Controlling Rheology via Boundary Conditions in Dense Granular Flows

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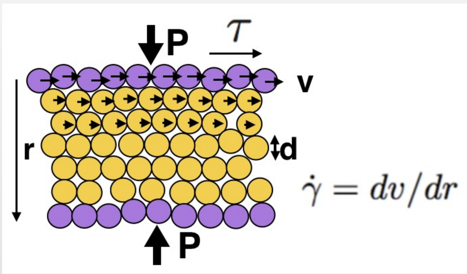
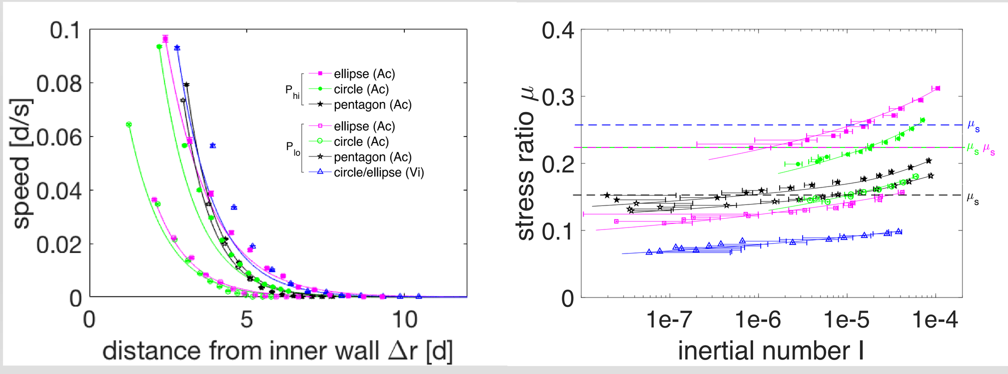
Background

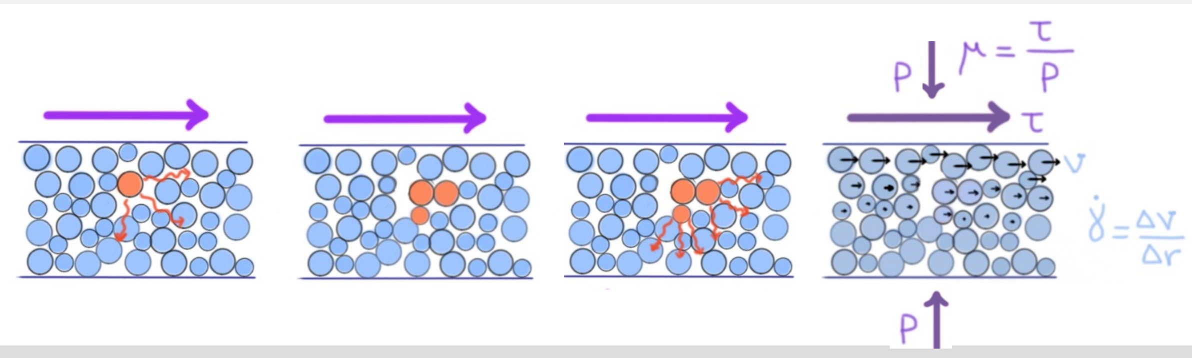
- Nonlocal rheologies [1] successfully model granular flows across different packing density, shape, size and shear rate [2,3]
- Flows are described by:
  - Flow speed $v(r)$
  - Stress ratio: ratio $\mu = \frac{\tau}{P}$ between
    Shear stress
    Normal pressure



- A set of 3 empirical material parameters describes each particle batch: yield stress μ_s, local parameter b and nonlocal parameter A [3]



Nonlocal rheology



Local rearrangement induces rearrangements in nearby particles [1]

fluidity $g \equiv \frac{\dot{\gamma}}{\mu}$

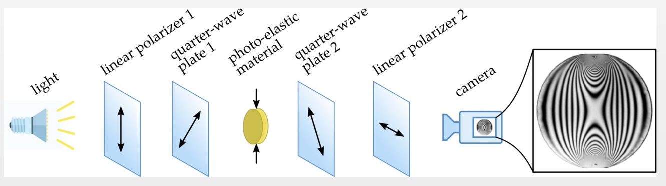
$\nabla^2 g = \frac{1}{\xi^2}(g - g_{loc})$

