Continuum modeling and simulation of dry and wet granular media

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Part 1:
Benefits of the simple – When can we get by with a simple granular rheology?

Part 2:
Extending this framework to handle fluid-saturated granular media

Salient properties of granular rheology

Basic level: Cohesionless granular media is an **elastic-plastic material** with a **frictional yield criterion** and **no tension**.
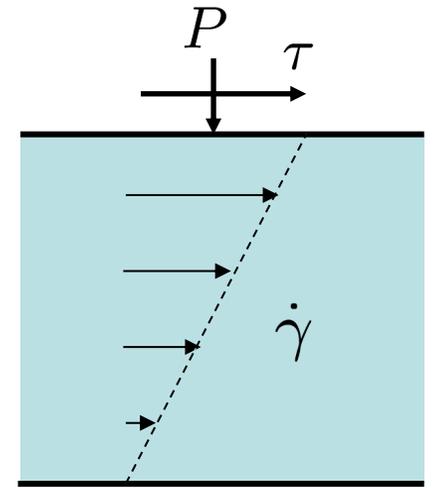
There are MANY tricky details.

a)   Dense state:
   • Dilation coupled to strength
   • Nontrivial yield surface shape
   • Internal state variables
   • Rate dependence
   • Particle size effects

b)   Dilute state:
   • Dissipative kinetic theory
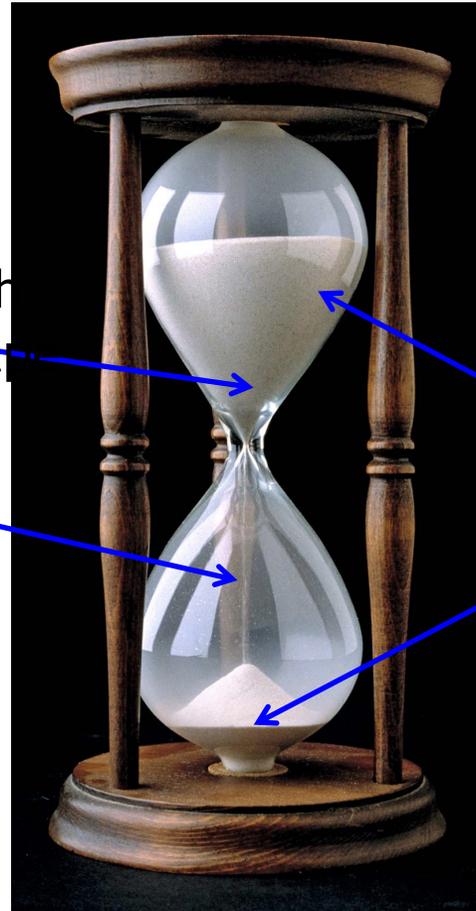
Liquid-like: Dense but flowing

Gas-like: separate grains

Solid-like: ~Zero flow

Simple granular continuum model:
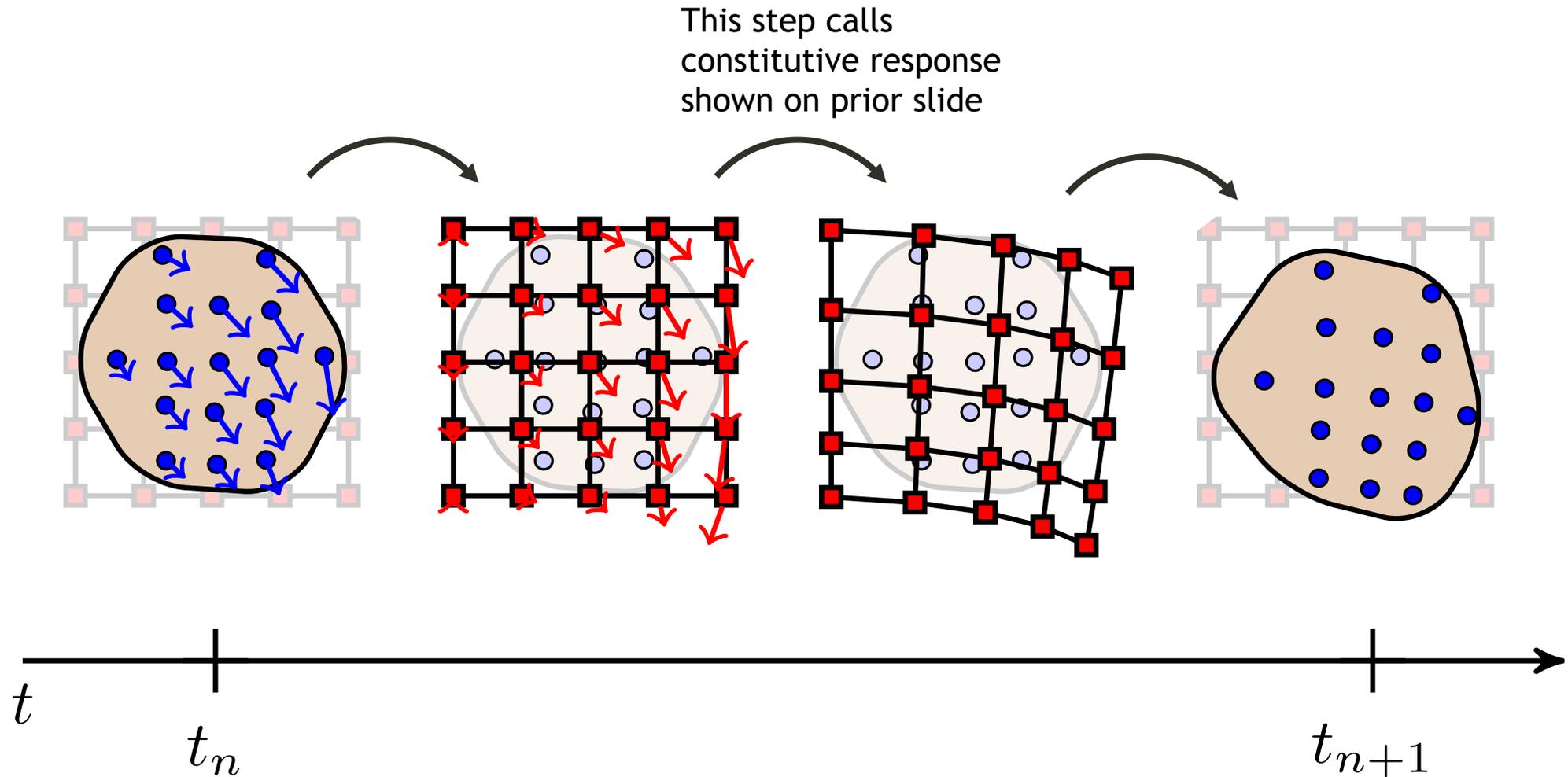## "Trans-Phase Frictional Plasticity"

Basic ingredients of the constitutive model:

- Frictional plastic flow: $\mu \equiv \tau/P = \mu_s$ if $\dot{\gamma} > 0, \quad \tau < \mu_s P$ otherwise

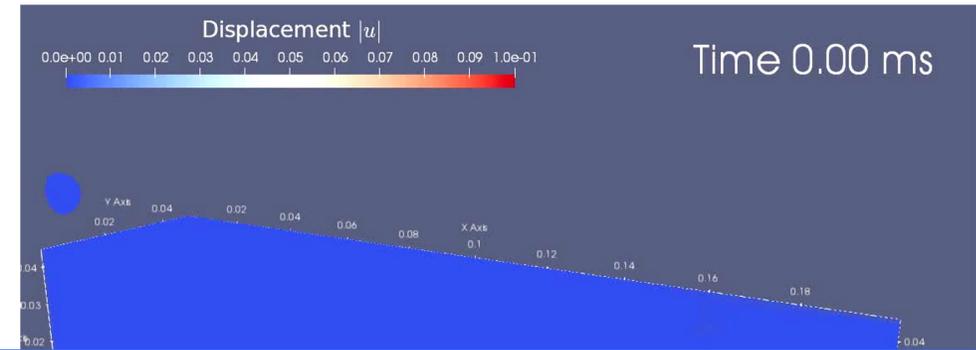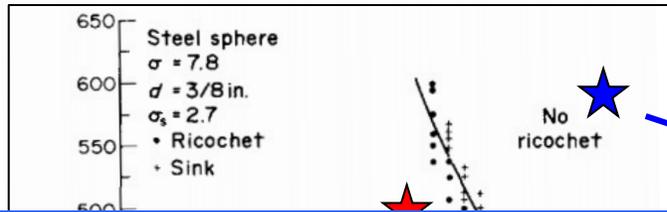- No tension: $P, \ \tau = 0$ whenever $\rho < \rho_c$

How to deal with huge deformations?

Answer: Use a mesh-free method. We use Material Point Method (MPM). (Goes back to Sulsky 1994)

This step calls constitutive response shown on prior slide

# Trans-Phase Frictional Plasticity: Implementation

Oblique impact, steel ball shot into sand. Exp data (Soliman et al 1976):

General observation: This model tends to do a good job predicting dynamics of (large) intruding objects near the free surface.

- Projectile impact (Dunatunga and Kamrin *JMPS* 2017)

- Locomotion and locomotive scaling laws (Slonaker et al *PRE* 2017, Zhang et al *PR Fluids* 2020, Agarwal et al *J Terra* 2019, Agarwal et al *Sci Adv* 2021)

- Explains the Resistive Force Theory of granular intrusion (Askari and Kamrin *Nature Mat* 2016, Agarwal et al *Sci Adv* 2021, Agarwal et al *PNAS* 2023)
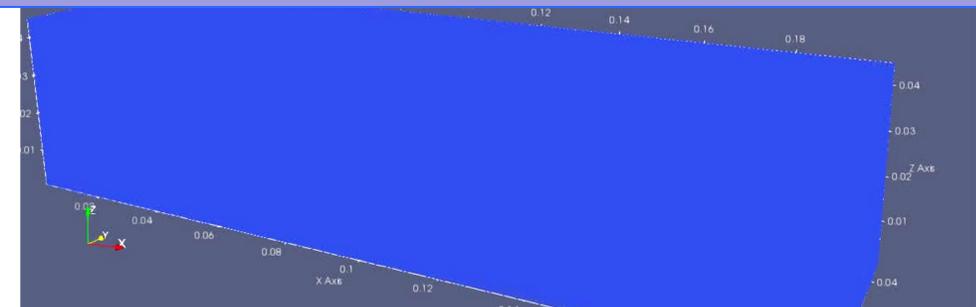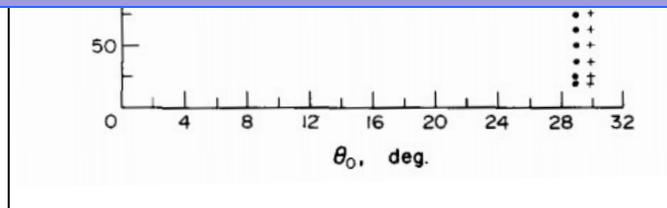
To the moon: Model the VIPER rover

