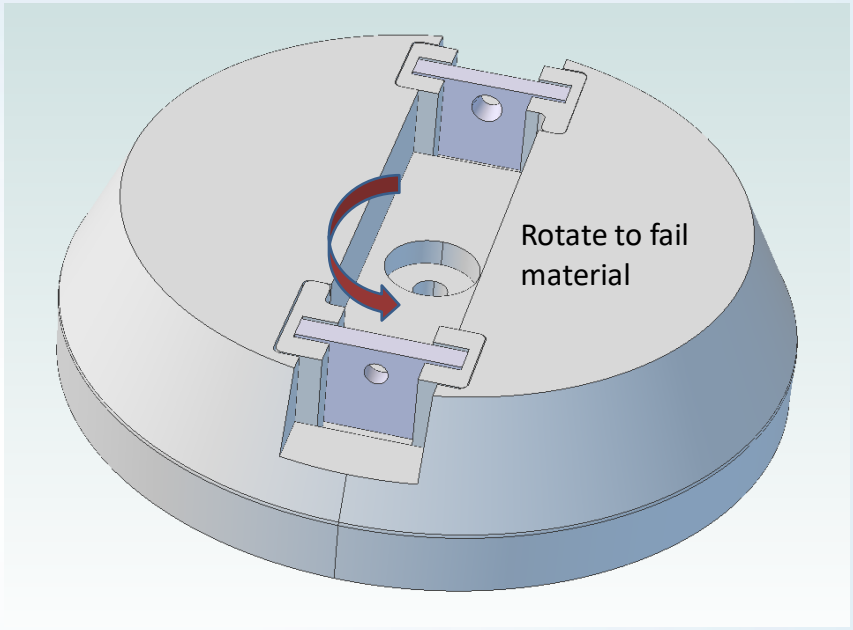
$$fc = \sum_A K_1 \cdot \frac{Dp_{50}}{Dp_A} + K_2 \cdot \left(\frac{Dp_{50}}{Dp_A}\right)^2 + \left[\left(K_1 \cdot \frac{Dp_{50}}{Dp_B} + K_2 \cdot \left(\frac{Dp_{50}}{Dp_B}\right)^2\right) \cdot n_{shear}\right]$$


Yield Strength (KPa)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
Percent Fine (%)
porosity of close
random packed
coarse particles
Dp Coarse = 467 micron
Dp Fine = 35 micron
Measured Sigma=20.0 KPa
Measured Sigma=15.9 KPa
Measured Sigma=11.6 KPa
Measured Sigma=7.3 KPa
Calculated Sigma=20.0 KPa
Calculated Sigma=15.9 KPa
Calculated Sigma=11.6 KPa
Calculated Sigma=7.3 KPa


Yield Strength (KPa)
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
Percent Fines (%)
D1
DA
D1
Dp Coarse = 467 micron
Dp Fine = 113 micron
Measured Sigma=20.0 KPa
Measured Sigma=15.9 KPa
Measured Sigma=11.6 KPa
Measured Sigma=7.3 KPa
Calculated Sigma=20.0 KPa
Calculated Sigma=15.9 KPa
Calculated Sigma=11.6 KPa
Calculated Sigma=7.3 KPa

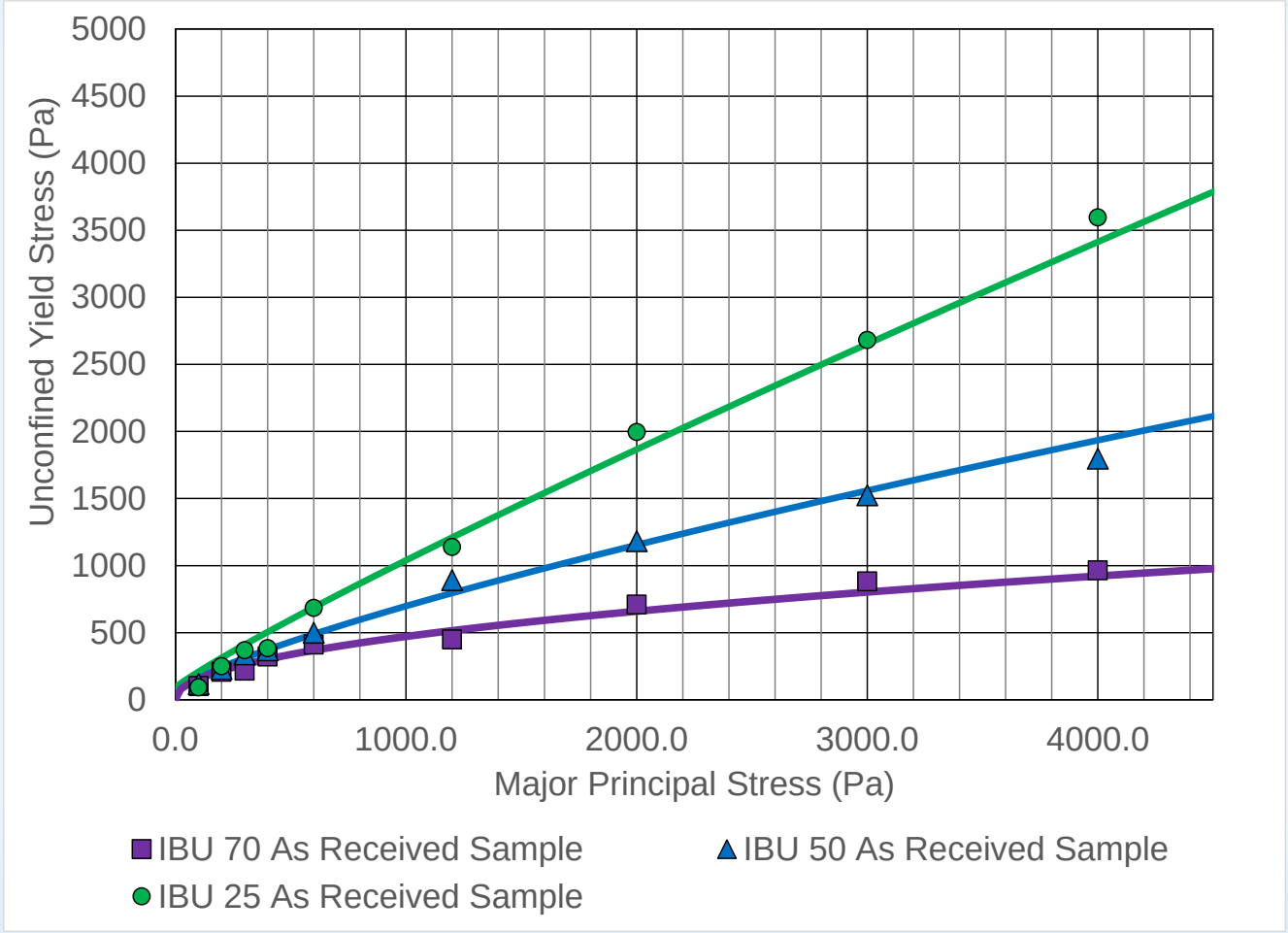
The New Generation Strength Tester



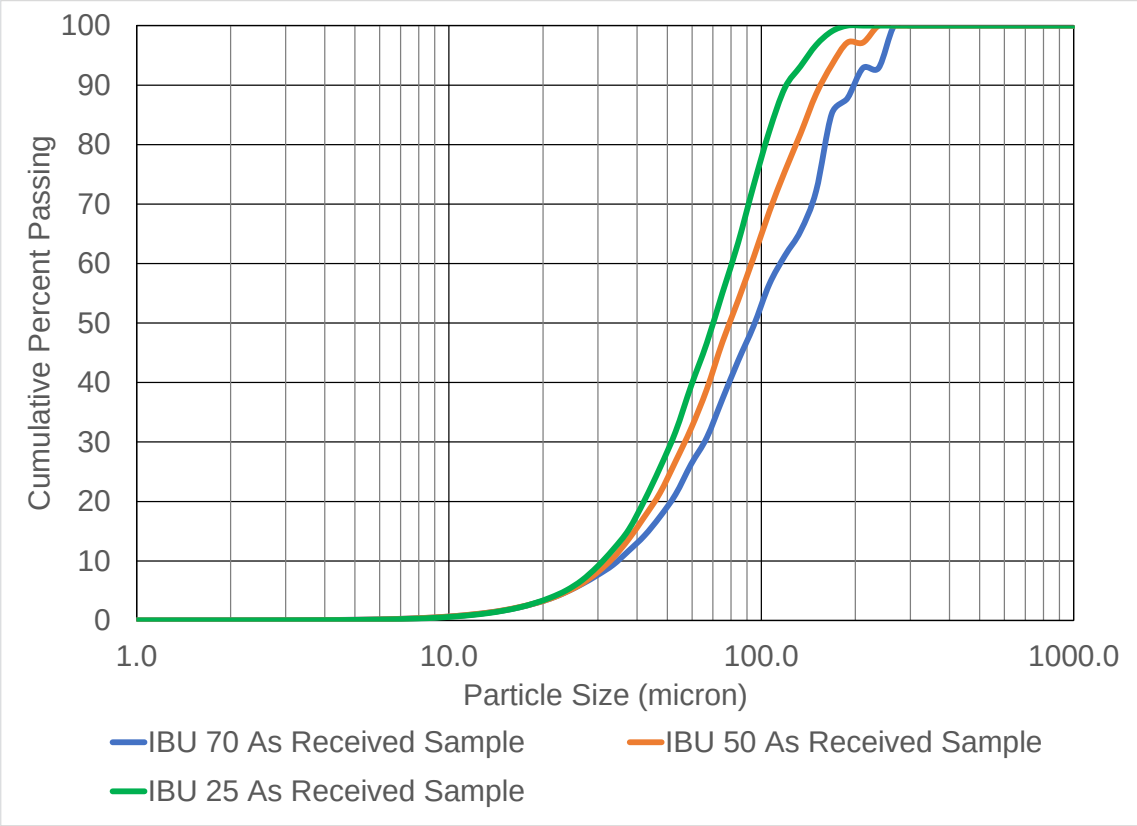


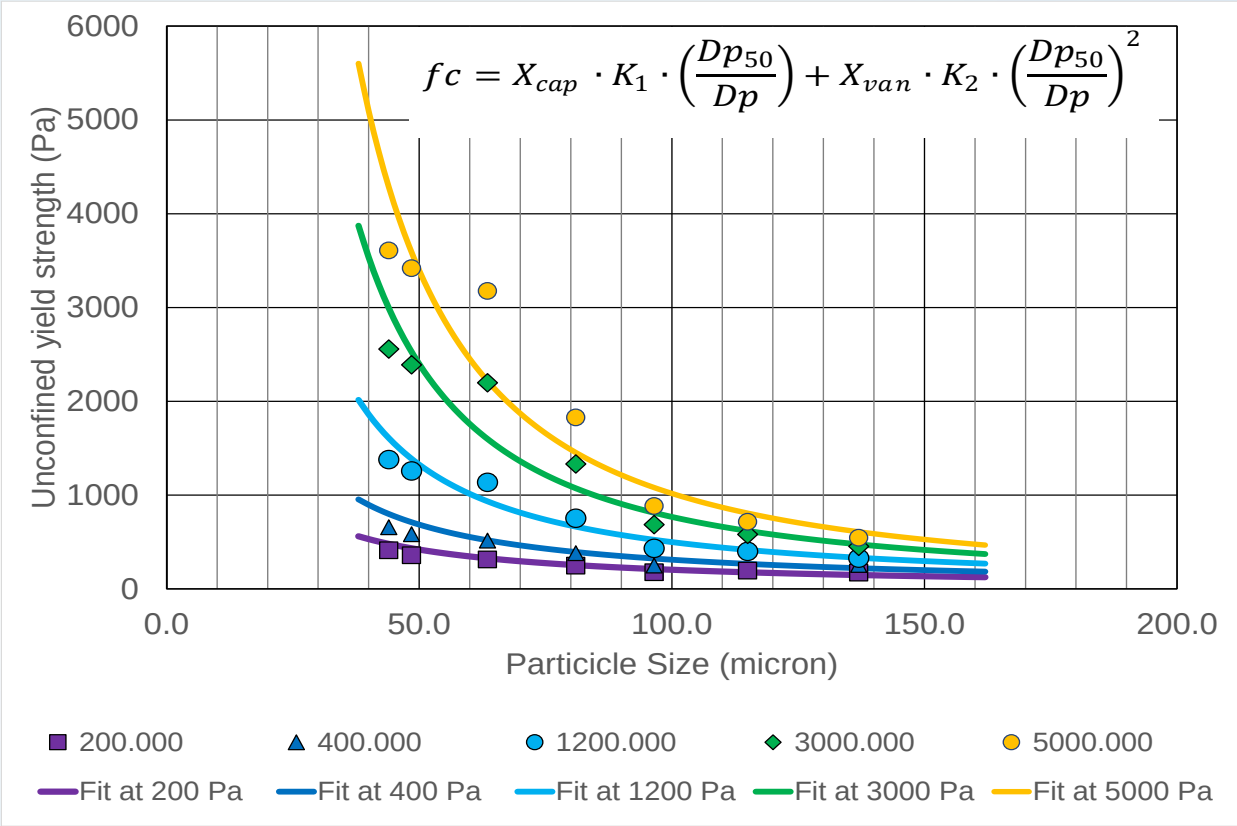
- Remove gates
- Spin rotor to generate forces to drive material from cell
- Laser detects free path
- Compute the unconfined yield strength