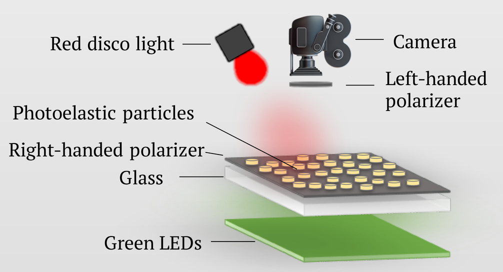
A Nonlocal parameter — b Local parameter — μ_s Yield criterion

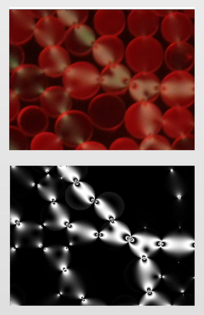
Open Questions

- How can we understand and predict $v(r)$ at the boundaries, analogous to fluid slip, as a function of roughness?
- Does boundary roughness control the force chains?
- Do these results carry over, across different geometries and for faster flows?
- What are the general principles which allow us to set the boundary conditions in nonlocal rheology models based on roughness and particle properties?

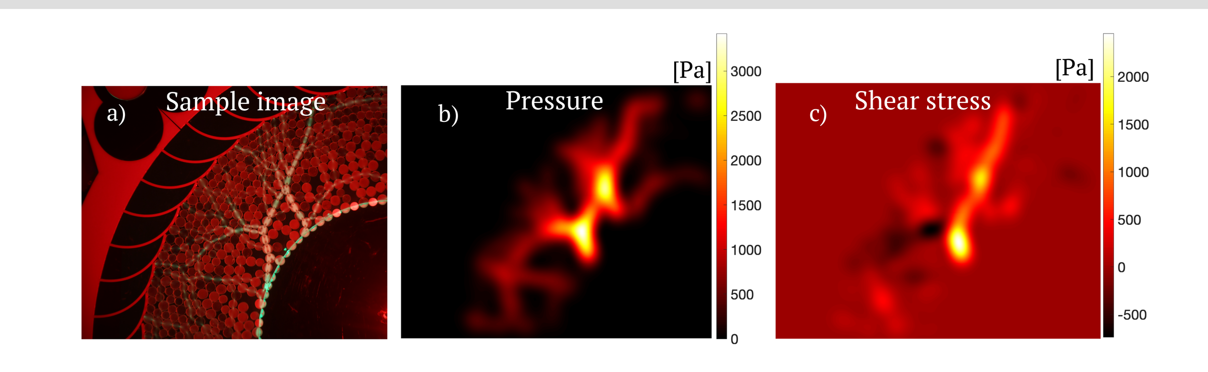
Photoelastic Method





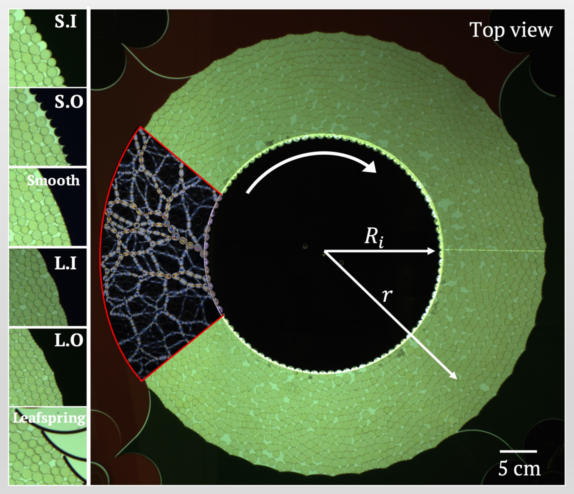


- The amount of birefringence at each point in the material depends on the local stress
- Force chains fluctuate over space and time, as the material is sheard
- Photoelastic disks measure the vector contact forces on each particles [4,5]
- Coarse-graining with a Lucy function [6] provides the stress tensor ($w = 1.3d$)