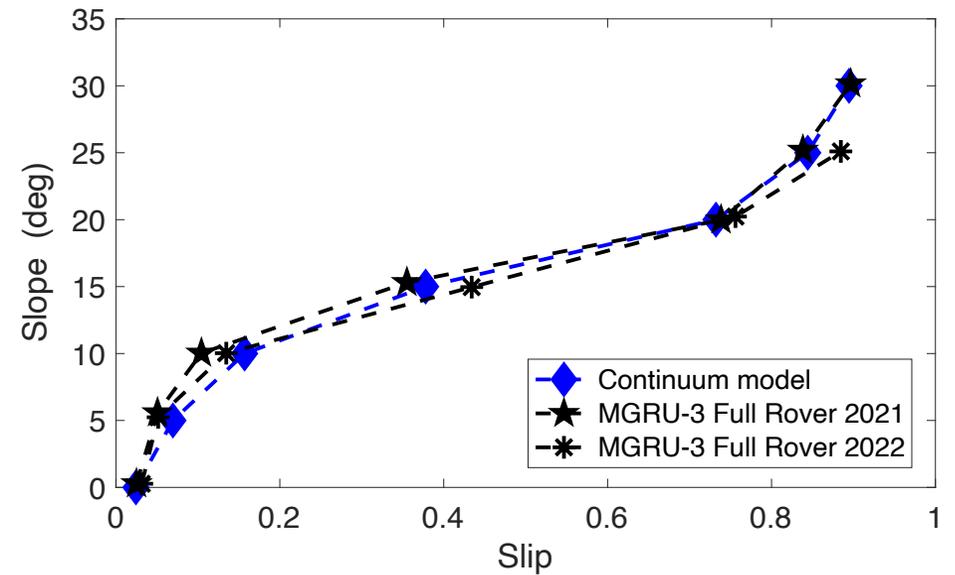Teaser: VIPER in trans-phase **continuum model** with **lunar gravity**.
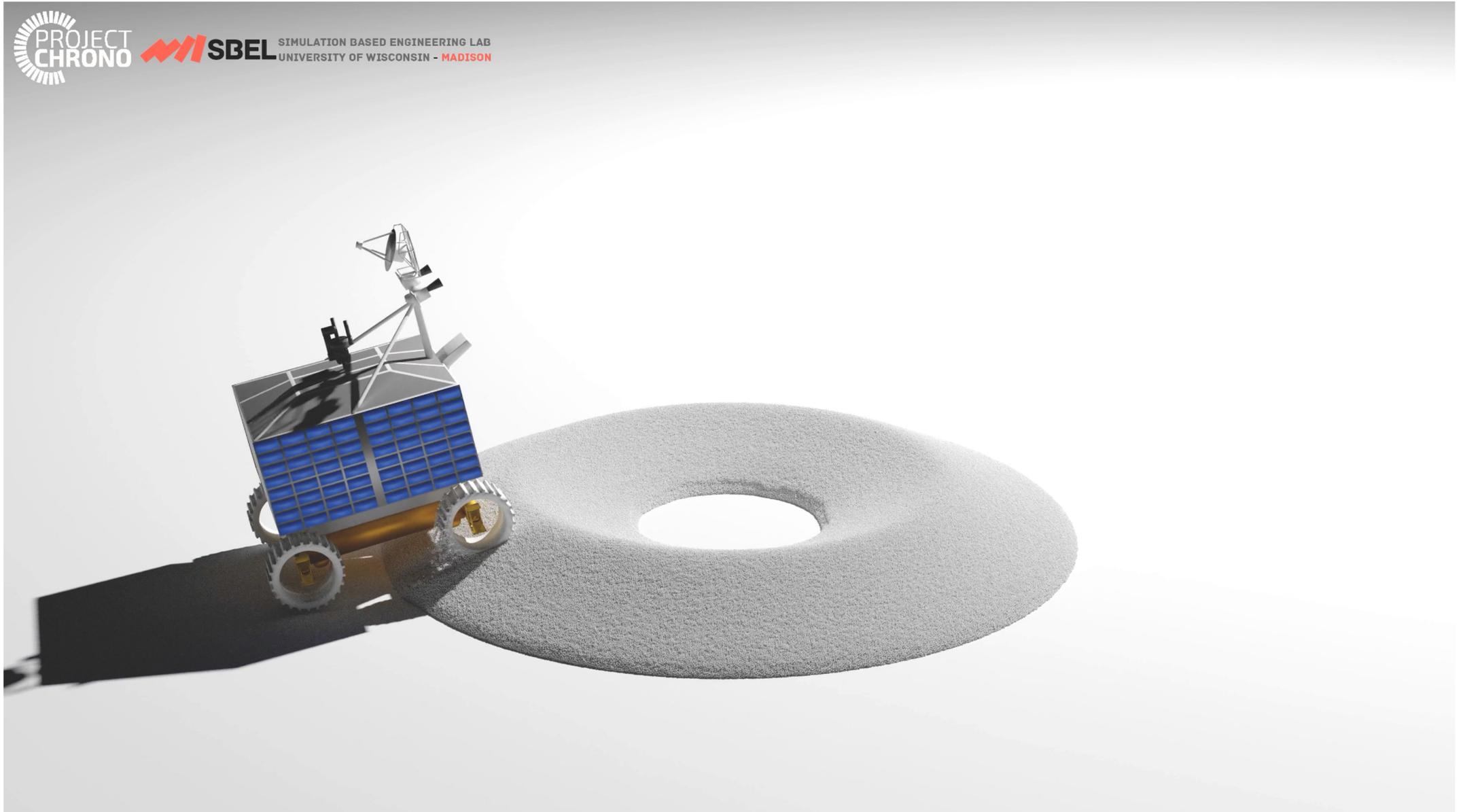
Compare model to earthbound experiments in simulant

MGRU-3 full rover test
(NASA Glenn Research Center)

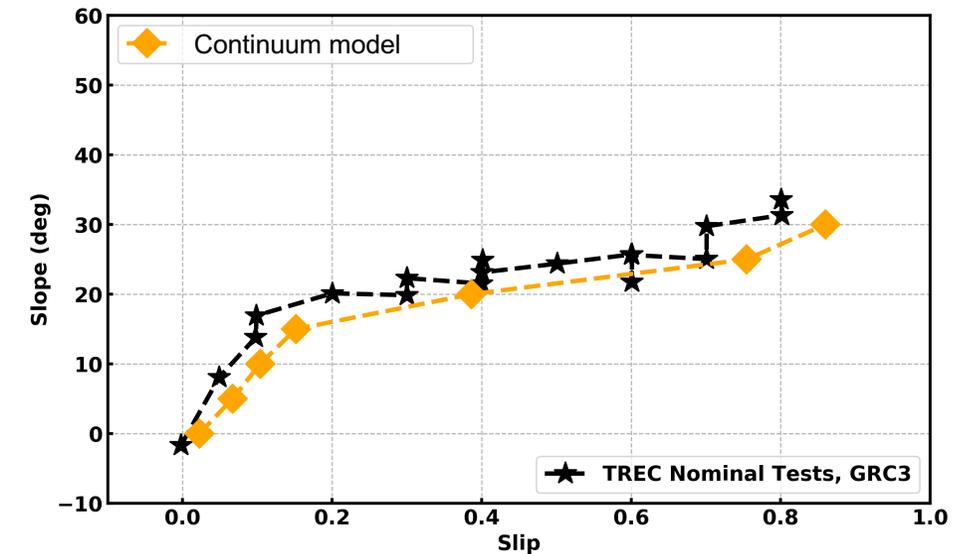Simulant: GRC-1

$\rho_c = 1730 \ \mathrm{kg/m}^3 \ , \ \mu_s = 0.76$

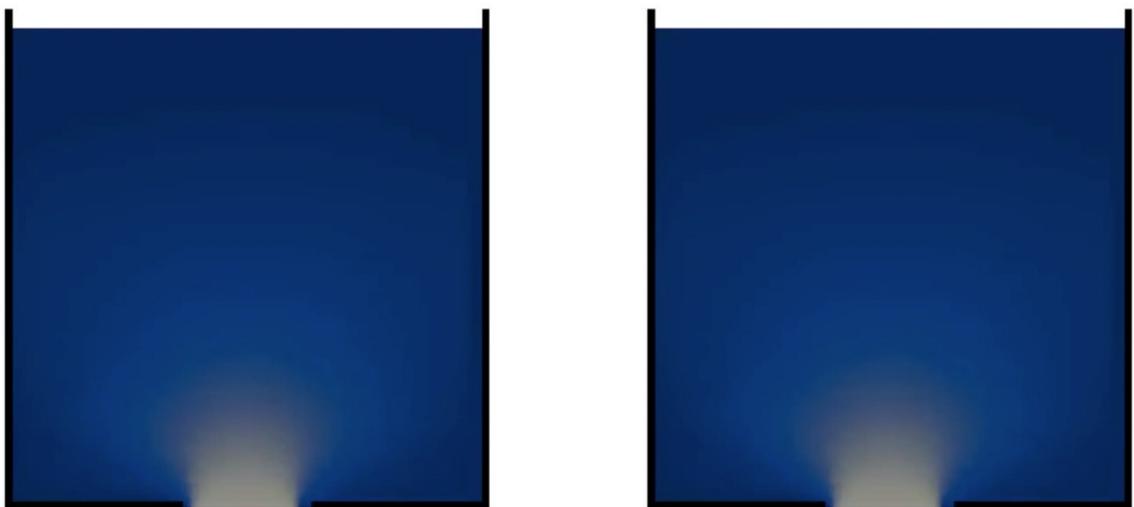

TREC wheel tests
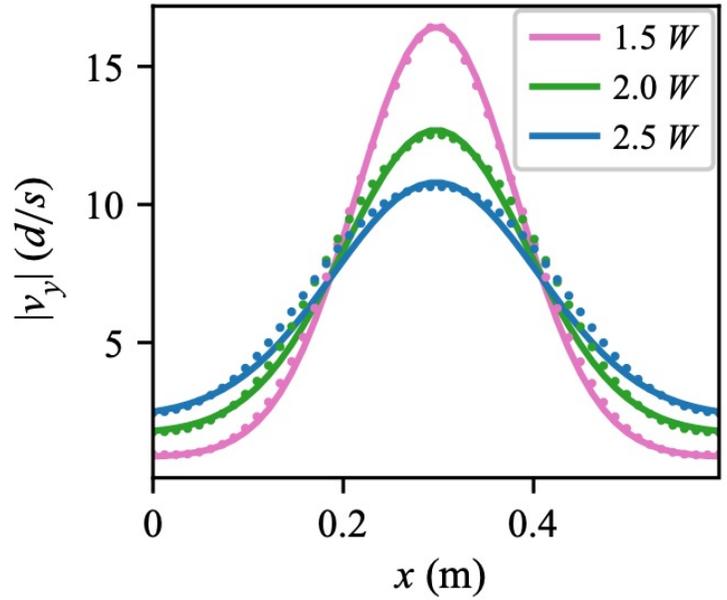(NASA Glenn Research Center)

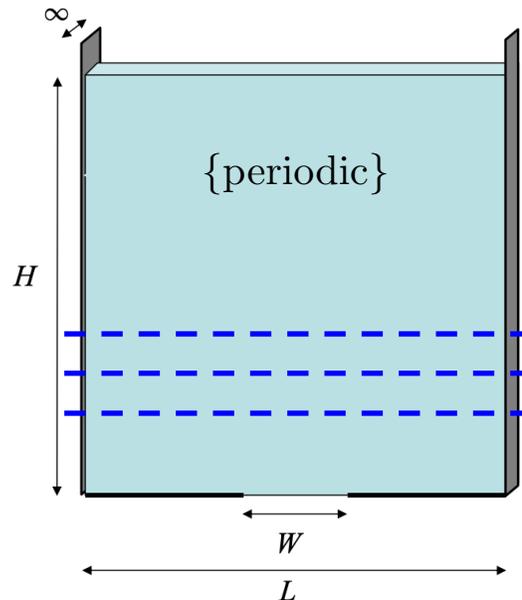Simulant: GRC-3

$\rho_c = 1734 \ \mathrm{kg/m}^3 \ , \ \mu_s = 0.90$

Aside: Adding a nonlocal effect...