Bulk Strength Measured by SSSpinTester



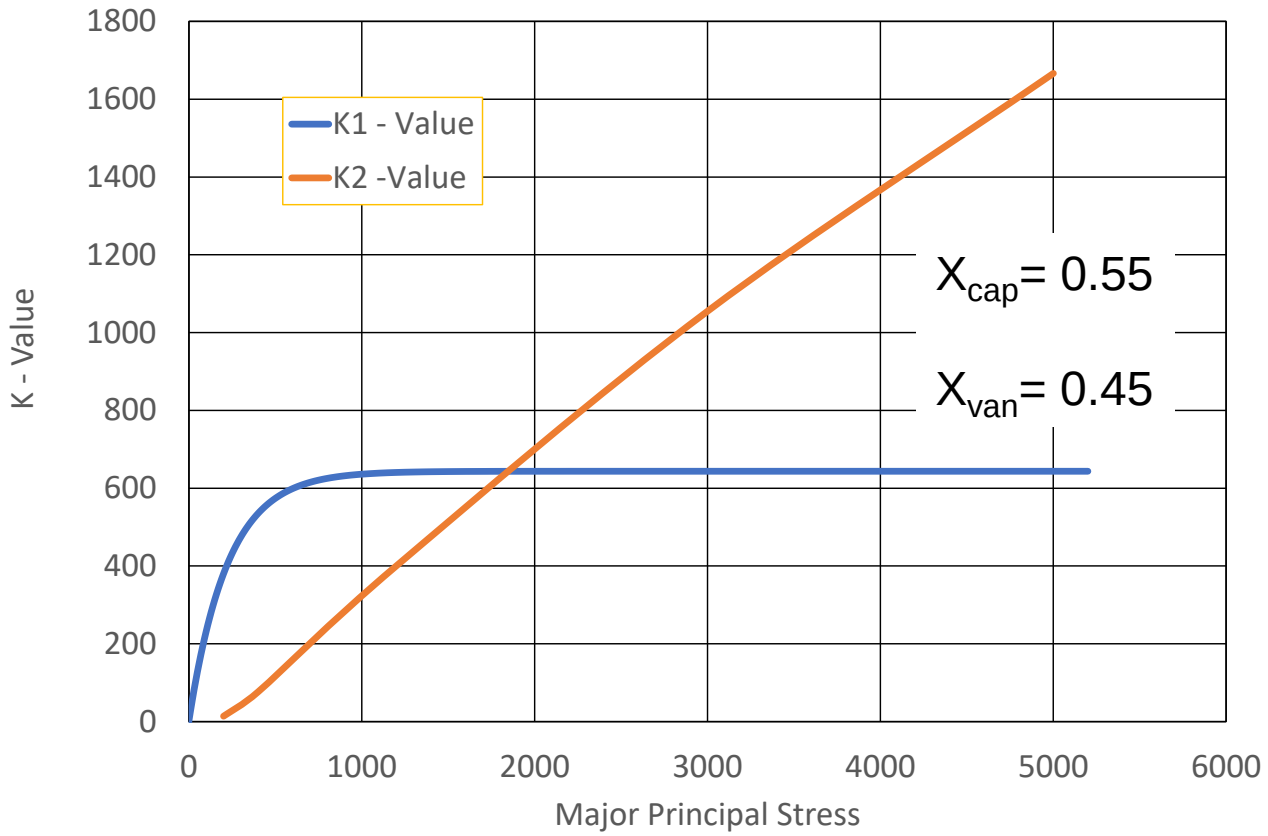
Particle Size Versus Strength





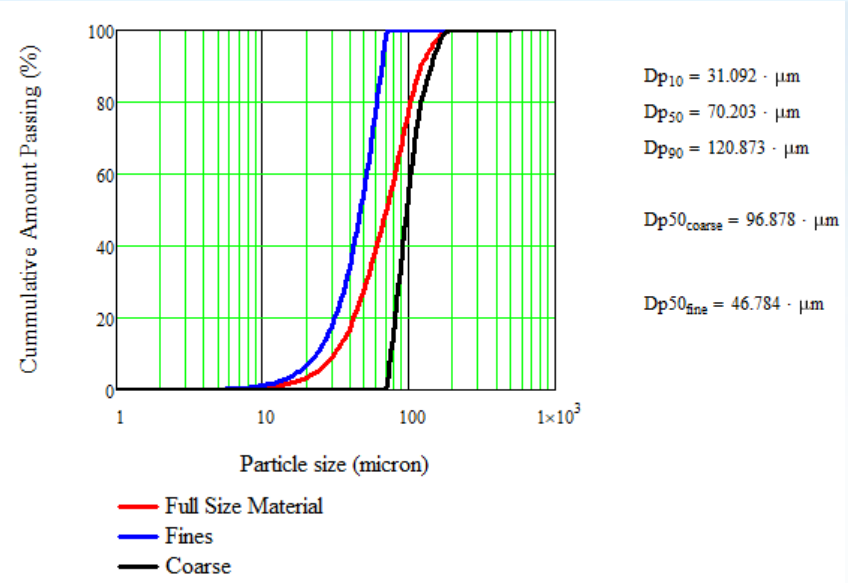
1. Divide bulk material to particle size bins narrow PSD
2. Measure Bulk Strength of Narrow PSD
3. Plot Bulk Strength versus Particle Size – Fit combined strength law

$$fc = X_{cap} \cdot K_1 \cdot \left(\frac{Dp_{50}}{Dp}\right) + X_{van} \cdot K_2 \cdot \left(\frac{Dp_{50}}{Dp}\right)^2$$

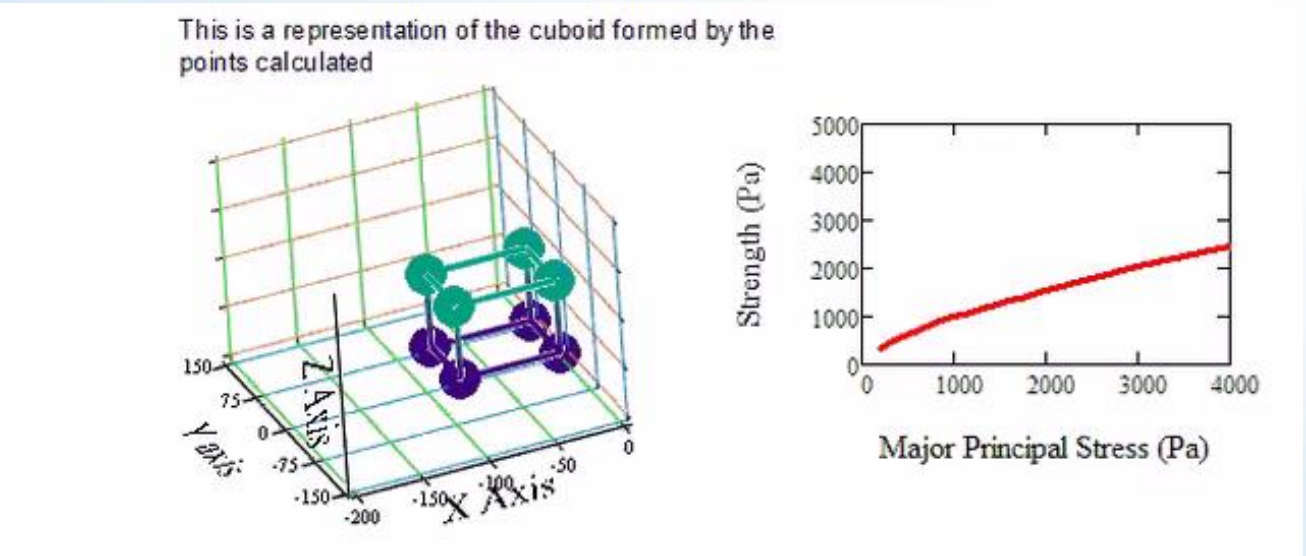


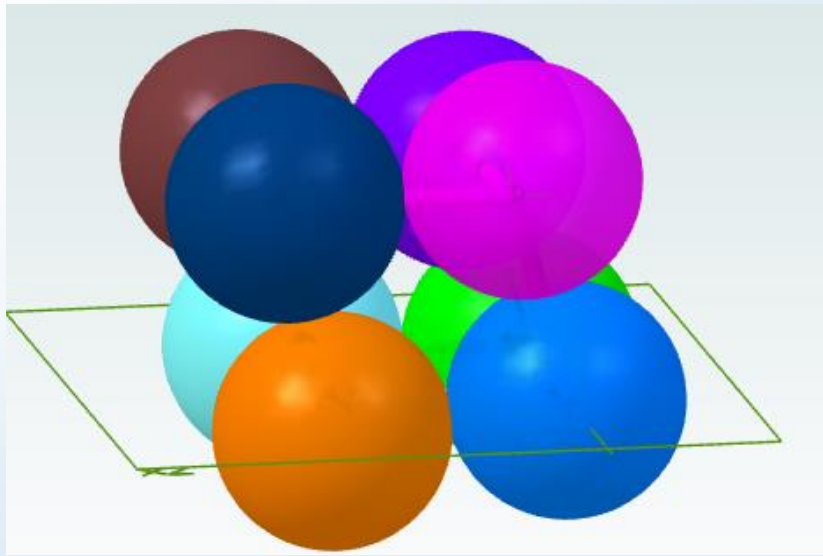
1. **Find K1 and K2 as a function of consolidation Stress**

2. **Find the partition coefficients**

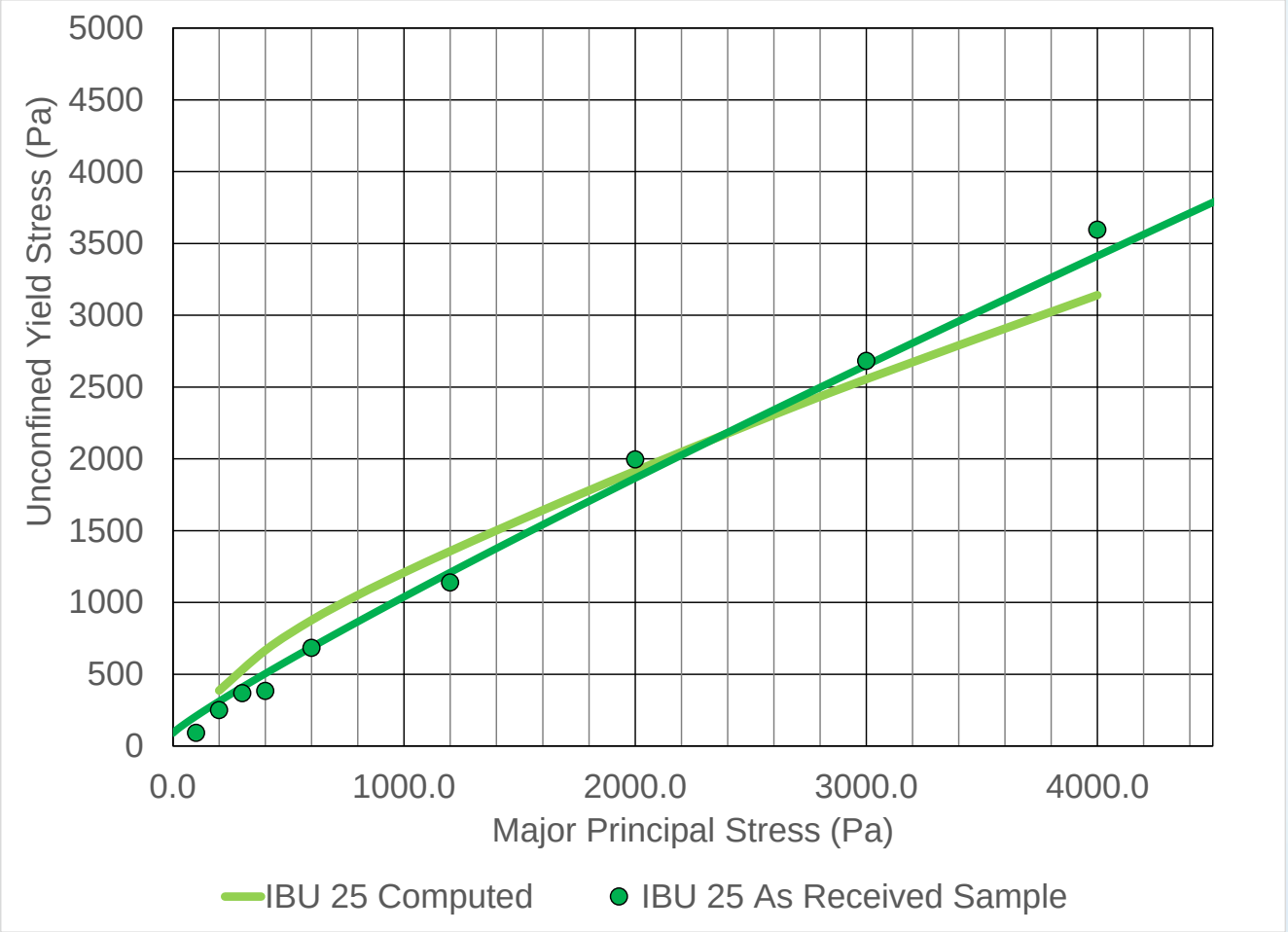


Create box of particles from coarse PSD and fill the void between the particles with particles from fine PSD. Compute the strength from poly disperse model.