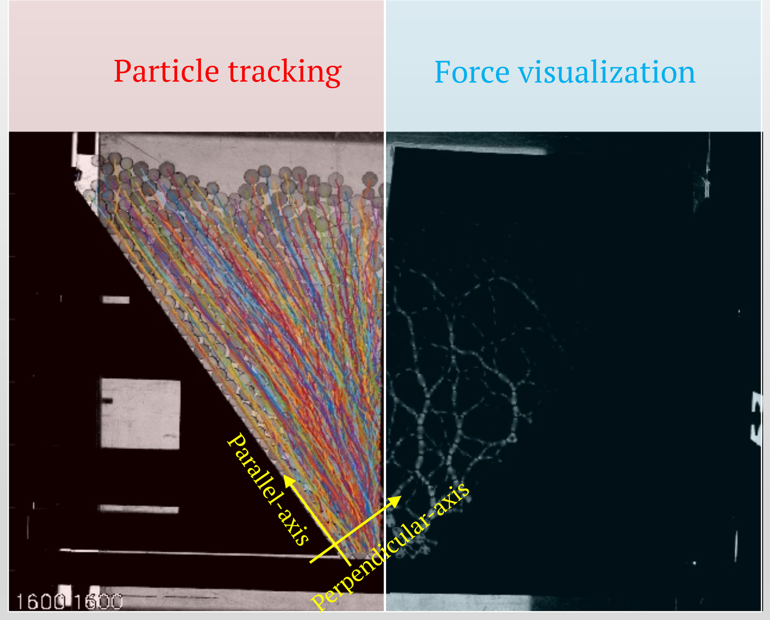


Experiment 1: Annular Rheometer

- Quasi-2D annular shear cell can run indefinitely [7]
- Photoelastic circular disks ($\mu_s \sim 0.25$)
- Rough rotating inner wall at constant velocity: $v = 1.1\ d/s$
- Stationary outer wall with 5 different roughnesses
- Torque sensor on the rotating shaft
- Constant packing density: $\phi = 0.65$
- Photoelastic stress visualization and particle-tracking happen in separate runs



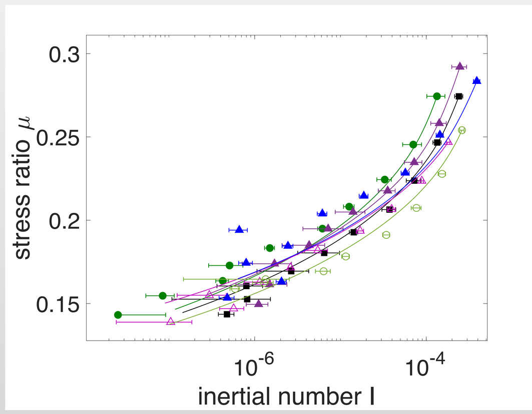
Experiment 2: Hopper



- Flipable, quasi-2D hopper, refurbished from Bob Behringer's lab, IFPRI report FRR 56-06 [8]
- 20-30 runs for each of 5 wall roughnesses
- High-speed video
- Particle-tracking on left; decompose into parallel and perpendicular axes
- Semi-quantitative force chains measurements on right
- Consider data taken during approximately stead-state flow rate