$d = W$

$d = W/5$

$p/\rho gH\,(-)$

$0 \qquad\qquad 0.75 \qquad\qquad 1.50$

{periodic}

H

W

L

$|v_y|\ (d/s)$

- 1.5 W
- 2.0 W
- 2.5 W

x (m)

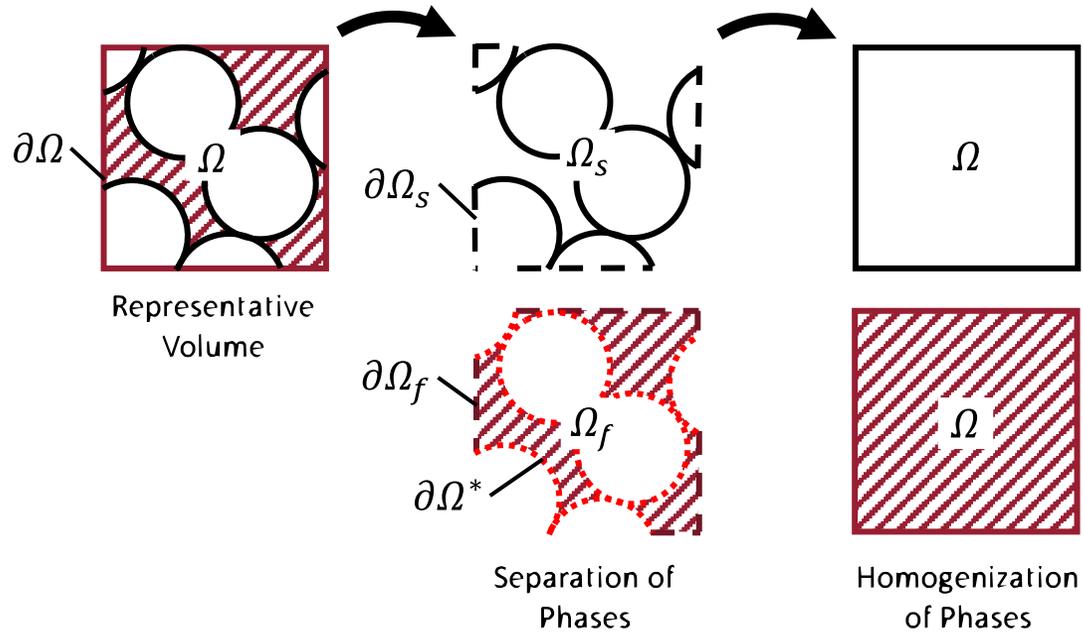Dunatunga and Kamrin, *JFM* (2022)
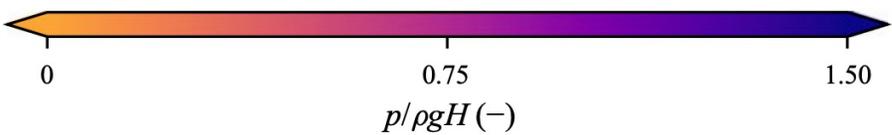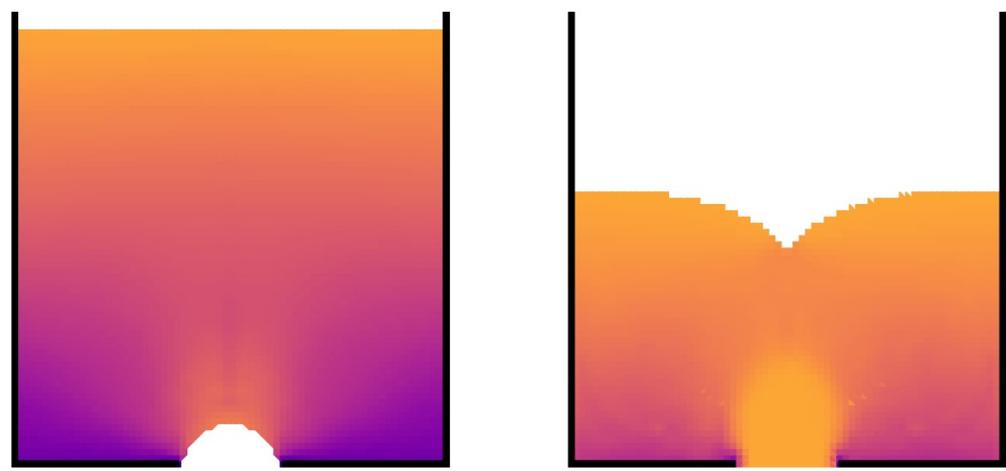
Part 2:
Extending and exploiting this framework to handle fluid-saturated granular media

Want: Continuum theory and simulation for fluid-sediment mixtures

Representative Volume

Separation of Phases

Homogenization of Phases

$$\rho = \bar{\rho}_s + \bar{\rho}_f \qquad \tau = \tilde{\tau} + \tau_f \qquad p = \tilde{p} + p_f$$

$$\bar{\rho}_f \frac{D^f \boldsymbol{v_f}}{Dt} = \operatorname{div}(\boldsymbol{\tau_f}) - n \operatorname{grad}(p_f) + \bar{\rho}_f \boldsymbol{g} + \boldsymbol{f_d}$$

$$\bar{\rho}_s \frac{D^s \boldsymbol{v_s}}{Dt} = \operatorname{div}(\tilde{\boldsymbol{\sigma}}) - (1-n)\operatorname{grad}(p_f) + \bar{\rho}_s \boldsymbol{g} - \boldsymbol{f_d}$$
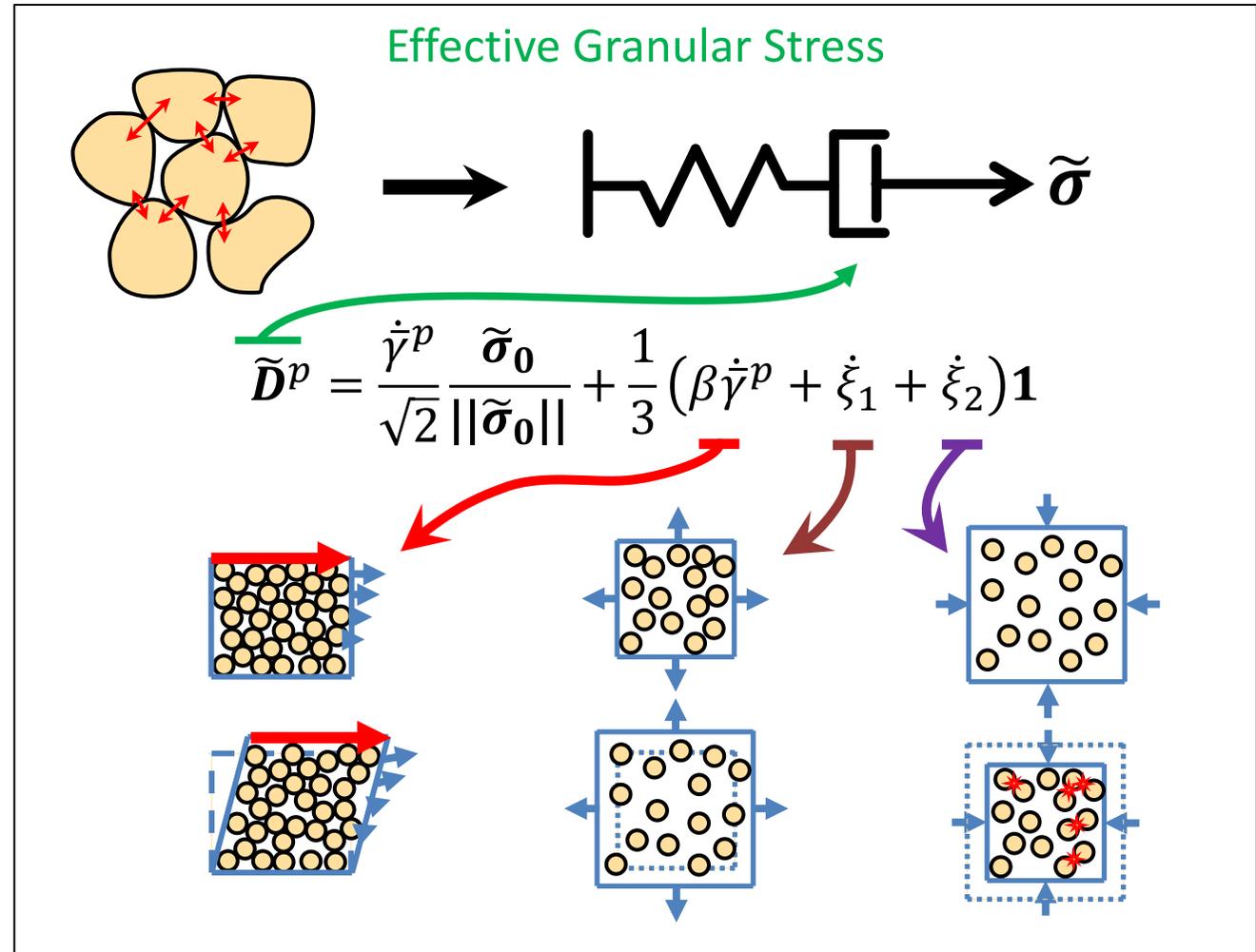