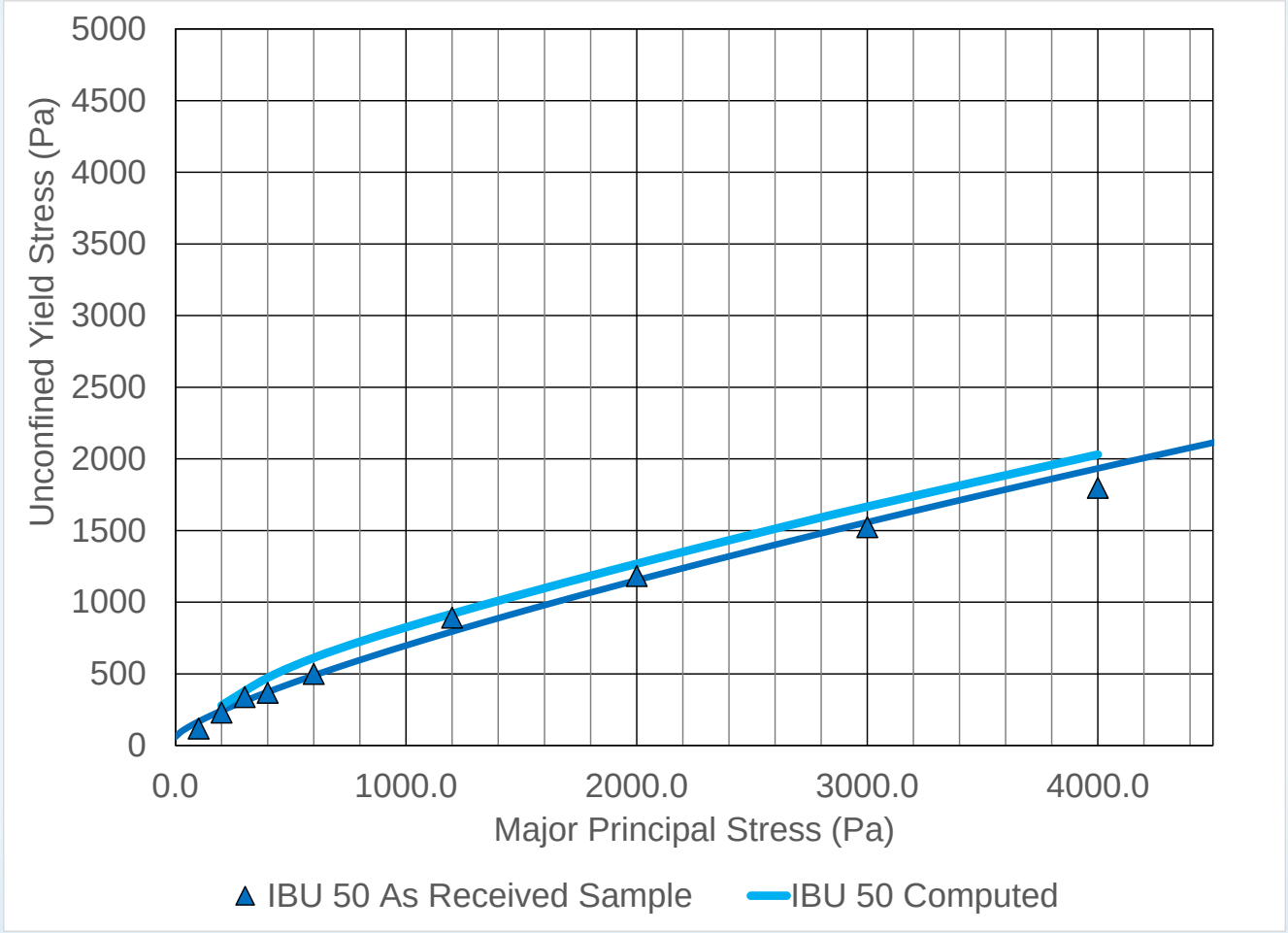




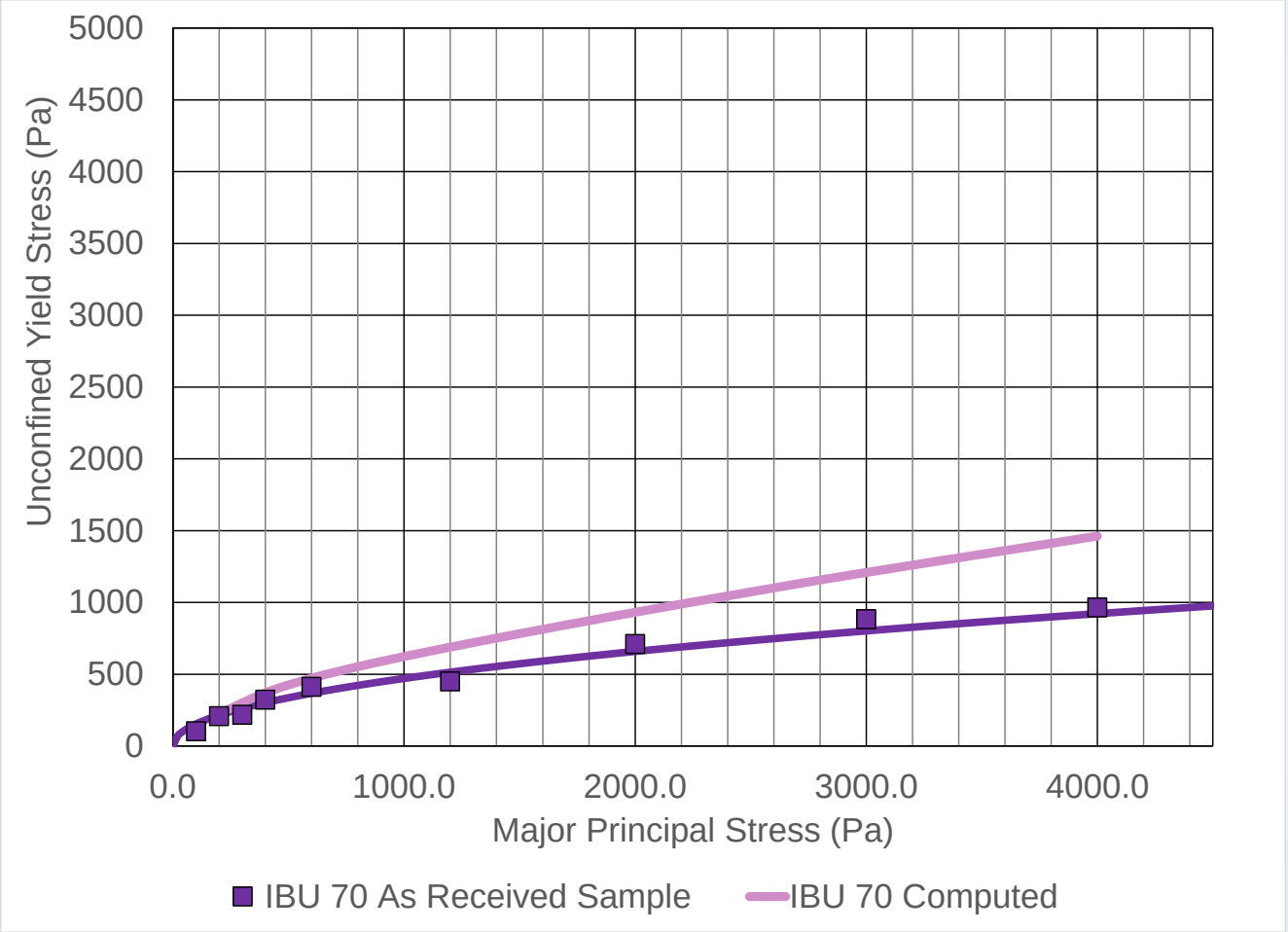
Measured Strength Compared to Computed Strength



Measured Strength Compared to Computed Strength



Measured Strength Compared to Computed Strength



Areas of study and exploration – Needed for Bulletin 321

- How does shape effect bulk strength
- How do elastic fibers affect bulk strength
- How does hydrophobic surfaces affect bulk strength
- What do I do about the strength of mixtures where:
  - ❑ I have different shapes
  - ❑ I have different PDS
  - ❑ I have different contact angle
  - ❑ I have different van der Walls constants
  - ❑ How does partition of liquid between particles affect strength
  - ❑ What role does a flow aid play in strength reduction
  - ❑ What roles does a flow aid play in caking and cementation reactions

What part of the sandbox do you want play in?

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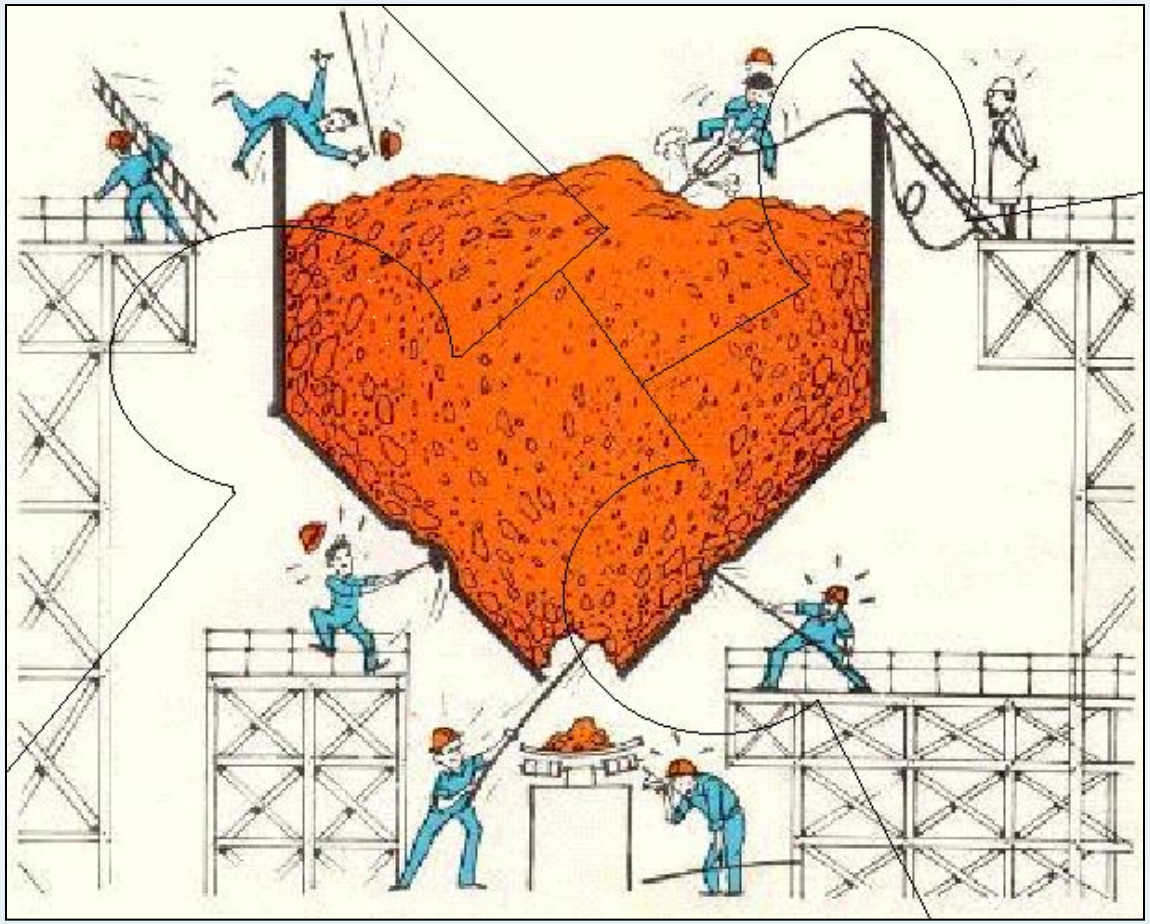
Extending and Filling the Gaps in Solids Radial Flow Theory

(or Bulletin 456: an extension of Bulletin 123)

Unconfined Yield Strength (fc) Causes Process Hang-ups



Save 50 Billion $ in Lost Revenue

Limiting Analysis Issues

- The work of Jenike and Johanson for computing arching, ratholing, and mass flow was based in a limit analysis.
- Predicts the limit of some behavior (i.e. the onset of funnel flow, the maximum stable arch, or the maximum stable rathole).

- Limit behavior is based on a certain set of assumptions.
- If assumptions violated result in the theory predicting poor results:
  - First, does the theory predict conservatively if the assumptions are violated?
  - Second, does the limit analysis presented in the original theory predict ultraconservative predictions forcing us into bad design choices?

Assumptions of the Radial Stress Theory

Question everything

1. Symmetric flow channel
2. Stress is symmetric about the centerline of process vessel
3. Slow - quasi static flow (no acceleration)
4. No external forces other than gravity
5. Stress is a linear function of the distance from the apex of hopper
6. The effective angle of internal friction goes through origin
7. Plastic conditions based of Mohr Coulomb failure criterion
8. Flow rule based on coincidence of direction of major principle stress and principle strain rate
9. No viscous effects
10. No gas press effects
11. No variable properties

Process Issue	Ranking - Predictive Ability (1-10)								
	Symmetric	Non - Symmetric	For 3D Geometry	Other Geometries besides bins	With Gas Effects	With external forces	For Elastic Solids	For Viscous materials	With variable properties
Rathole Prediction	7	3	1	1	6	3	3	1	1
Arching Prediction	8	4	1	2	7	3	3	1	6
Mass Flow	8	5	2	2	6	3	2	2	4
Velocity Pattern	7	5	2	2	6	3	2	2	4
Segregation	5	5	2	2	3	3	3	2	4
Flow rate	8	4	2	4	4	4	4	3	4

Red is Jenike's Contribution Green are areas of urgent need

Arching Issues

- Stresses near arch are orders of magnitude lower than can be measured with traditional testers.
- Theory assumes flow first analysis for flow factor computation.
- Theory assumes radial stress is the only stress that causes steady flow.
- Theory does not include the effect of geometry induced stress gradients.
- Theory does not include gas effects.
- Stress predicted from flow factor method gives stresses that are not possible in small diameter bins.

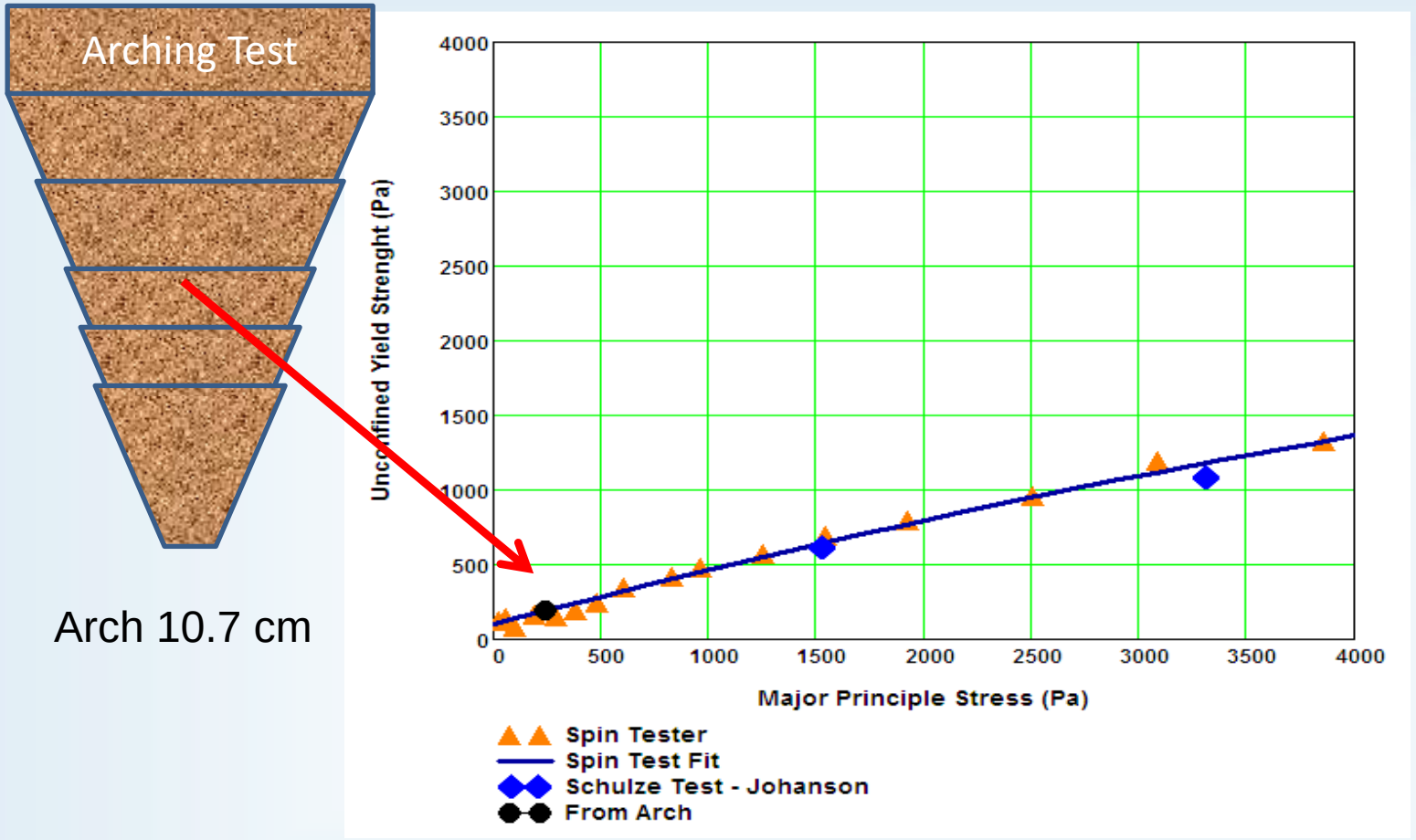
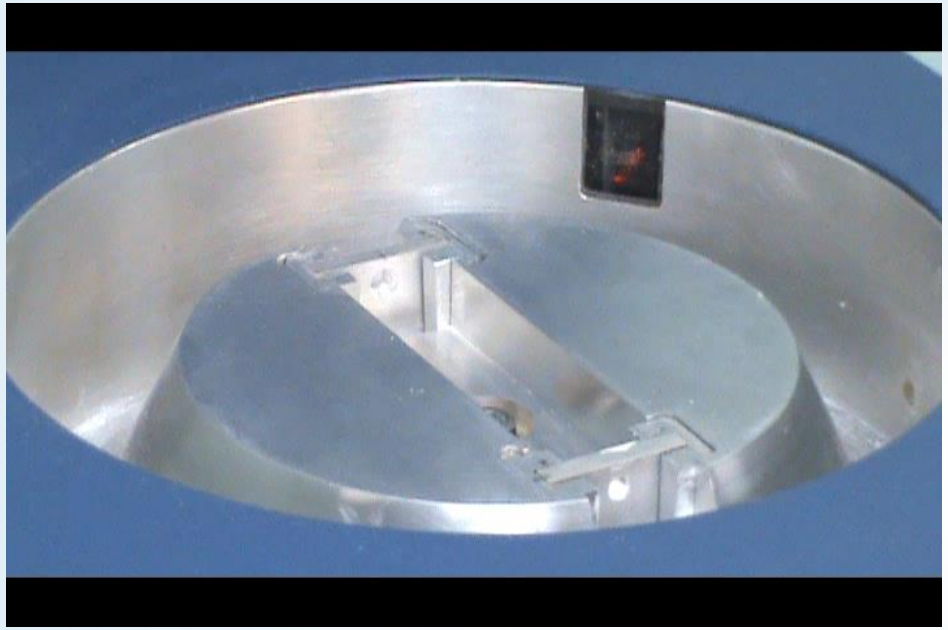
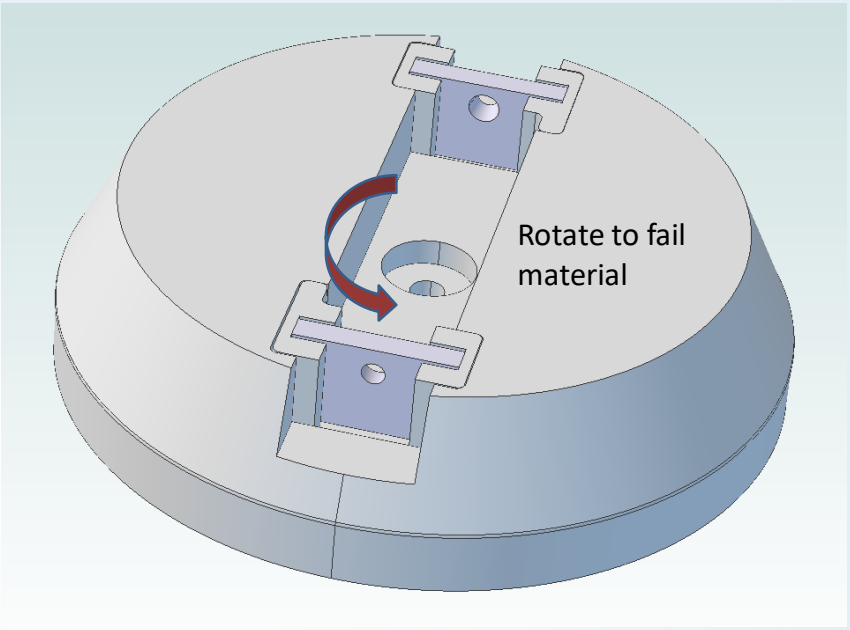