Success of nonlocal rheology for various boundary properties

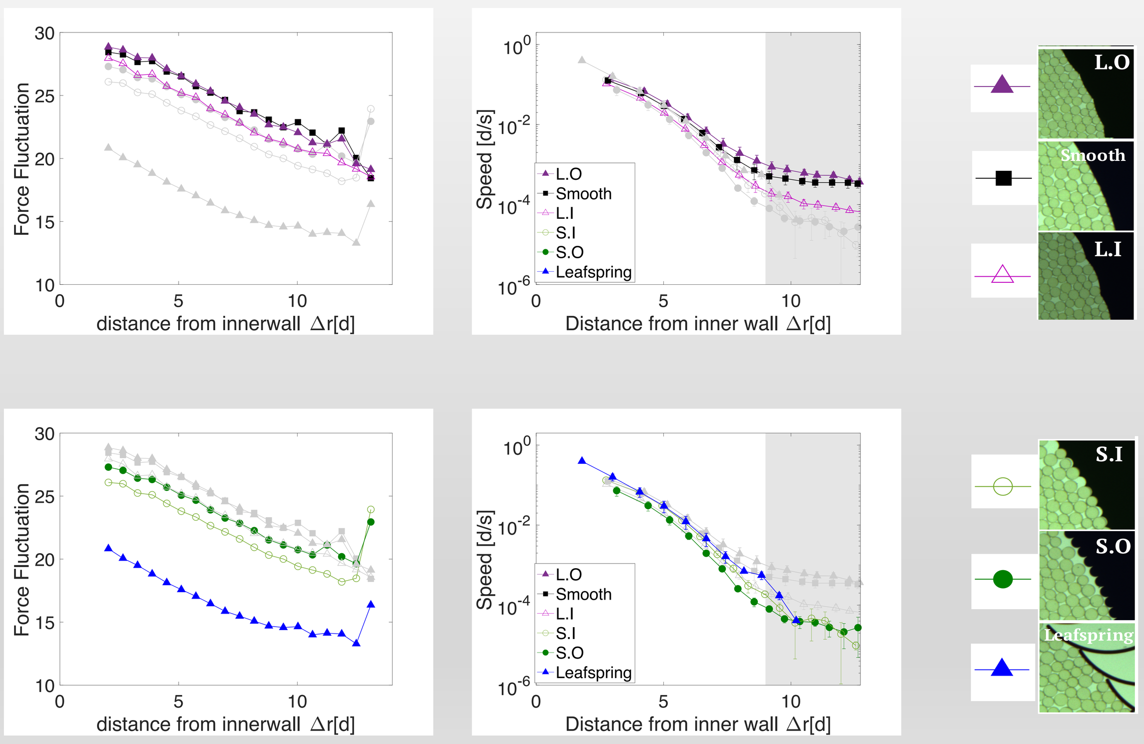
- Photoelasticity: measure fluidity g throughout system
- Particle-tracking: Measure outer wall velocity v_{slip}
- Both: calculate μ and I throughout system
- Integrate nonlocal rheology from outer wall
- Optimize A, b, μ_s to best fit the data



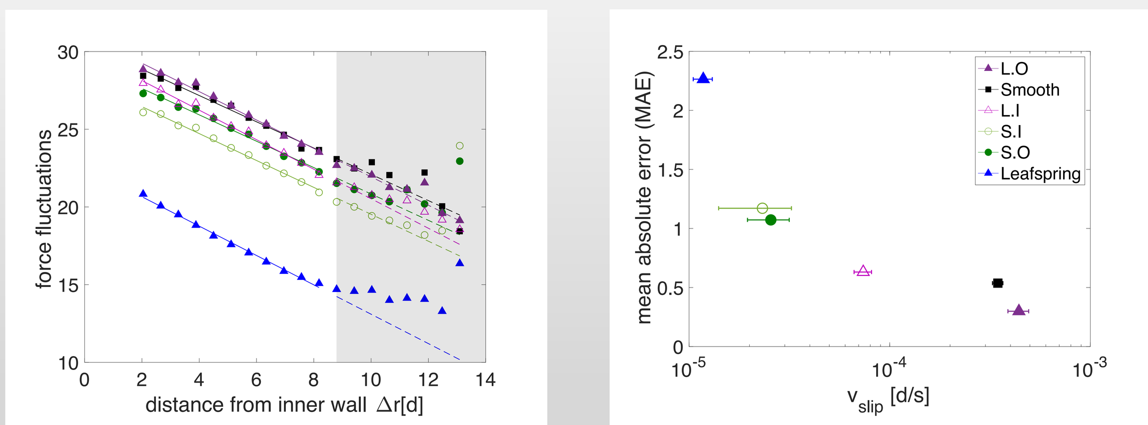
A	b	μ_s
0.22 ± 0.03	1.1 ± 0.5	0.25 ± 0.01