$$0 < n \le 1$$
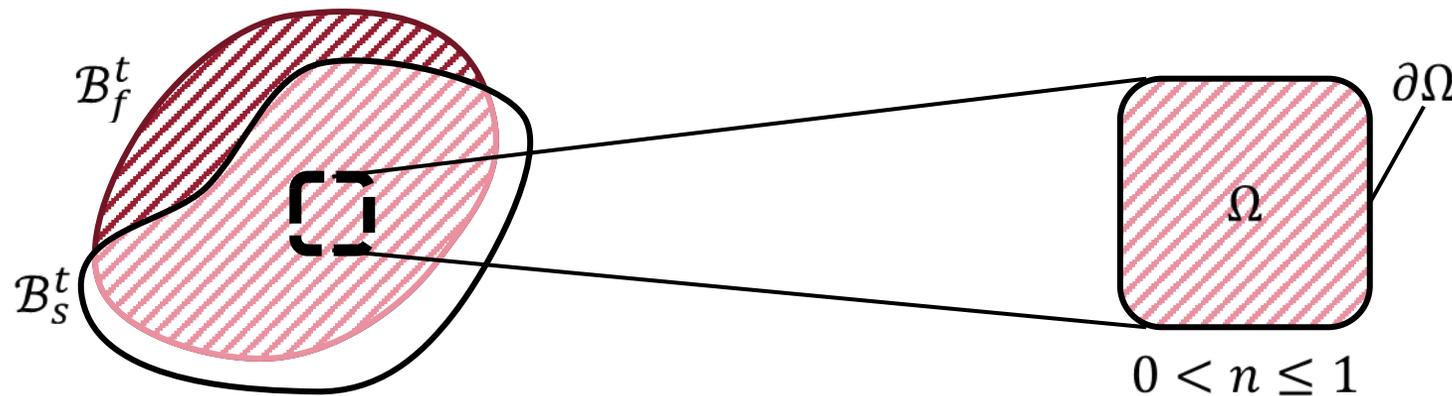
Teaser:
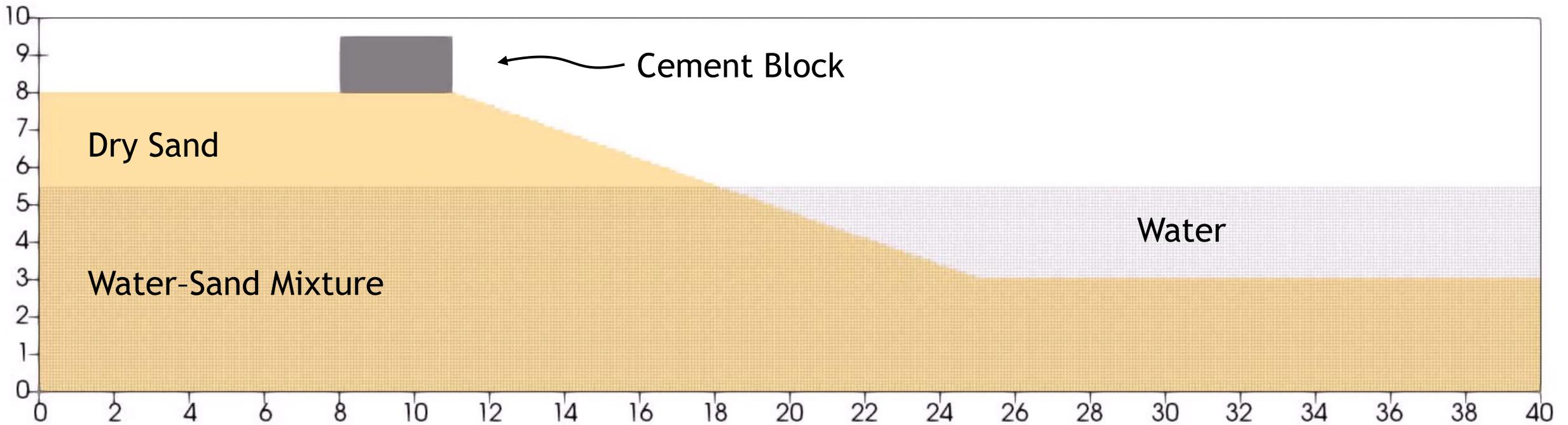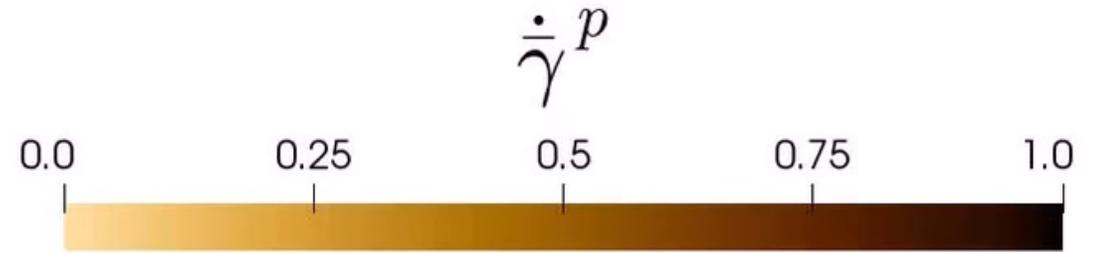
Constitutive Modeling

$$\bar{\rho}_f \frac{D^f \boldsymbol{v}_f}{Dt} = \mathrm{div}(\boldsymbol{\tau}_f) - n\,\mathrm{grad}(p_f) + \bar{\rho}_f \boldsymbol{g} + \boldsymbol{f_d}$$

$$\bar{\rho}_s \frac{D^s \boldsymbol{v}_s}{Dt} = \mathrm{div}(\widetilde{\boldsymbol{\sigma}}) - (1-n)\mathrm{grad}(p_f) + \bar{\rho}_s \boldsymbol{g} - \boldsymbol{f_d}$$

Effective Granular Stress

$$\widetilde{\boldsymbol{D}}^p = \frac{\dot{\gamma}^p}{\sqrt{2}} \frac{\widetilde{\boldsymbol{\sigma}}_0}{\|\widetilde{\boldsymbol{\sigma}}_0\|} + \frac{1}{3}\left(\beta\dot{\gamma}^p + \dot{\xi}_1 + \dot{\xi}_2\right)\mathbf{1}$$
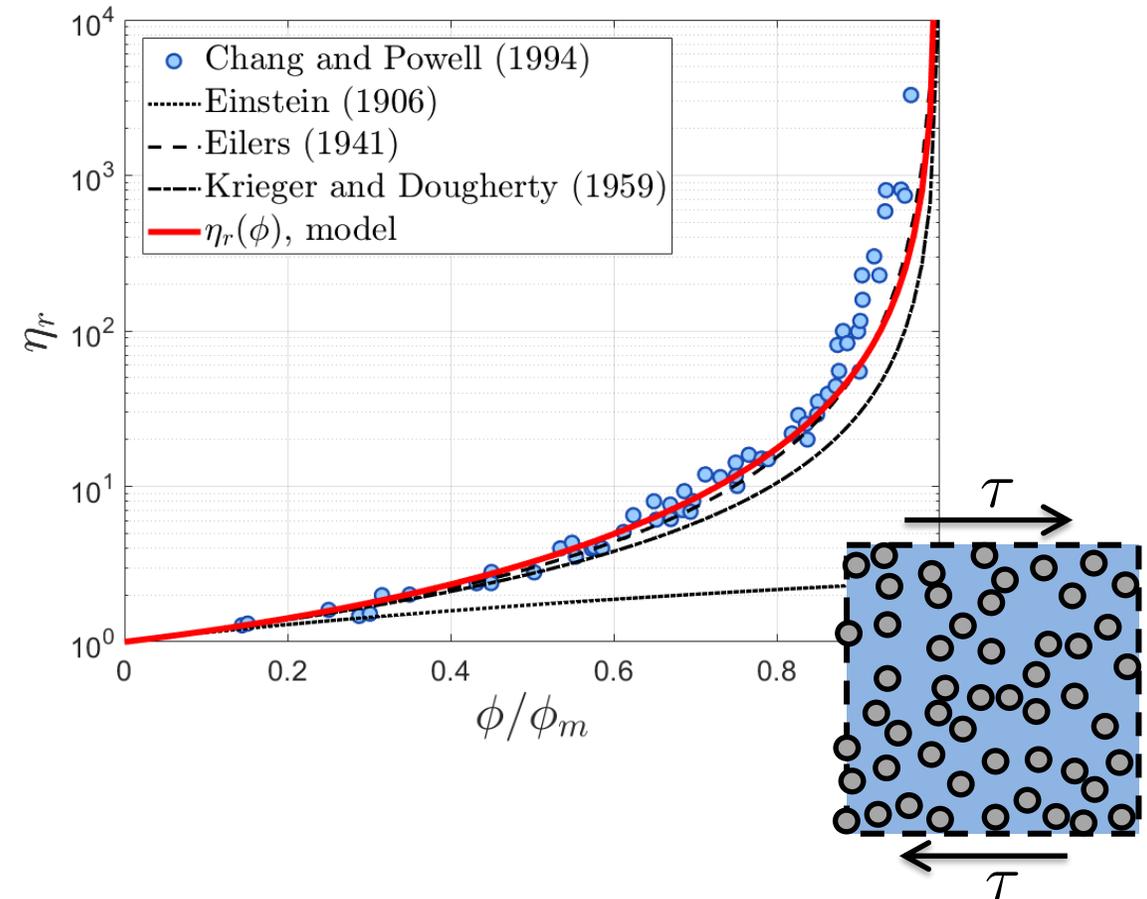
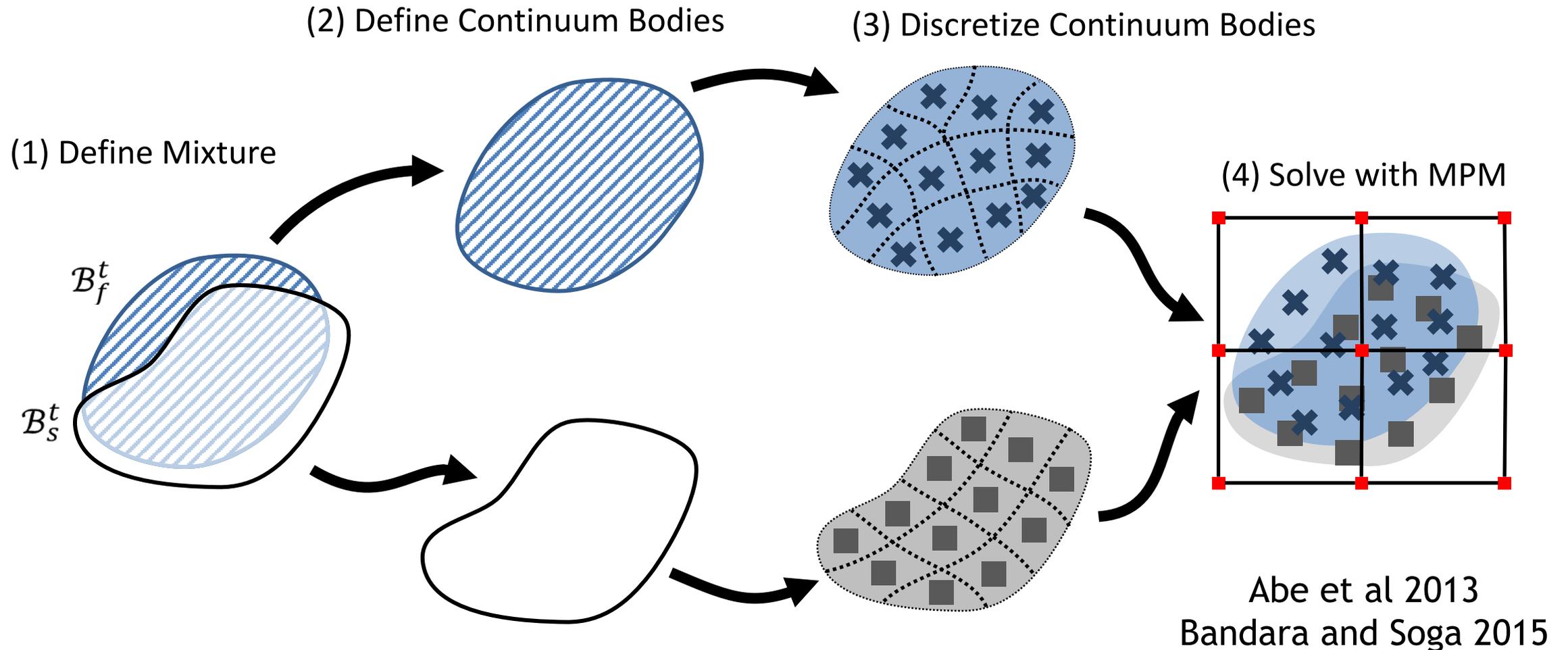# Co-moving simple shearing captures correct rheologies