Figure 5. ***Comparison of the lower stress level unconfined yield strength of Argo corn starch measured with the Schulze direct shear method and the new test technique (SSSpinTester)***

Strength Measurement at Low Stress





- Remove gates.
- Spin rotor to generate forces to drive material from cell.
- Laser detects free path.
- Compute the unconfined yield strength.

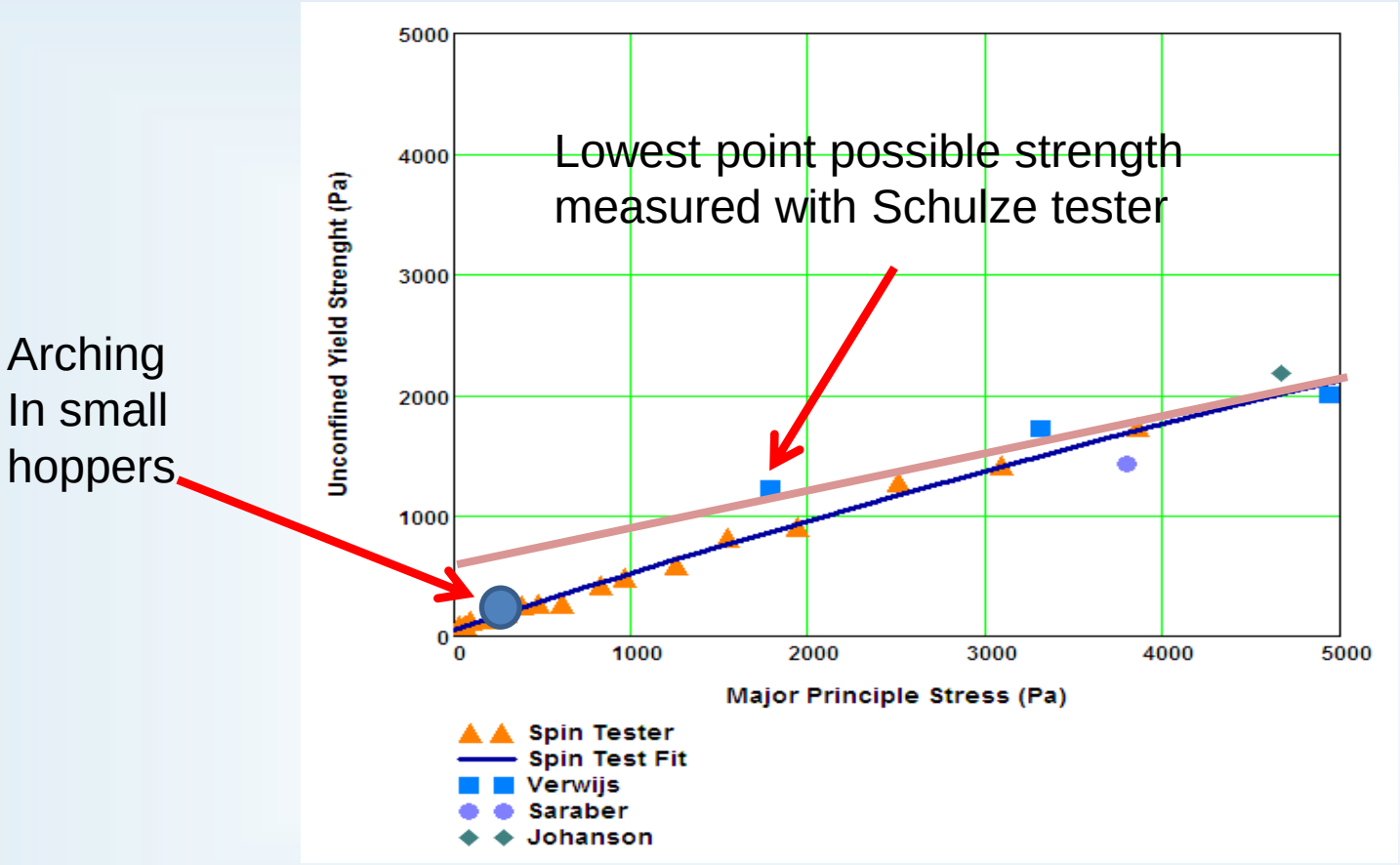
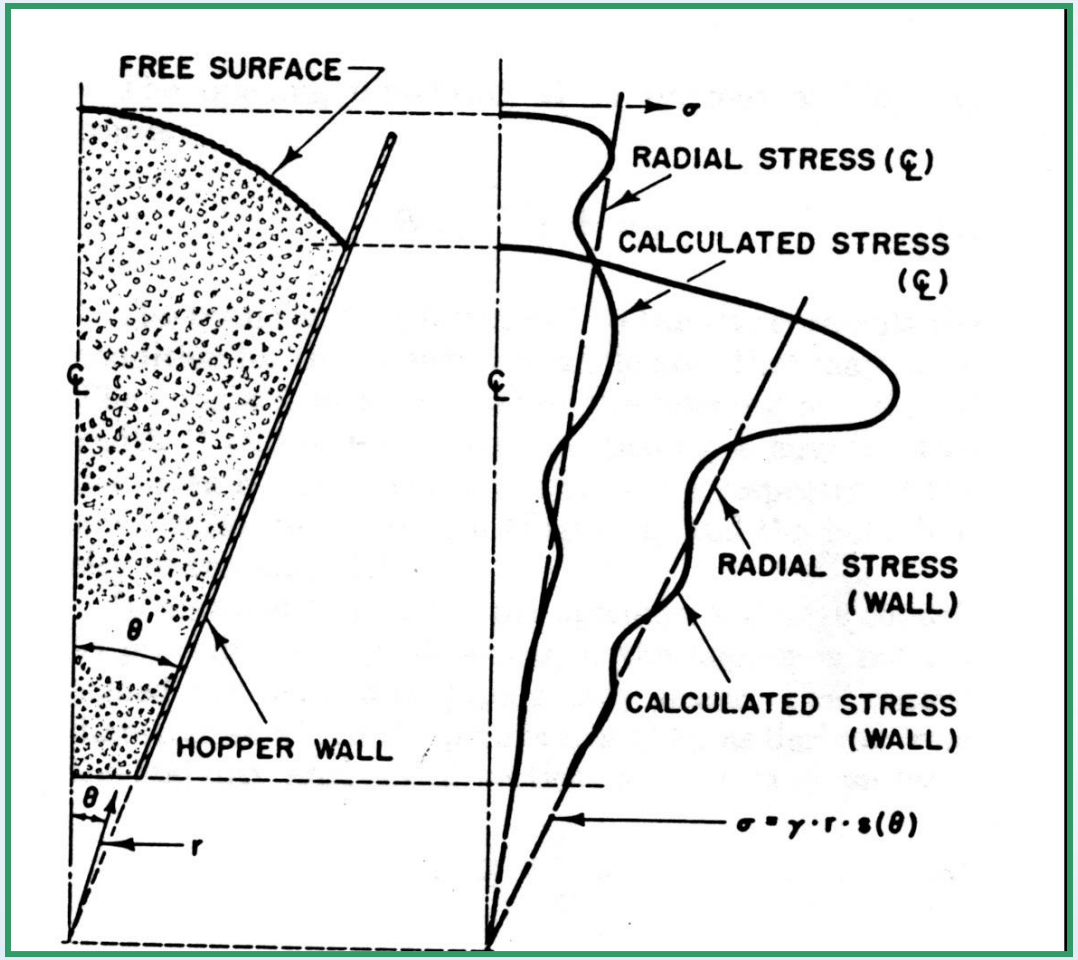


Figure 3. Comparison of low stress level BCR limestone data generated from three different studies and new test technique (SSSpinTester)

Review of Radial Stress Equations



The stress computed in the hopper section tends to a linear function based on distance from the apex of the cone.

$$\sigma = \gamma \; r \; s(\theta)$$

Radial Stress

Find flow factor ff = q(δ,θ,ϕ) for bin or hopper

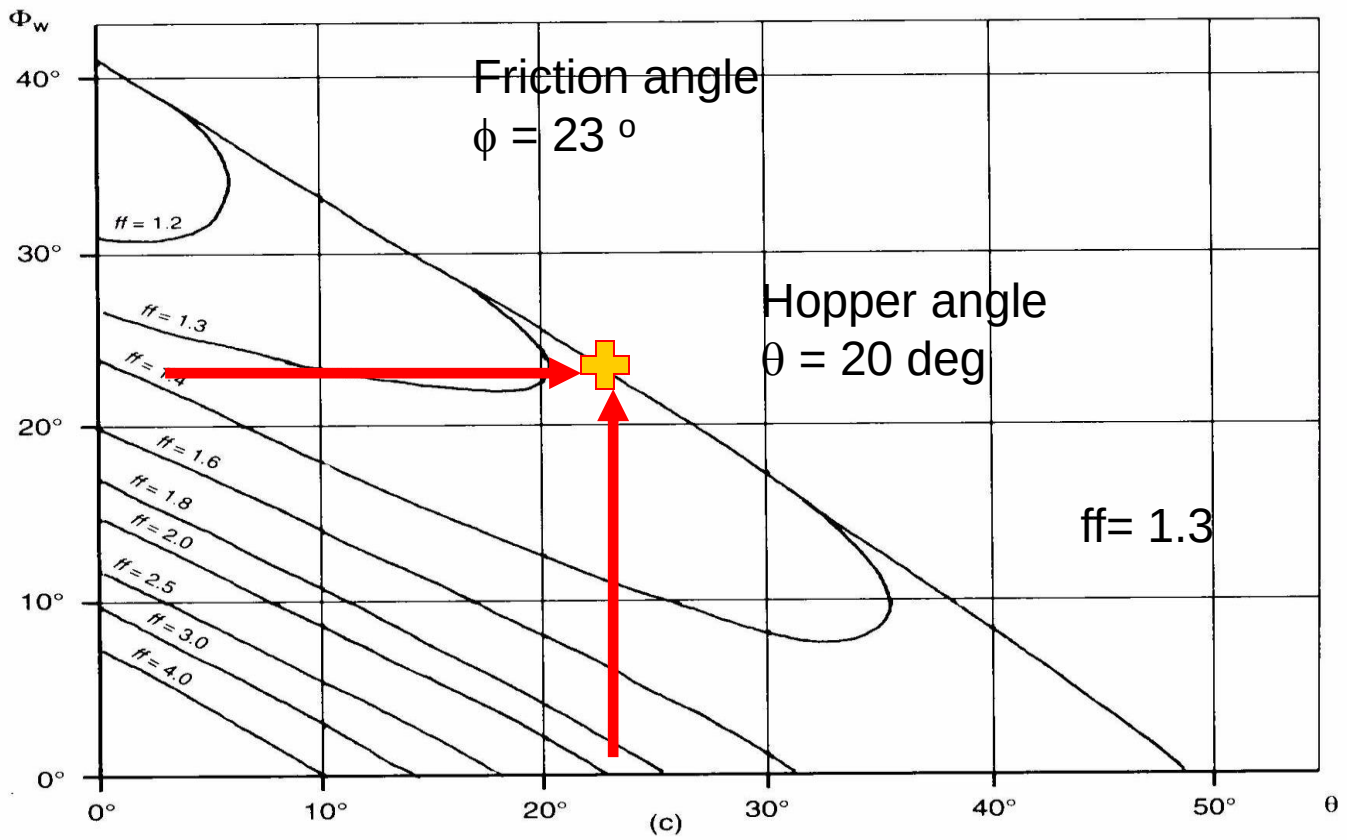
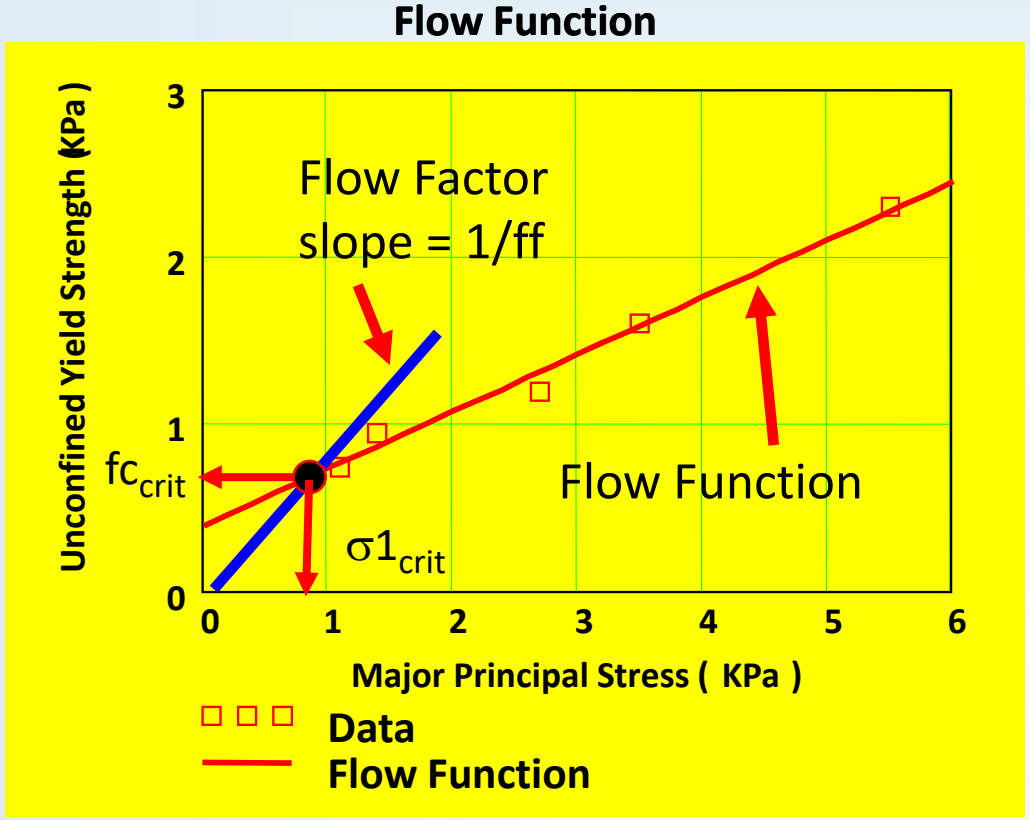