One set of parameters fits all boundary conditions

Rough vs. Smooth Boundaries

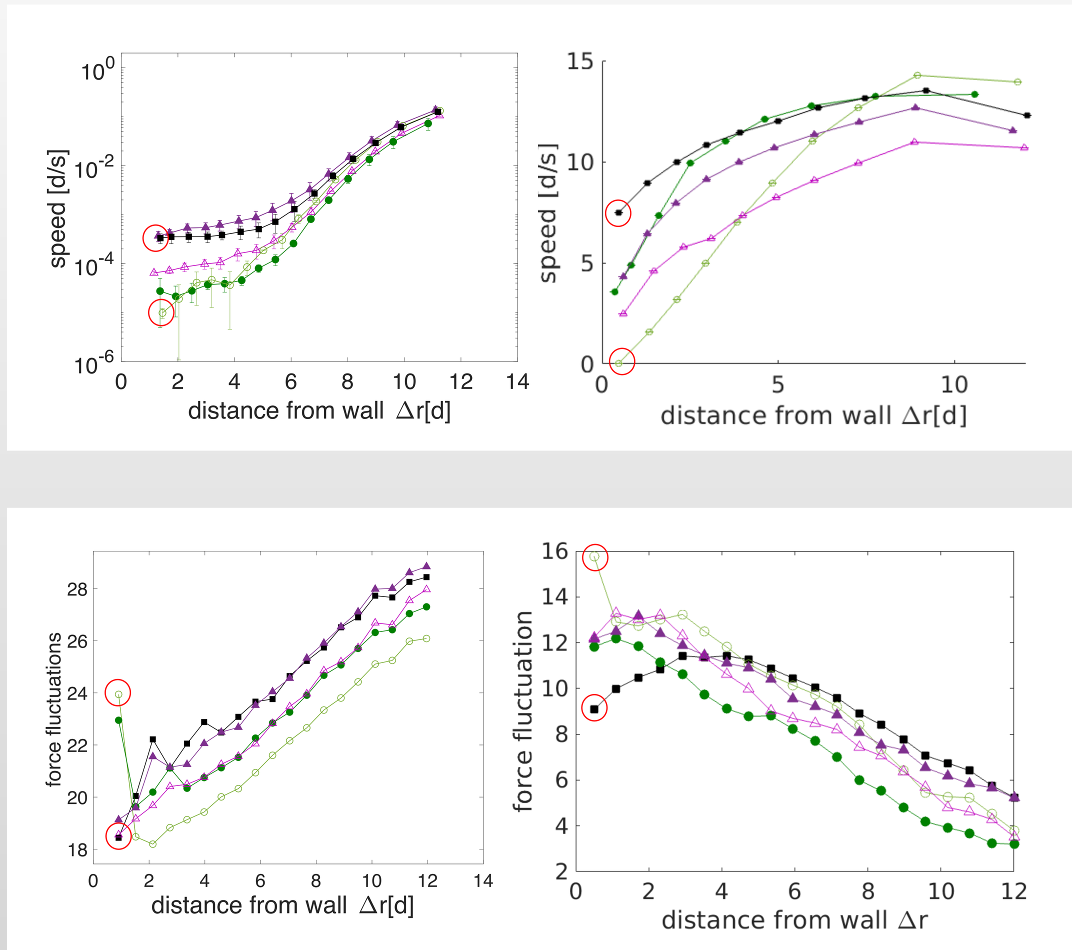


Boundaries control force chain fluctuations and slip velocity



$$MAE = \frac{\sum |predicted\ value - measured\ value|}{number\ of\ data\ points}$$

Annulus vs. Hopper



- Similar, but not identical, trends in both geometries, comparing force fluctuations, wall slip, and roughness vs smoothness
- In extreme cases (very smooth and very rough boundaries) flow behavior is similar near the wall, but geometry affects the flow characteristics

Conclusions

Nonlocal rheology successfully models granular flows across different:
- ✓ packing densities
- ✓ shear rates
- ✓ particle shapes and materials
- ✓ boundary roughness and compliance

while maintaining a consistent set of A, b, μ_s that fits each set of particles

We qualitatively predict wall slip associated with wall properties using force fluctuations

Next steps: examine a continuum of roughness and particle shape to establish v_{slip}, A, b, μ_s on a predictive footing

[1] Kamrin & Koval. *Physical Review Letters* (2012)
[2] Tang, Brzinski, Shearer, Daniels. *Soft Matter* (2018)
[3] Fazelpour, Tang, Daniels. *Soft Matter* (2022)
[4] Abed Zadeh et al. *Physical Review E* (2019)
[5] Daniels, Puckett, Kollmer. *Rev. of Sci. Inst.* (2017) https://github.com/jekollmer/PEGS
[6] Weinhart, Hartkamp, Thornton & Luding. *Physics of Fluids* (2013)
[7] Brzinski & Daniels. *Physical Review Letters* (2018)
[8] Tang & Behringer. *Europhysics Letters* (2016)