$\mu(I_v)$ - Viscous granular rheology
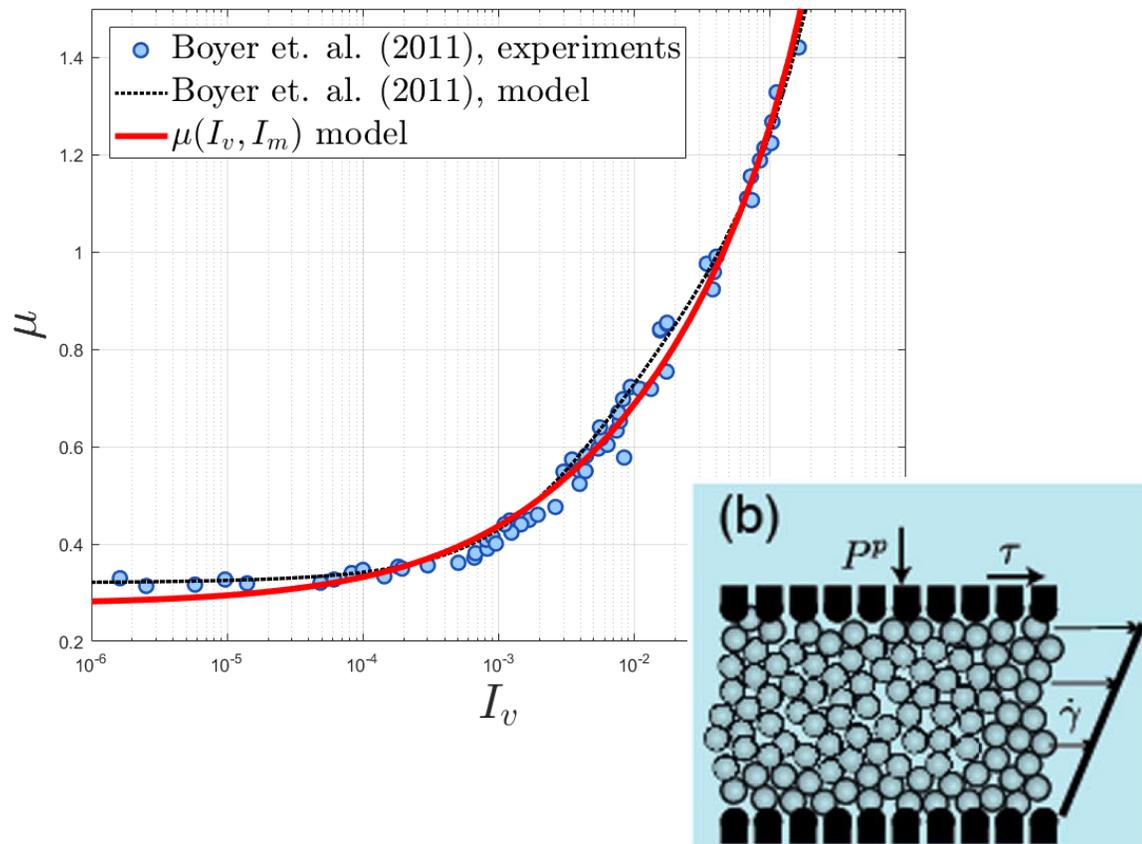
$\eta_r(\phi)$ - (Extended) Einstein suspension rheology

Material point method used to solve governing equations

(1) Define Mixture

(2) Define Continuum Bodies

(3) Discretize Continuum Bodies

(4) Solve with MPM

$\mathcal{B}_f^t$

$\mathcal{B}_s^t$

Abe et al 2013
Bandara and Soga 2015

Model applied to granular column collapse

- Calibrate model to wet beads based on submerged avalanching data in Pailha and Pouliquen (2009).

- Rondon et. al. (2011) analyze the effect of initial packing fraction on the dynamics of a submerged granular column collapse.

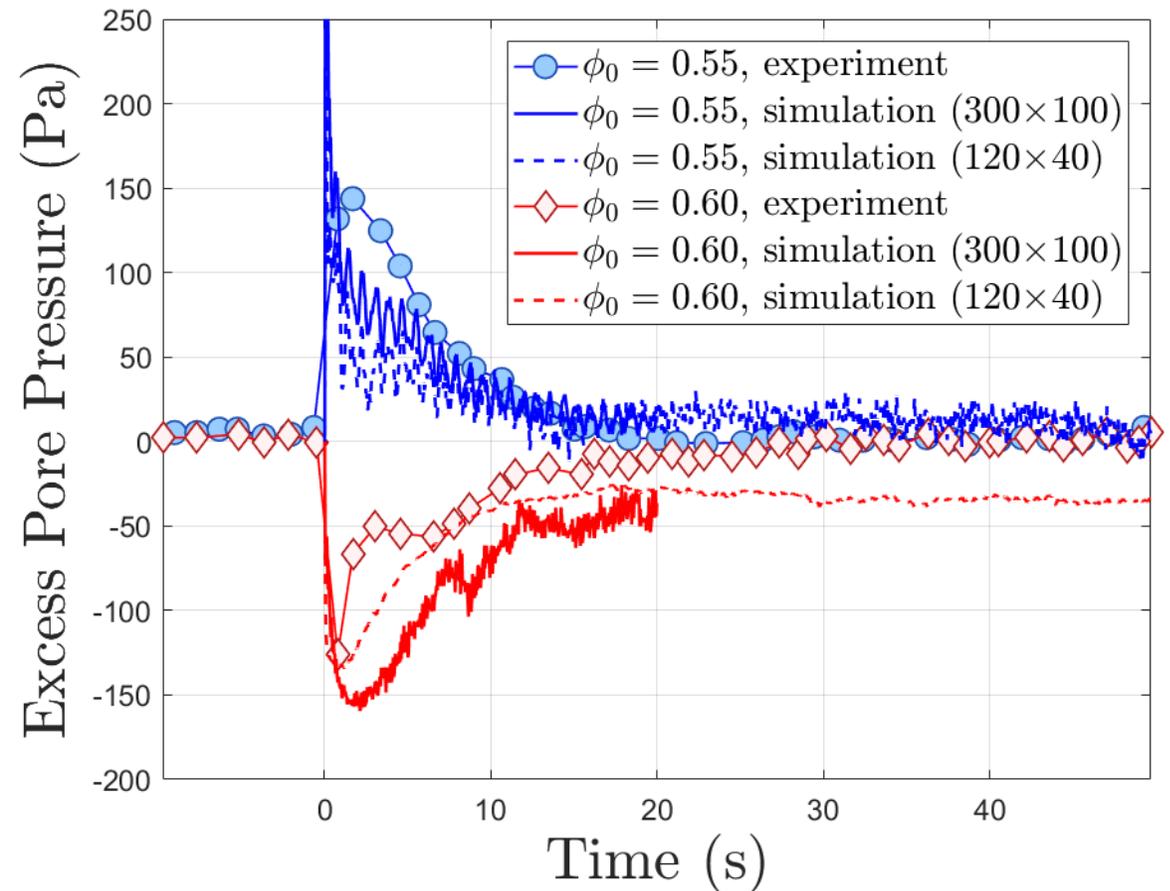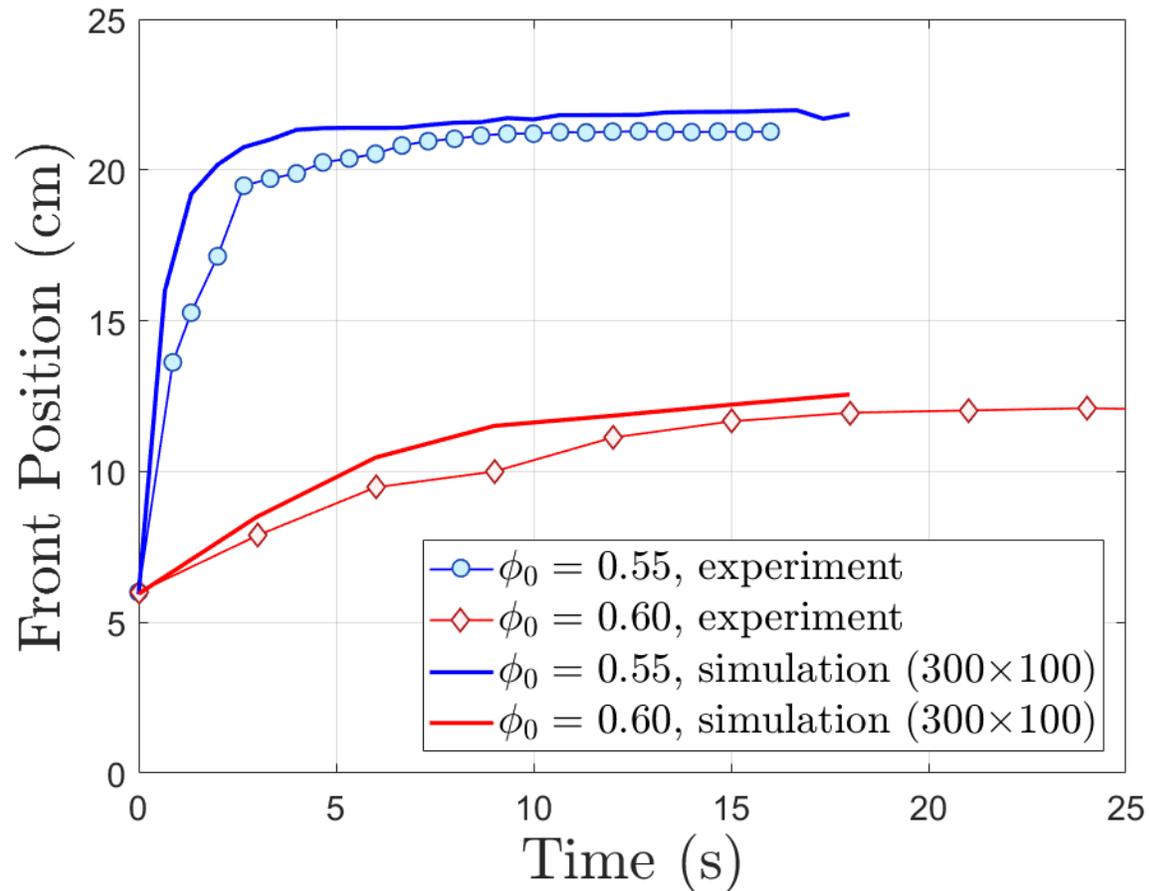- Measure front propagation and pore pressure within column for 225μm glass beads
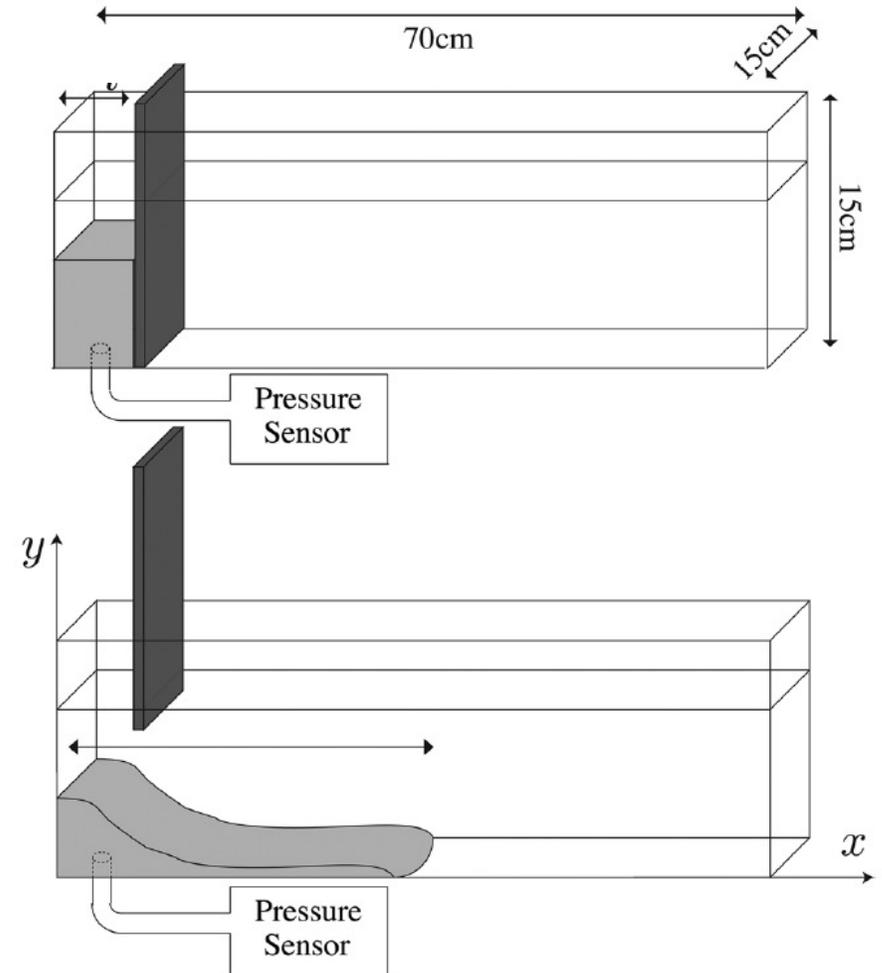
Granular column collapse simulation video