Figure 8.18 (c) Hopper flow factor values for conical channels, $\delta = 50°$

Arching Calculation from intersection of Flow Function and Flow Factor (*ff*)



Compute the critical strength from the intersection of the flow factor ff=1.3
and measured flow function. Then calculate the arch span (AI) from the equation below. H_θ equals 2.3 for 20 deg conical bin.

$$AI = \frac{H_\theta \cdot fc}{\gamma \cdot g}$$

Flow Factor: $f_c = \sigma_1 / ff$,
ff =1.2 Typical mass flow
ff = 1.7 for funnel flow

$$ff = \frac{\sigma_1}{f_c}$$

Whoops we have a problem:

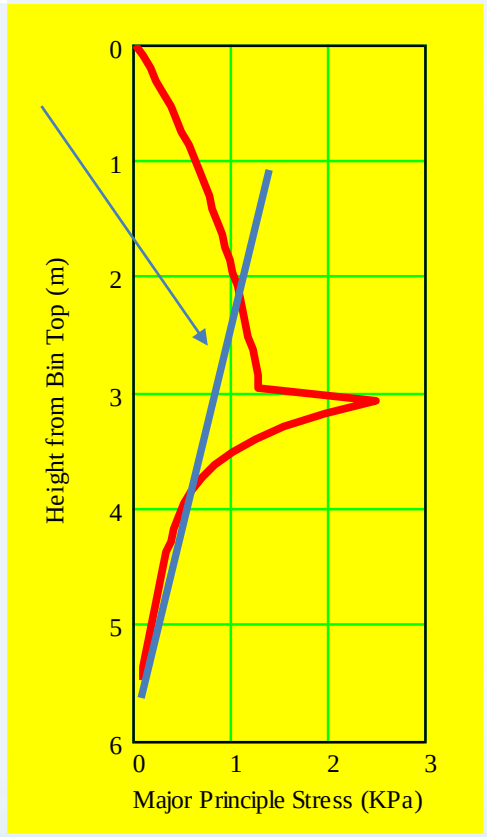
Associated with fc_{crit} is $\sigma 1_{crit}$ – But what happens if the hopper is small and cannot develop stresses that large? Does an arch form?

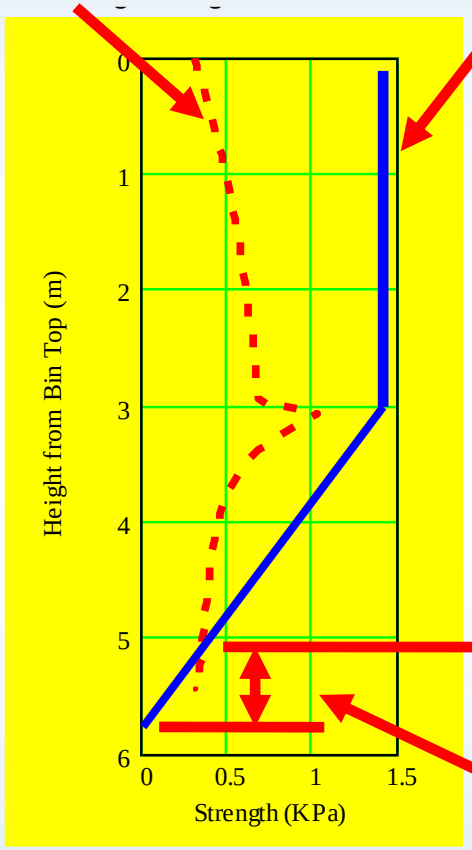
Arching in the Hopper - When the Strength Exceeds the Stress in the Arch

Strength

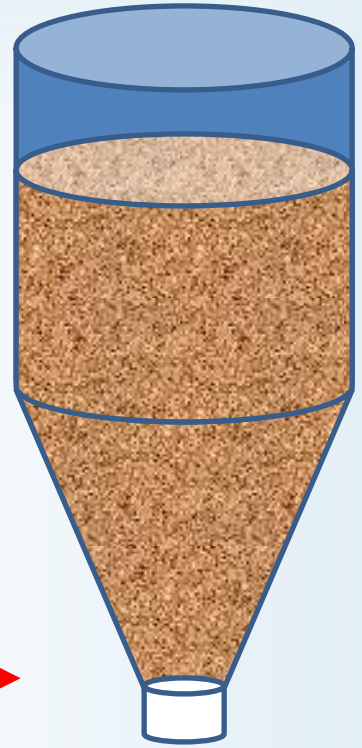
Stress in Arch

Radial stress

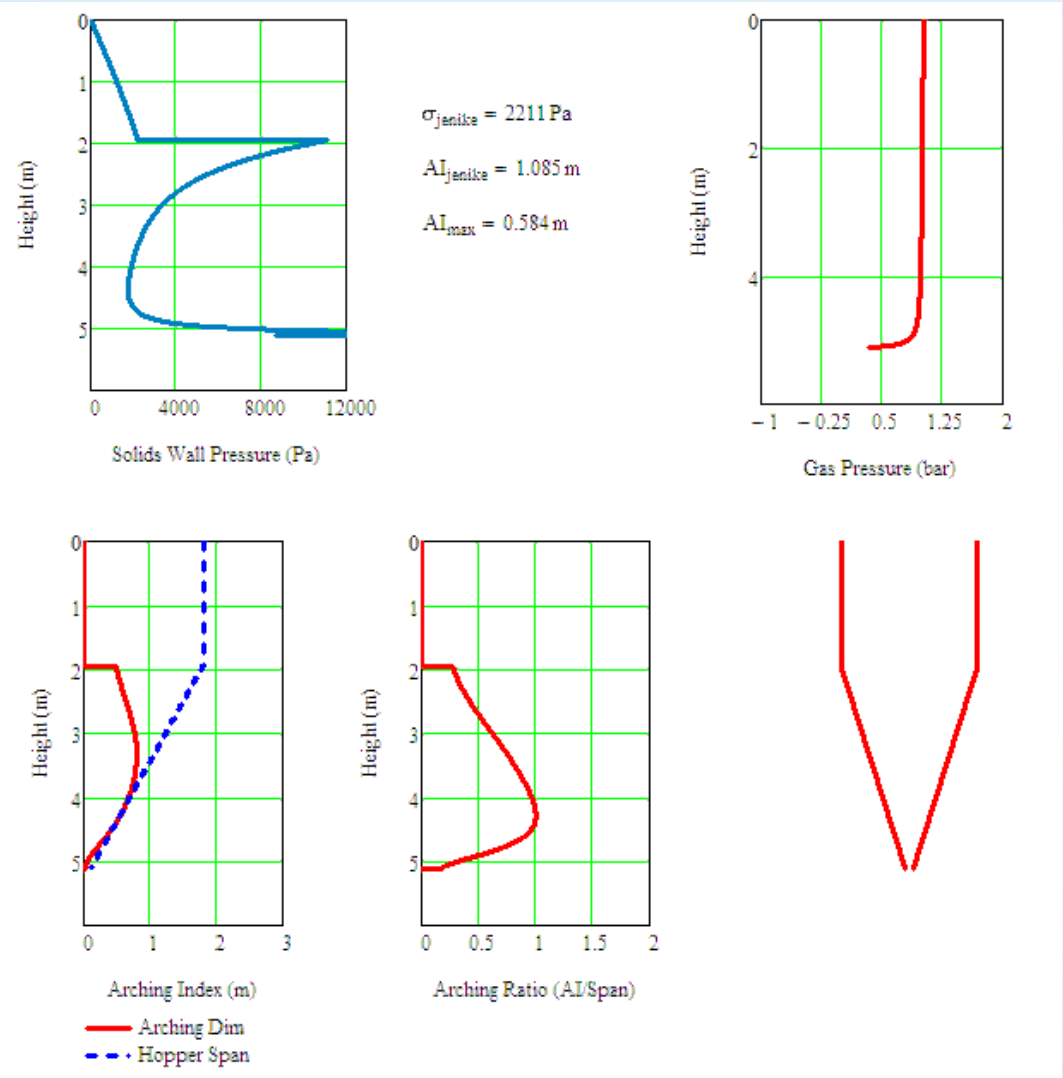




Arch



Zone where strength exceeds stress. Outlet size must exceed max span to prevent stable arch.



Sometimes we give the product a push by inducing gas pressure gradients to induce flow from outlets

1. Where does radial stress apply?
2. What is the effect of gas pressure?
3. Does it help or hurt?

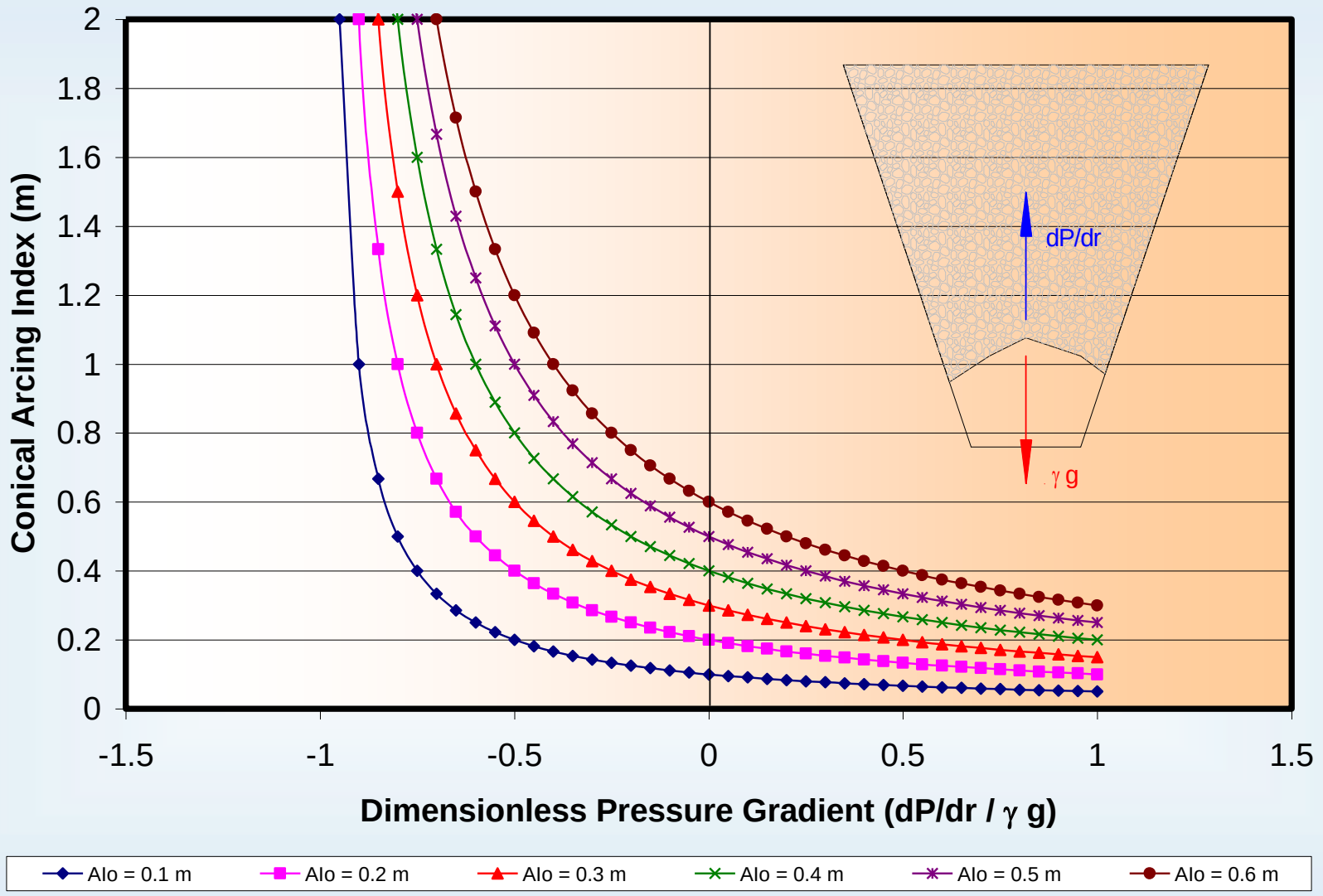
$$AI = \frac{H_\theta \cdot fc}{\gamma \cdot g}$$

Without gas effects

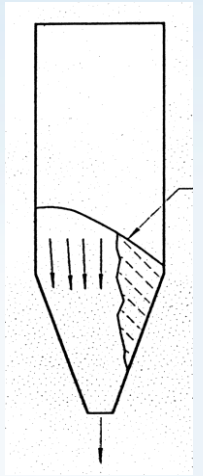
$$AI = \frac{H_\theta \cdot fc}{\left(\gamma \cdot g - \left(\frac{dP}{dz}\right)\right)}$$

With gas effects

So, if the gas pressure below the outlet is larger than the gas pressure above, the gas pressure gradient increases the critical arching dimension.

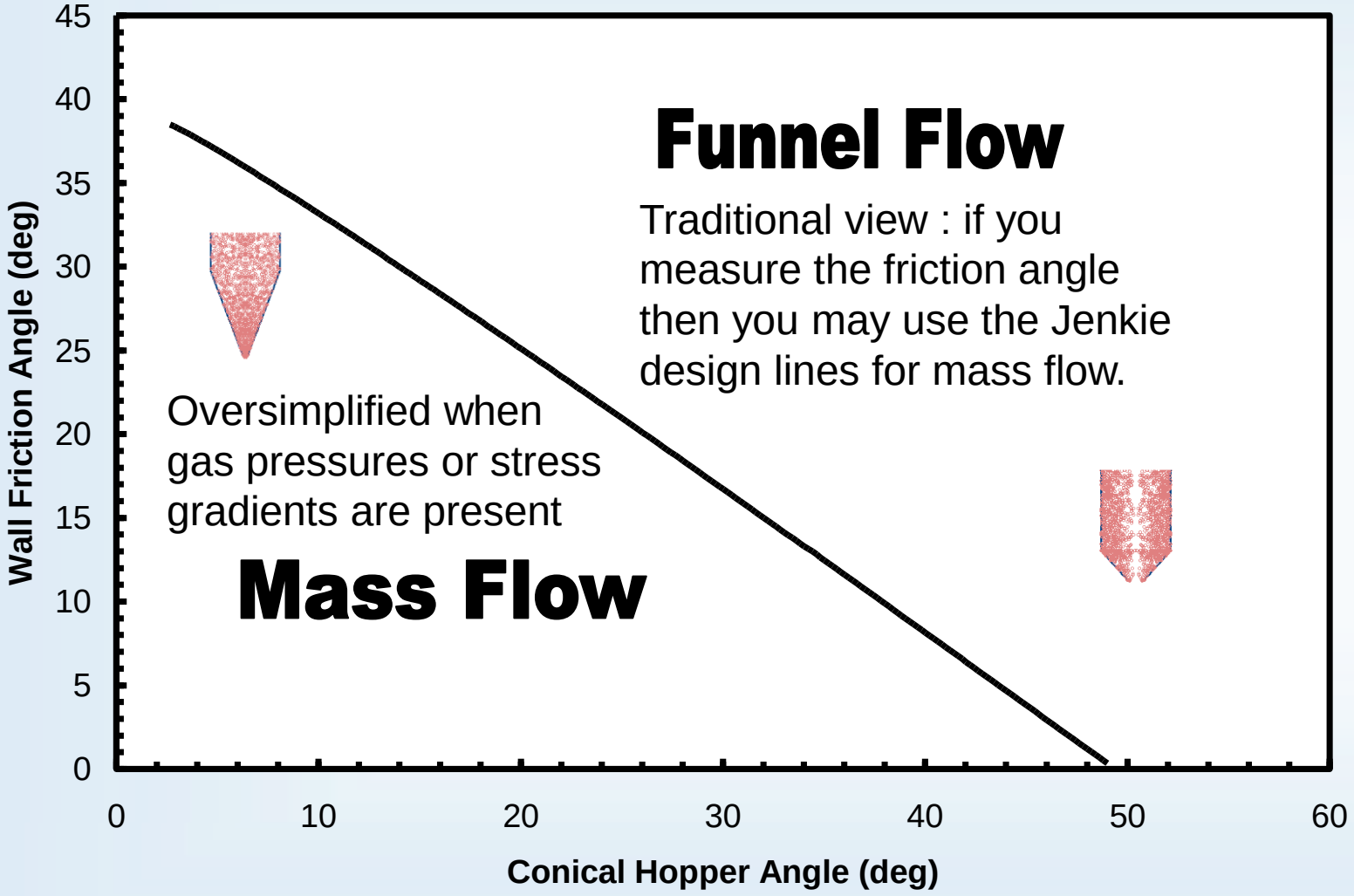

Conical Arcing Index (m)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1.5
-1
-0.5
0
0.5
1
1.5
Dimensionless Pressure Gradient (dP/dr / γ g)
dP/dr
γ g
Alo = 0.1 m
Alo = 0.2 m
Alo = 0.3 m
Alo = 0.4 m
Alo = 0.5 m
Alo = 0.6 m

Mass Flow Issues



- Theory based on symmetric flow
- Theory based on radial stress
- Theory based on radial velocity
- Theory does not include gas effects
- Theory assumes constant friction angle
- Friction angle of equipment and measurement can be different
- Friction angle assumed to be invariant with strain rate
- Does not take in to account time adhesion effects
- Need to extend theory to include inserts
- Theory falls apart with low effective internal friction angles
- Velocity profiles in bins not measured accurately
- Mass flow a function of gas and stress gradients

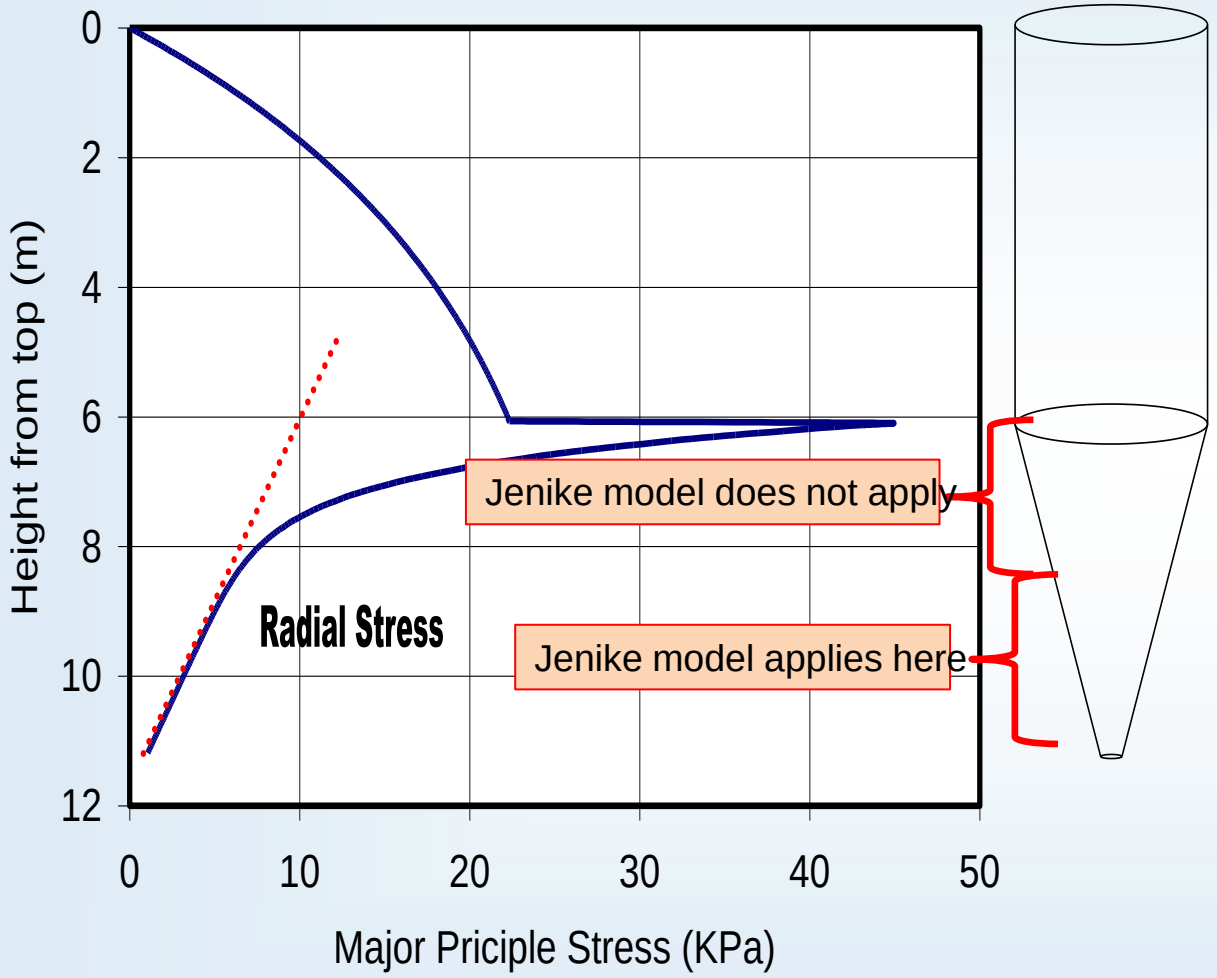
Determine Flow Mode of Material in Process



Need

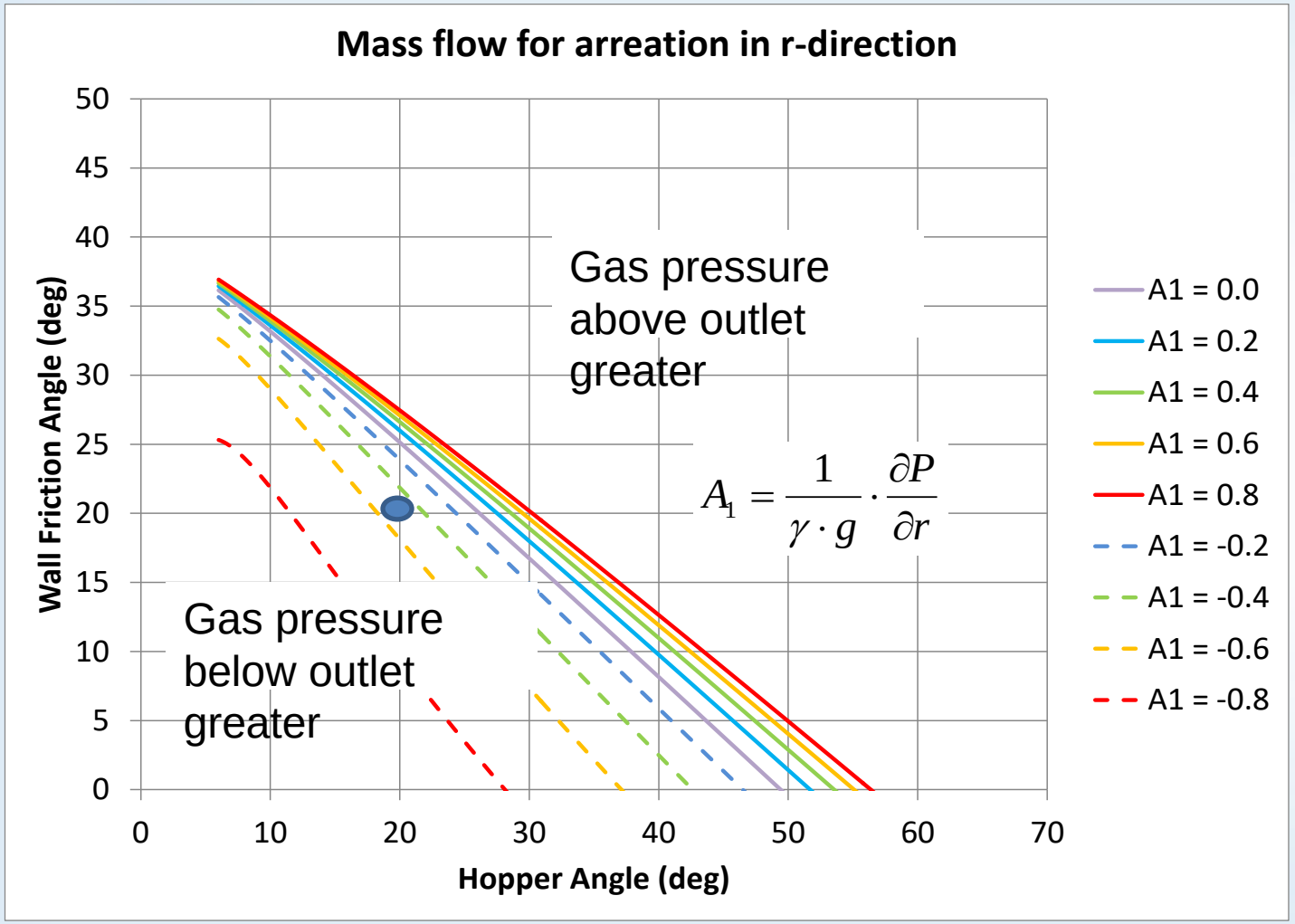
1. Compute limiting flow condition from radial stress theory: i.e. mass flow or funnel flow
2. Compute velocities in equipment from radial stress theory
3. Extend the radial stress theory to include gas pressure gradients and stress gradients and non-symmetric effects

Research Interest 3: Extending the Radial Stress Flow Theory



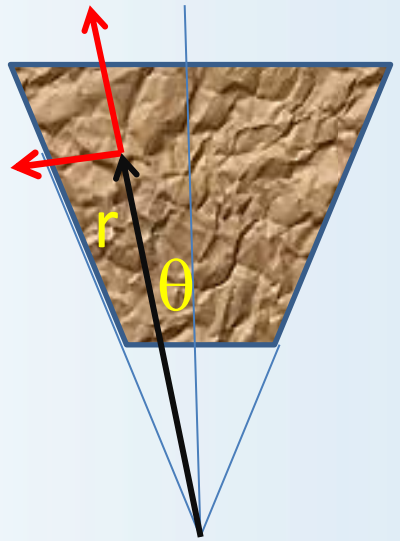
Radial Stress Theory		
Assumption	**Jenike and Johanson Work**	**Added or needs to be added**
Symmetric Stress State	Yes	need offset pile
Symetric process geomtery	Yes	Need offset cone (in process)
Complex Geometry	Conical and Plane Flow	Cone-in-cone added 2006 to 2010 Diamondback hopper in process Inverted baffle in process Pyramidal hopper
No Gas pressure gradients	No	Added 2017 to 2022
No Stress gradients (overpressures)	No (stress was linear with distance from apex)	Added 2022
No viscosity effects	No	In process (apply to tar sands)
No effect of strength in mass flow limits	No	In process (apply to cohesive materials