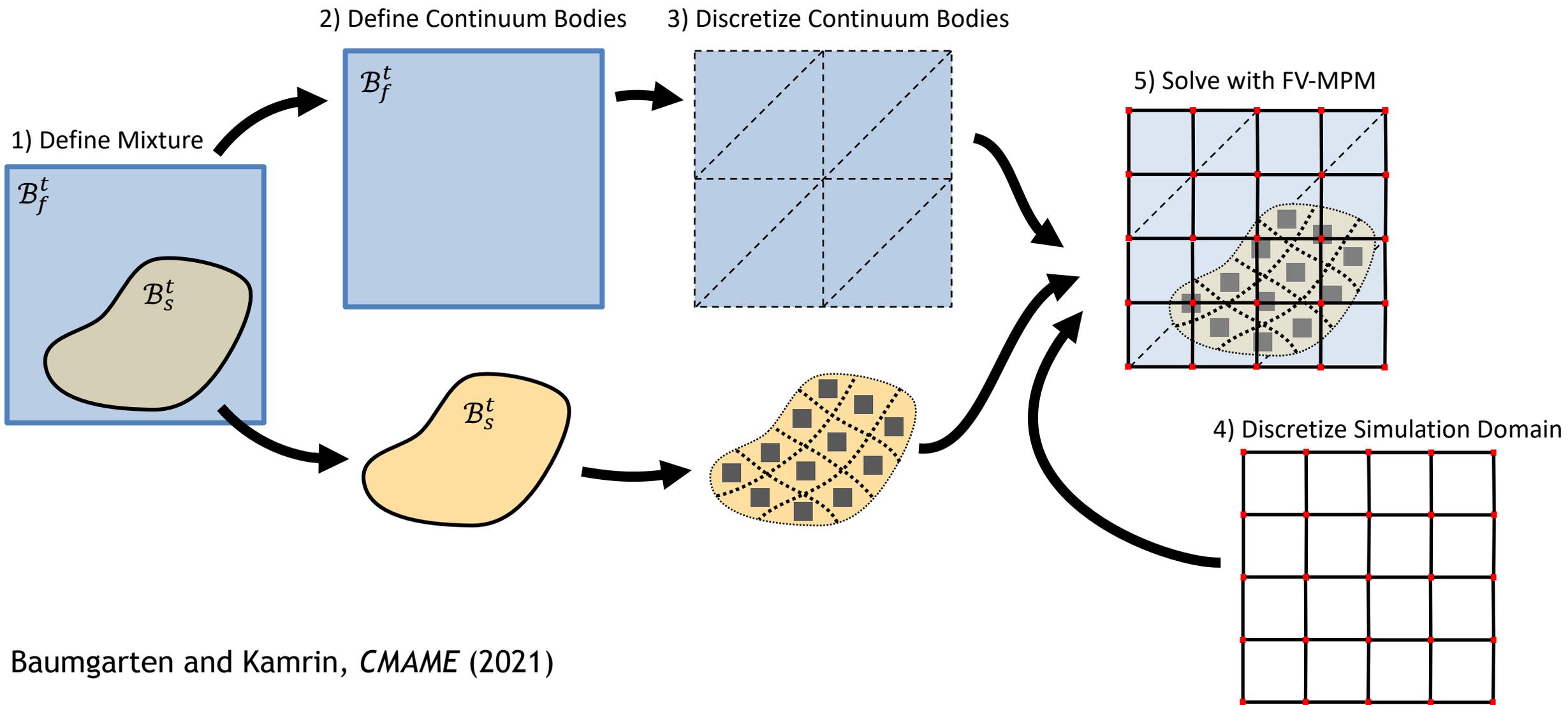Baumgarten and
Kamrin *JFM* 2019
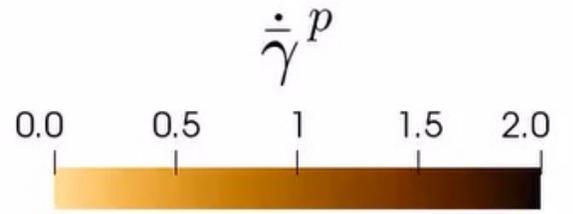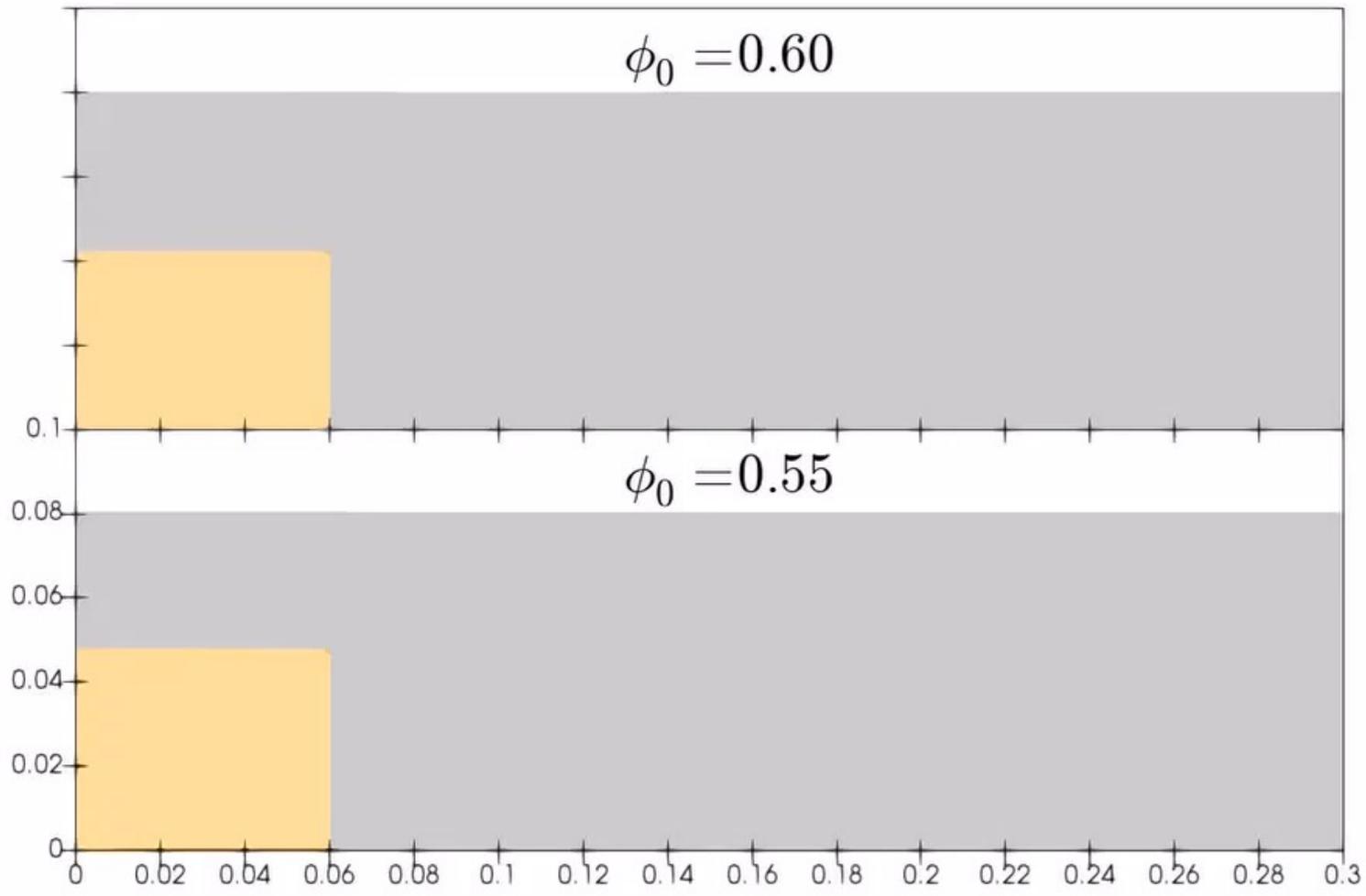
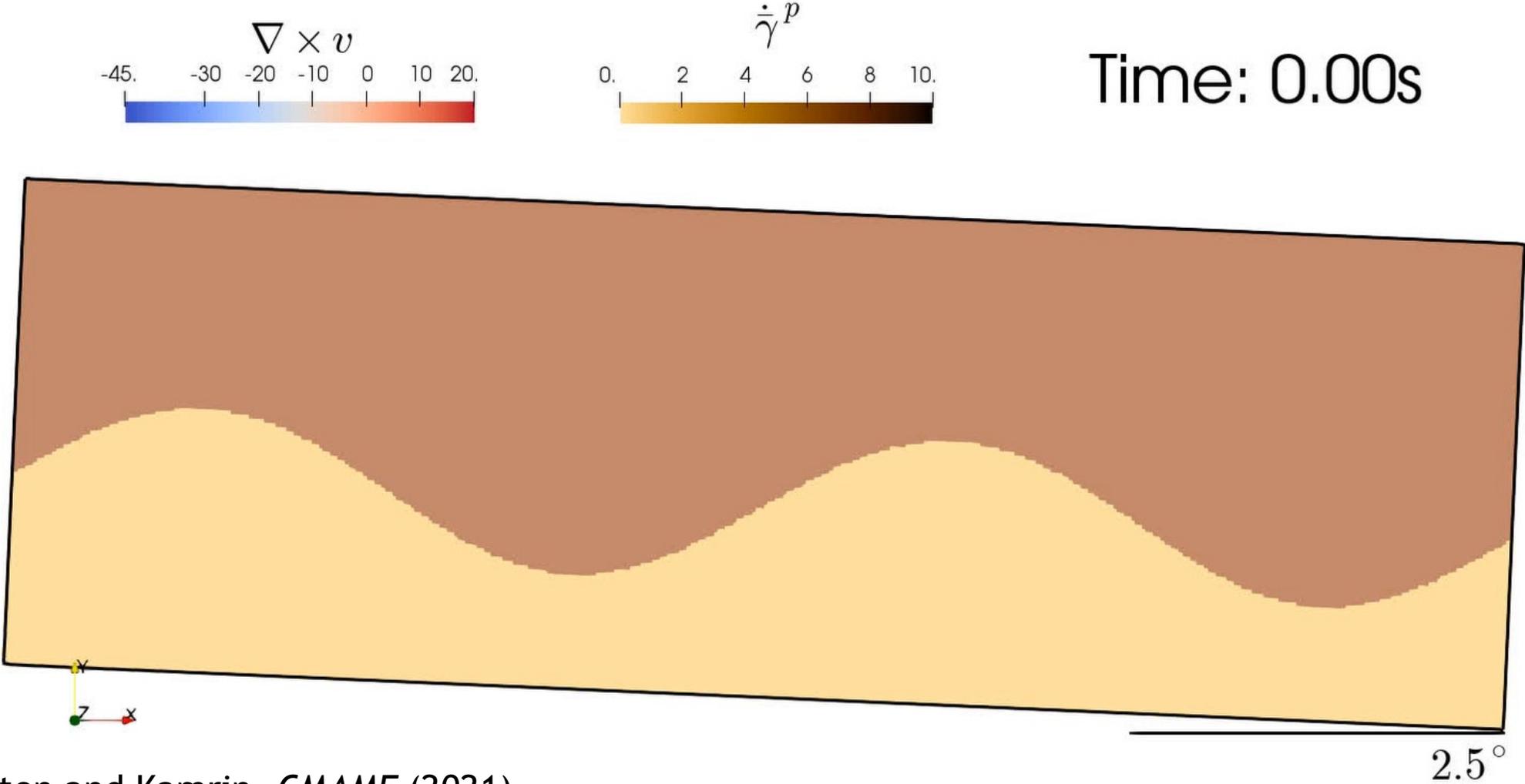# Pressure and run-out values show similar quantitative behavior