Add Gas Pressure Gradient to Radial Stress Flow Theory

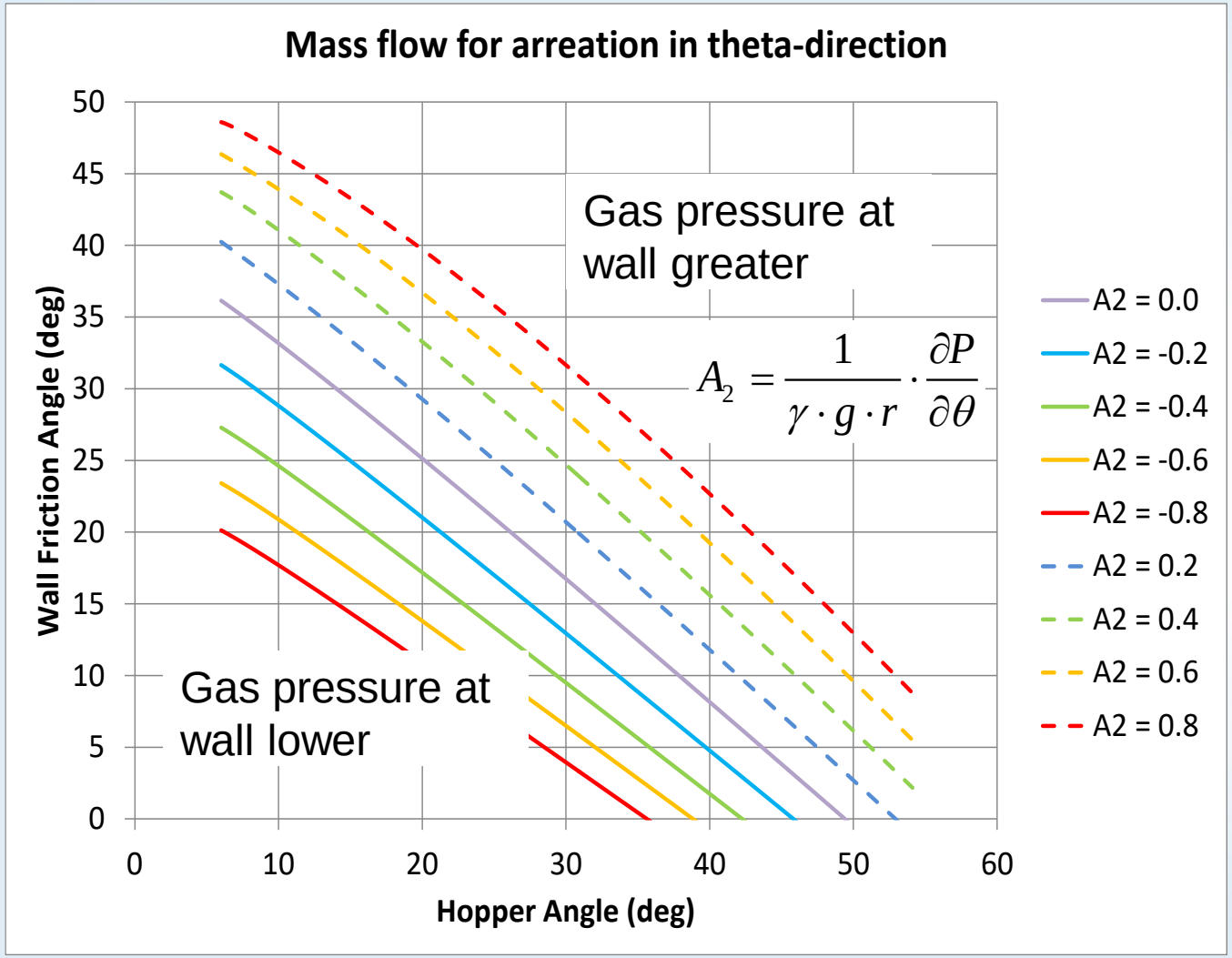


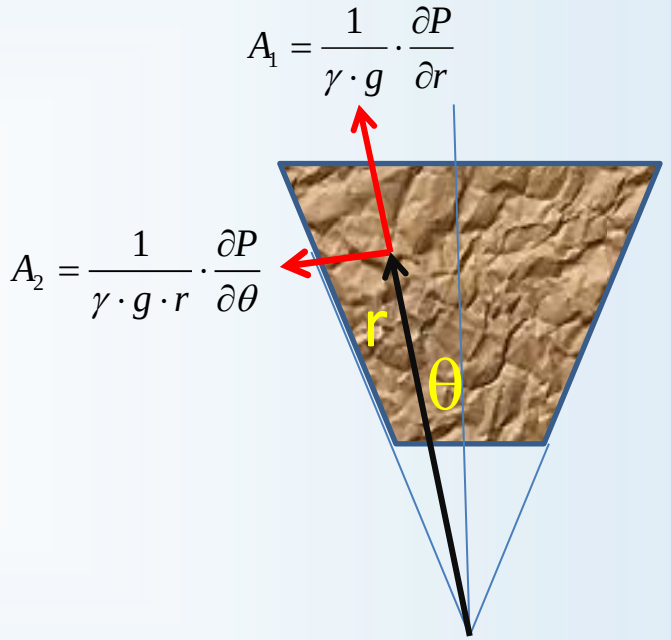
$$A_1 = \frac{1}{\gamma \cdot g} \cdot \frac{\partial P}{\partial r}$$

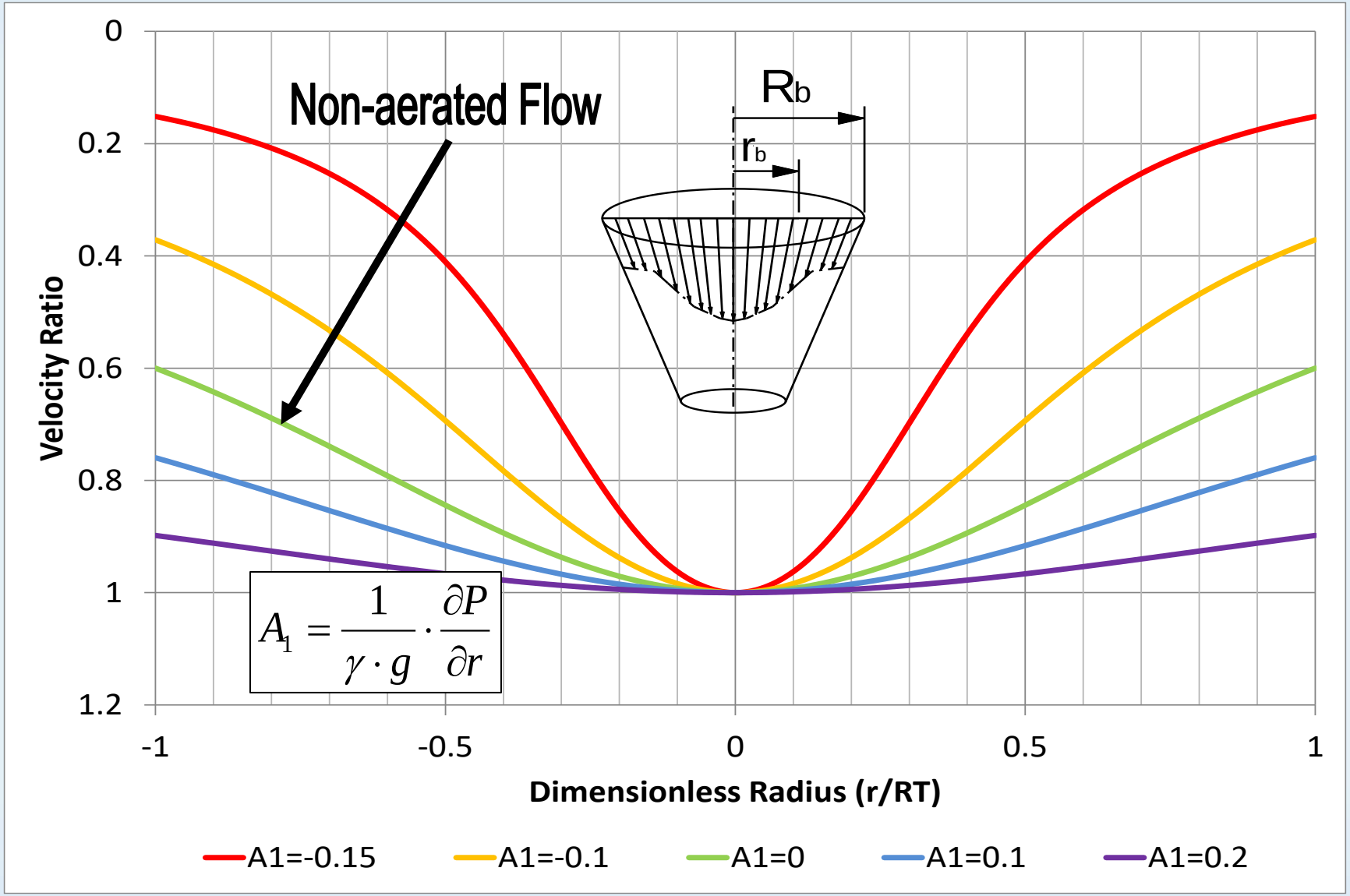
$$A_2 = \frac{1}{\gamma \cdot g \cdot r} \cdot \frac{\partial P}{\partial \theta}$$



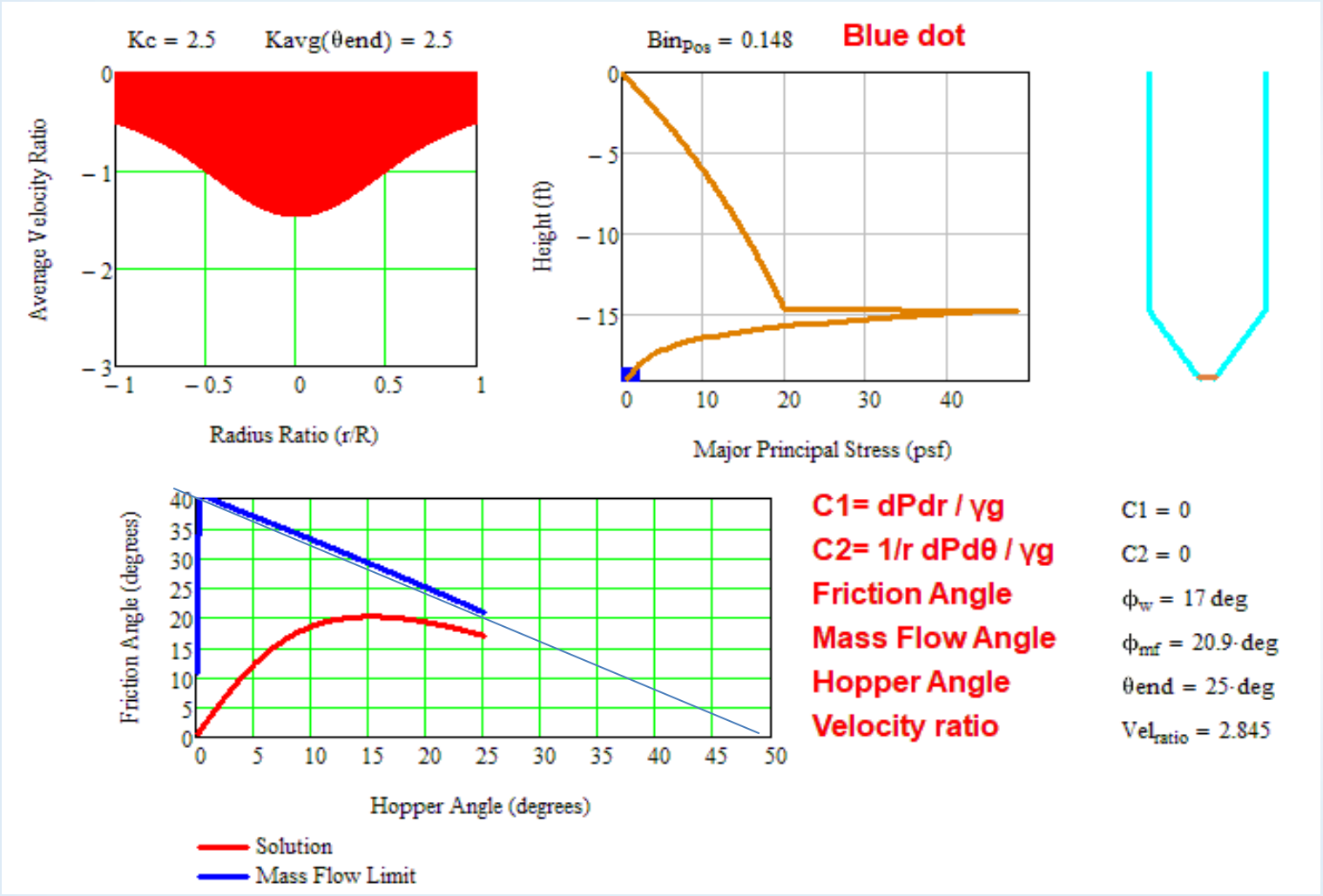
Add Gas Pressure Gradient to Radial Stress Flow Theory






Non-aerated Flow
Rb
rb
Velocity Ratio
0
0.2
0.4
0.6
0.8
1
1.2
-1
-0.5
0
0.5
1
Dimensionless Radius (r/RT)
$A_1 = \frac{1}{\gamma \cdot g} \cdot \frac{\partial P}{\partial r}$
A1=-0.15
A1=-0.1
A1=0
A1=0.1
A1=0.2

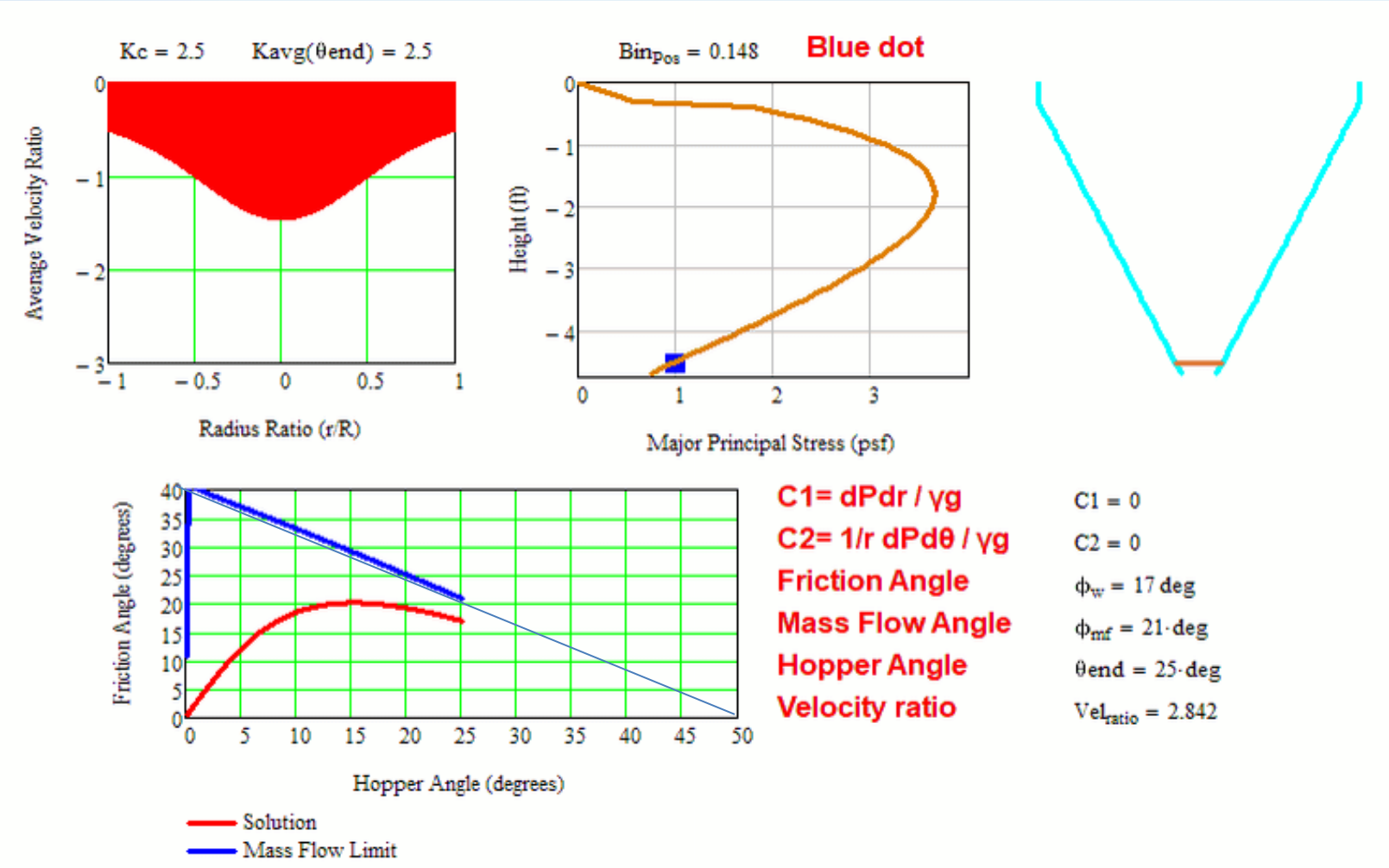
Stress Gradient in Tall Hopper



1. Velocity ratio is between 2.8 and 3.0, depending on the stress gradient

2. As stress gradient becomes larger, the mass flow limit moves slightly to the left and down making mass flow less likely

Stress Gradient in Short Hopper



1. Velocity ratio is between 2.0 and 2.5, depending on the stress gradient

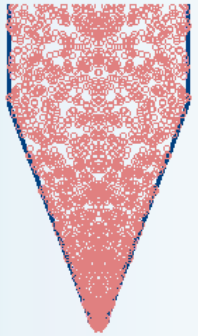
2. As stress gradient reverses direction, the mass flow limit moves to the right and up making mass flow more likely

Conclusions On New Radial Stress Theory

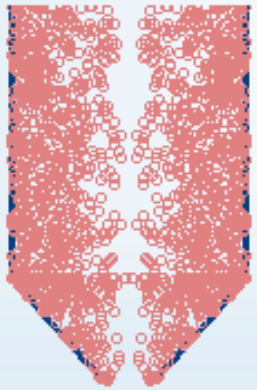
1. The short hopper is very sensitive to changes in both stress and gas pressure gradients
2. The profiles for the stress gradient in the lower hopper near the outlet are similar to those in the traditional radial stress theory
3. Gas pressure gradients significantly change the mass flow limit and the velocity ratio

Velocities in Hoppers Determine when Particles Leave the Bin

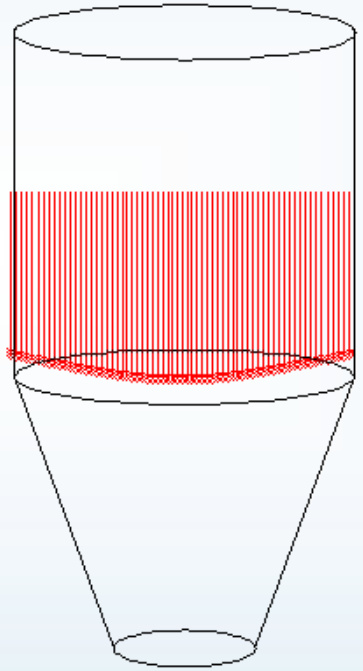
If segregation exists, then different velocity profiles can cause different segregation patterns leaving bin



Mass flow – all material in motion



Funnel flow– only central flow channel

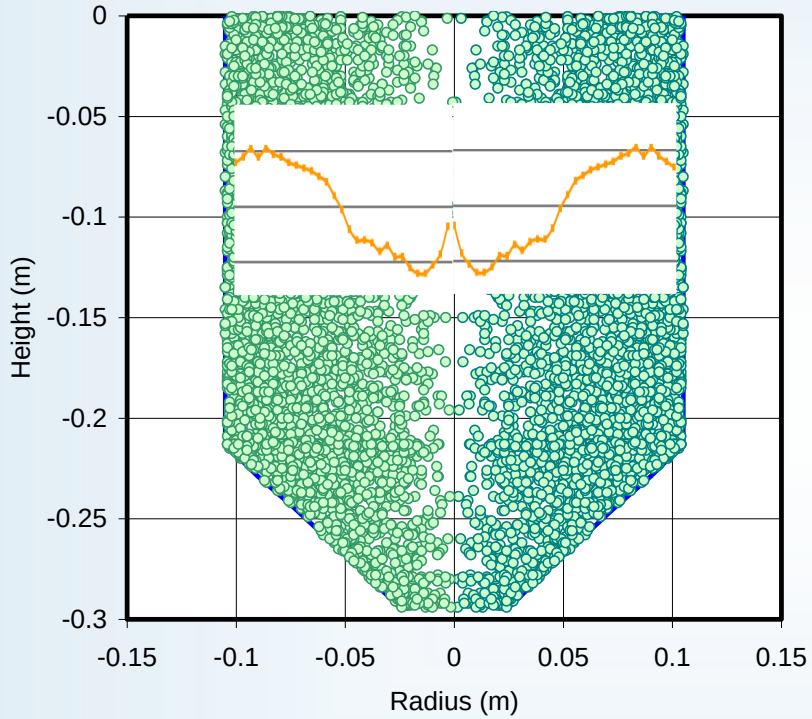


The material velocity in the cone depends on the friction along the conical hopper wall. The more frictional the wall the steeper the velocity profile. If the friction angle becomes too high flow stops entirely at the hopper wall resulting in funnel flow. Observe the velocity profiles in the cone as the friction angle is changed.

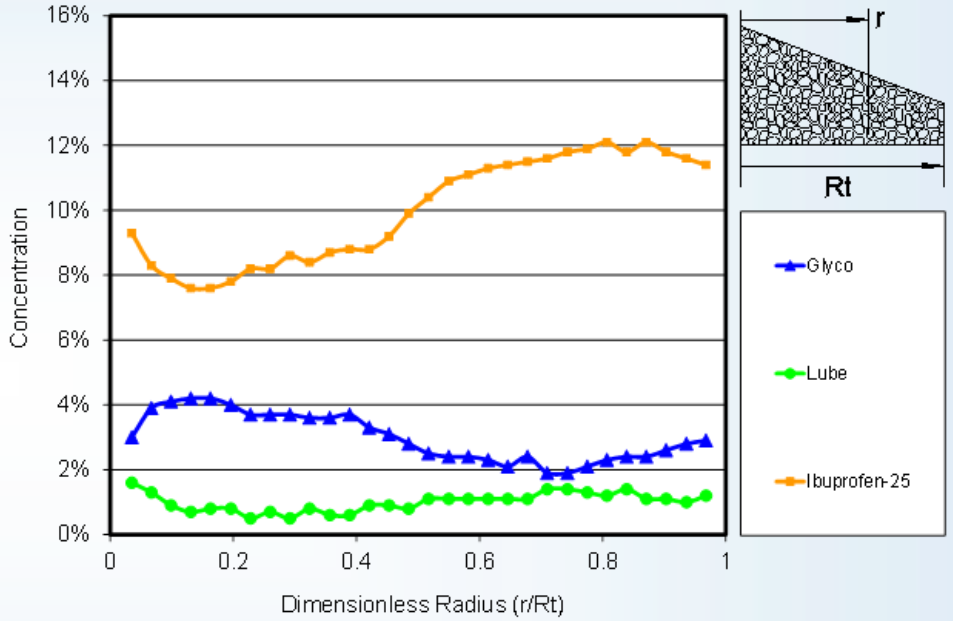
Wall Friction Angle

$\phi_{Wall_{FRAME}} = 5\ deg$

Assign the Concentration Based on Radial Position in Bin



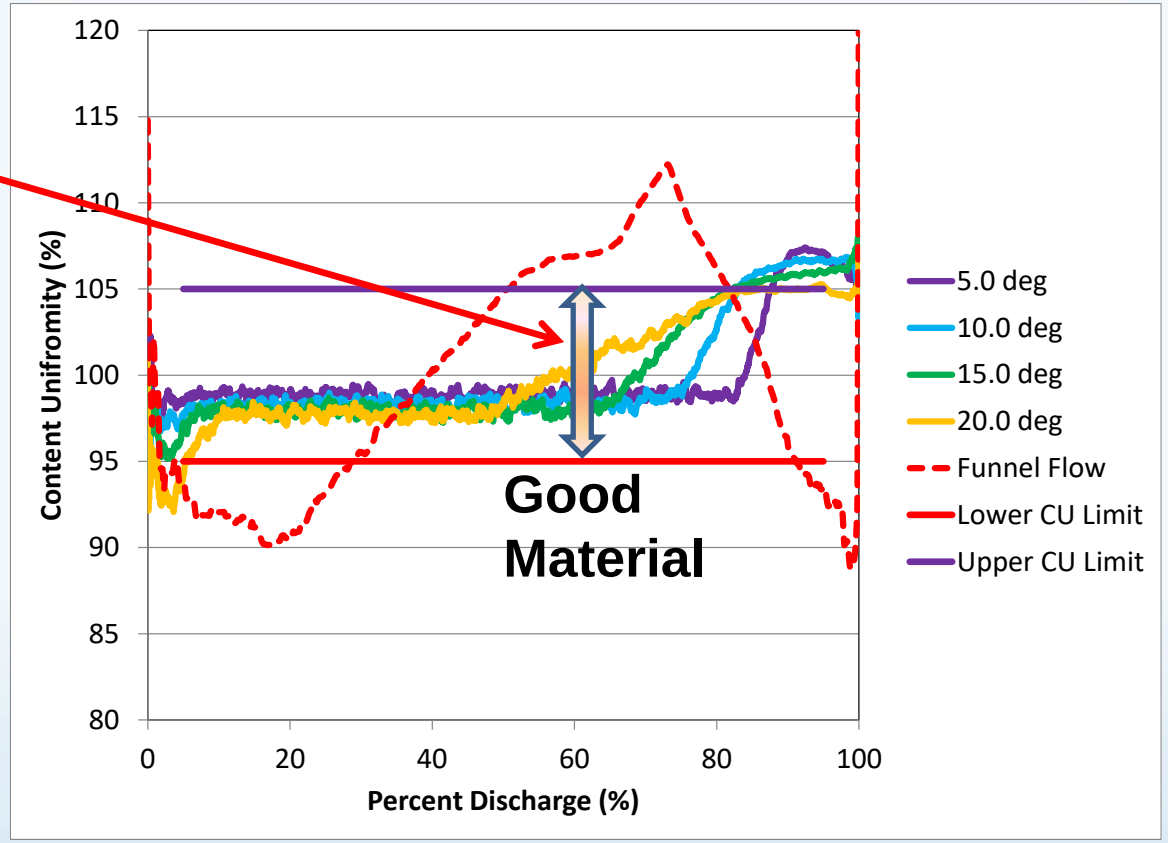
Initial clusters in bin



Component concentration for 10% API mixtures

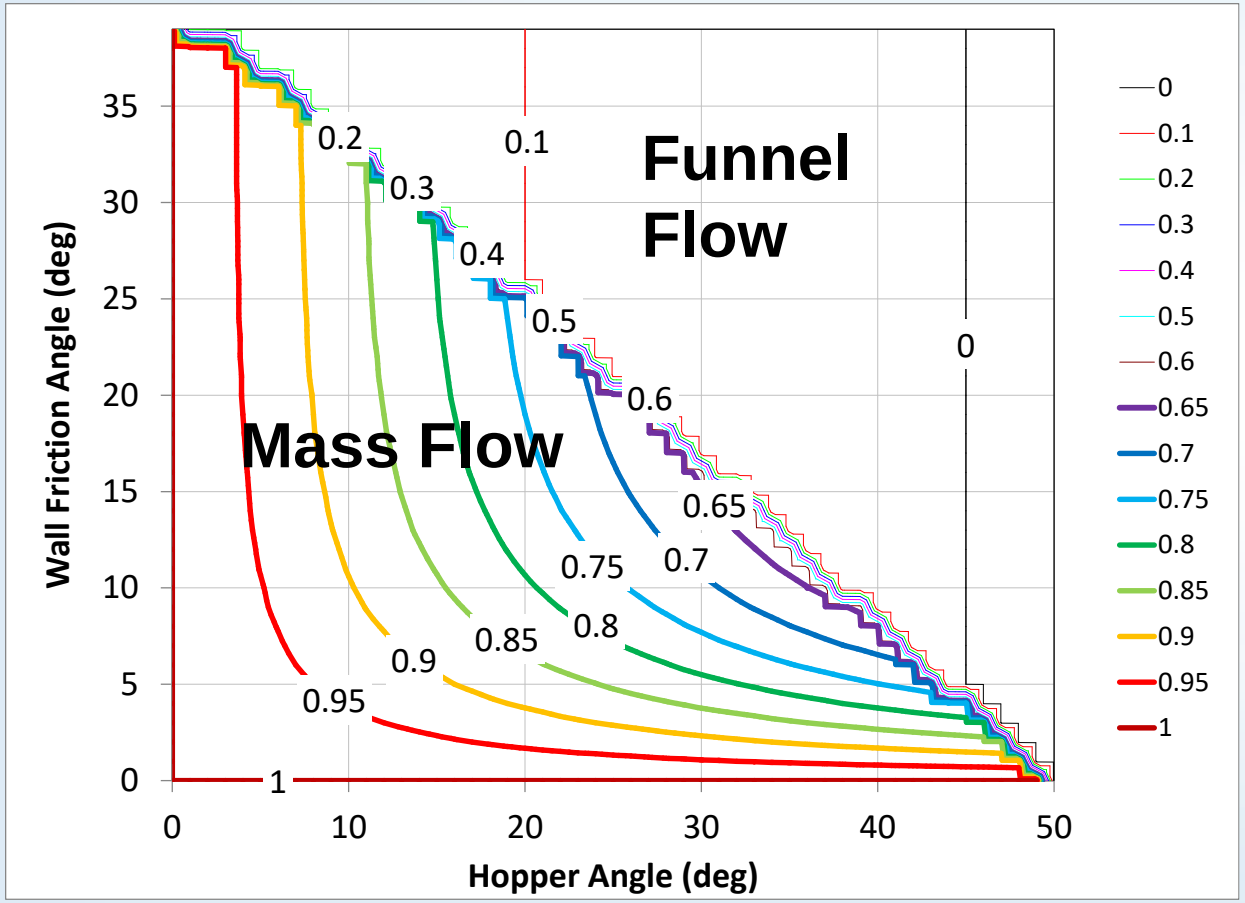
We can define a reasonable variation off the mean concentration (±5%) and ask the question: How much of the material is outside the range of these acceptable limits?

Sum the fraction of the curves that fall in the good zone 100% would be perfect operation



The percent of bad material exiting the hopper depends on the type of flow

Segregation Design Space for a Material in a Conical Hopper: Fill-then-Empty Mode



- We need to rethink how we do design work.
- Intelligent design would ask: what does my process need to do to solve the problem at hand?
- We cannot do intelligent design from a limit analysis – Mass flow versus funnel flow. Intelligent design (design with a purpose) requires one to identify the need and then ask: how can the application of the theory fill that need?

Still a lot of work to be done!

Process Issue	Ranking - Predictive Ability (1-10)								
	Symmetric	Non - Symmetric	For 3D Geometry	Other Geometries besides bins	With Gas Effects	With external forces	For Elastic Solids	For Viscous materials	With variable properties
Rathole Prediction	7	3	1	1	6	3	3	1	1
Arching Prediction	8	4	1	2	7	3	3	1	6
Mass Flow	8	5	2	2	6	3	2	2	4
Velocity Pattern	7	5	2	2	6	3	2	2	4
Segregation	5	5	2	2	3	3	3	2	4
Flow rate	8	4	2	4	4	4	4	3	4

What part of the sandbox do you want play in?

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Red is Jenike's Contribution Green is areas of urgent need

Thank You for Your Attention

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