For air-grain simulation: Use new two-phase FV-MPM framework

2) Define Continuum Bodies

3) Discretize Continuum Bodies

1) Define Mixture

5) Solve with FV-MPM

4) Discretize Simulation Domain

$\mathcal{B}_f^t$

$\mathcal{B}_s^t$

$\mathcal{B}_s^t$

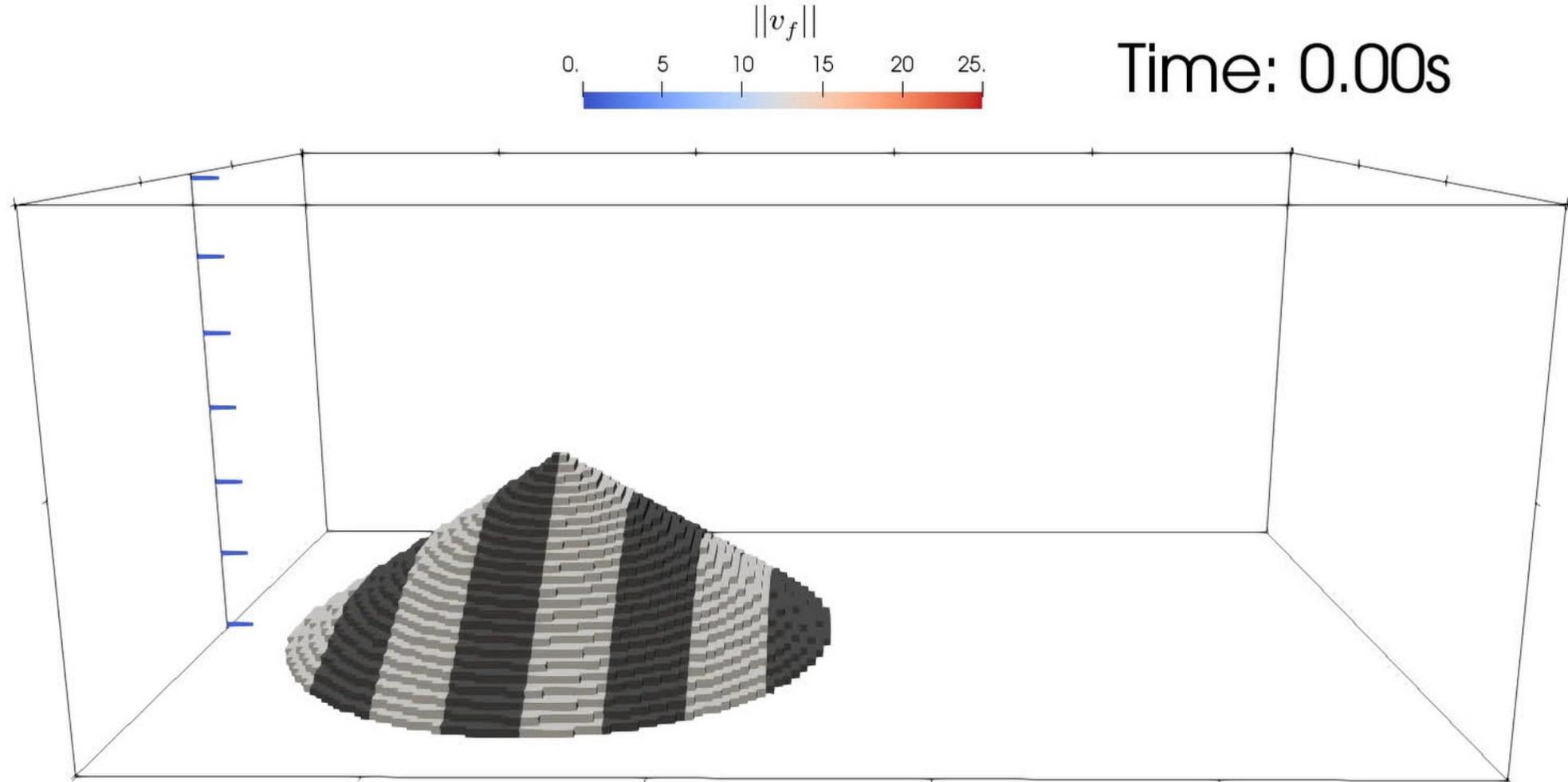Baumgarten and Kamrin, *CMAME* (2021)

# Numerical simulation of flow in an infinite pipeline
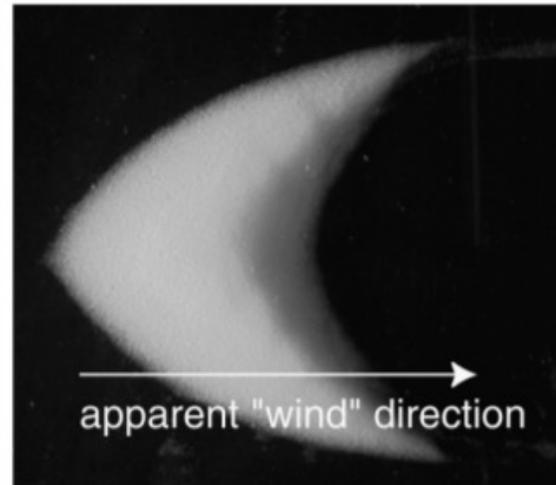

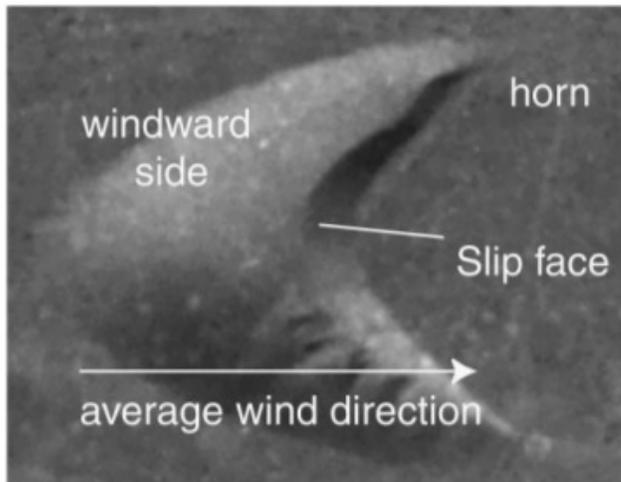Baumgarten and Kamrin, *CMAME* (2021)

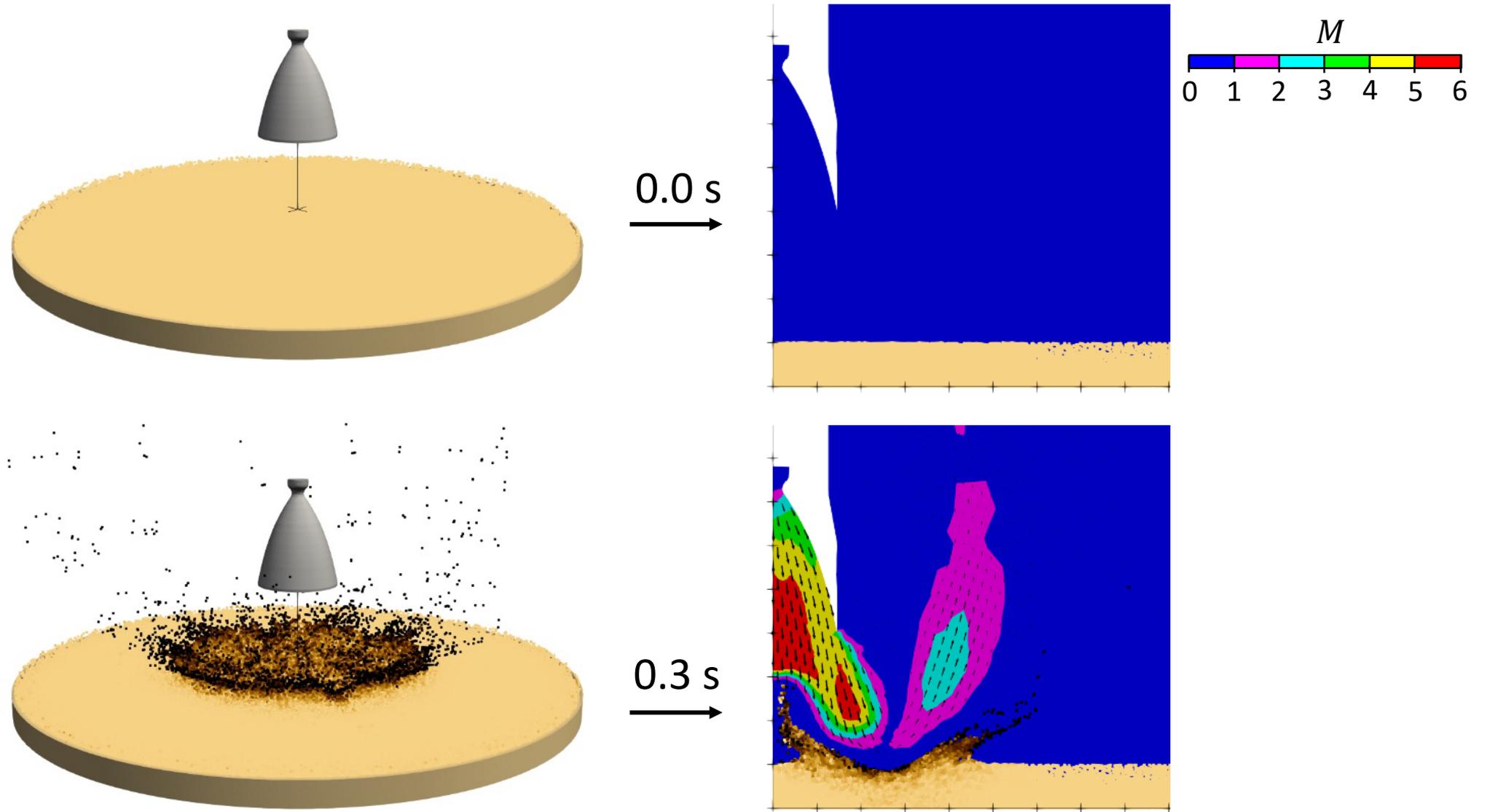# Numerical simulation of sand pile erosion in wind tunnel

Final shape of eroded sand pile resembles barchan dune shape

- Porosity contours ($n = 0.8$) help visualize dune shape

- Crescent shape with steep back slope characteristic of barchan dunes
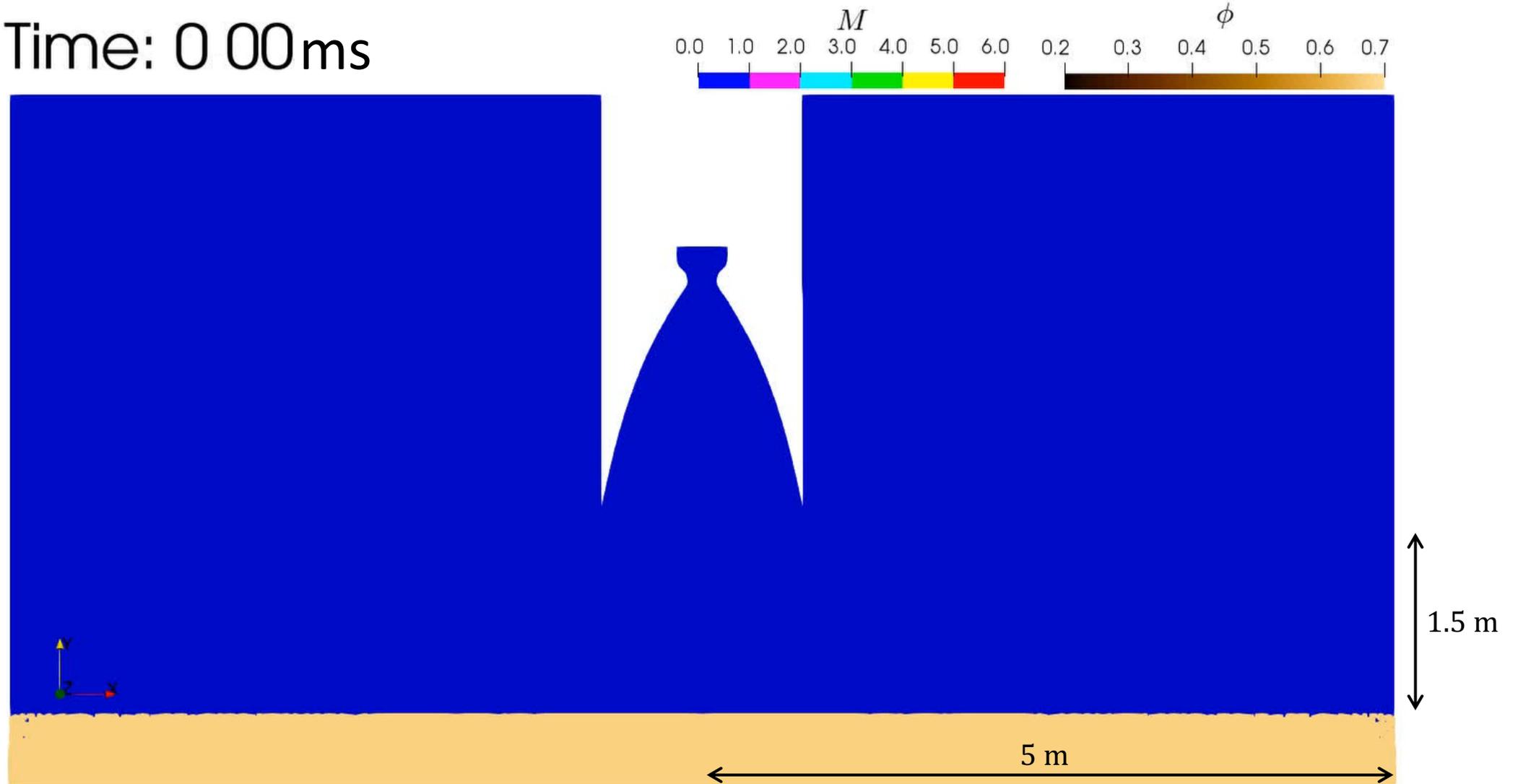

Hersen et al. (2002)

Time: 0.00s

Time: 0.50s

Time: 2.00s
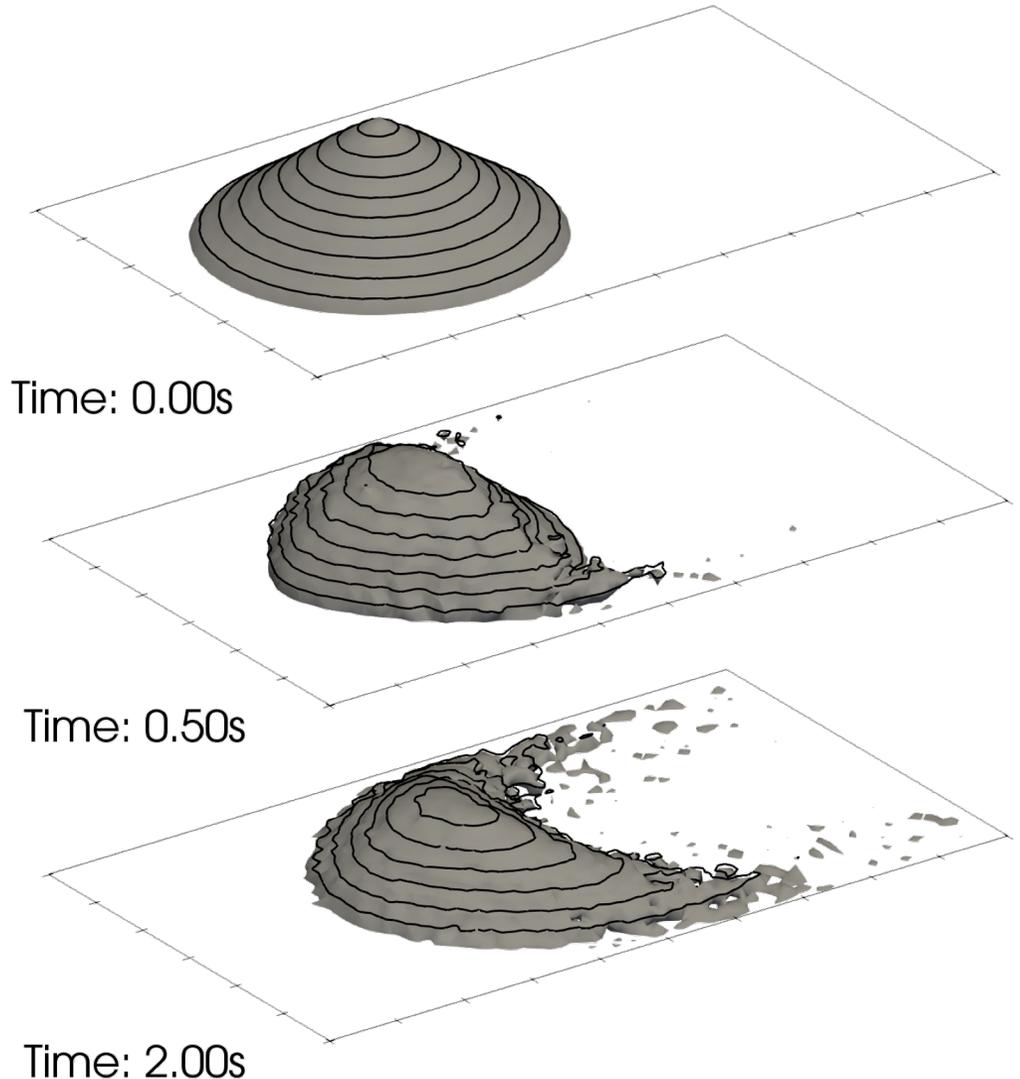
Numerical simulation of cratering of Martian surface below LMDE

Numerical simulation of cratering of Martian surface below LMDE

Conclusions

Part 1: Basic dry granular rheology and MPM implementation

- Simple model with frictional yielding and no-tension can work in a family of intrusion problems.

Part 2: Continuum modeling fluid-grain mixtures

- Implemented a dilative mixture constitutive model for fluid-sediment mixtures based on two-phase MPM.
- Validated our model against multiple large deformation, complex flows across flow regimes for simple grains in Newtonian liquid.
- Have introduced a new method that is more appropriate for gas-grain flows (FV-MPM) with large fluid domains.
- Represents a potentially high-Reynolds, thermal fluid phase (gas) with Eulerian FV, which avoids issues from using Lagrangian MPM in the fluid.
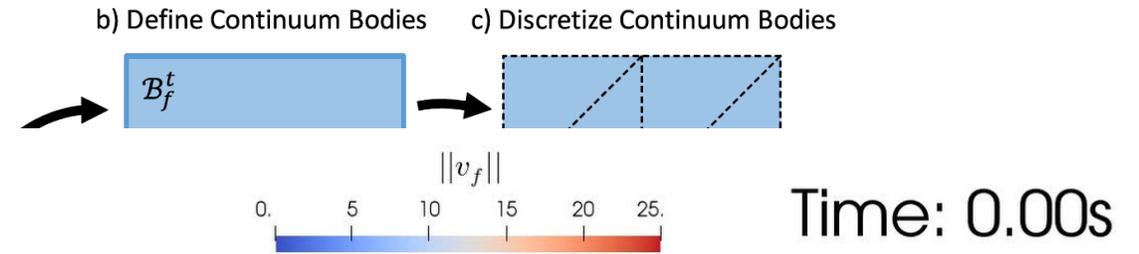
Questions?

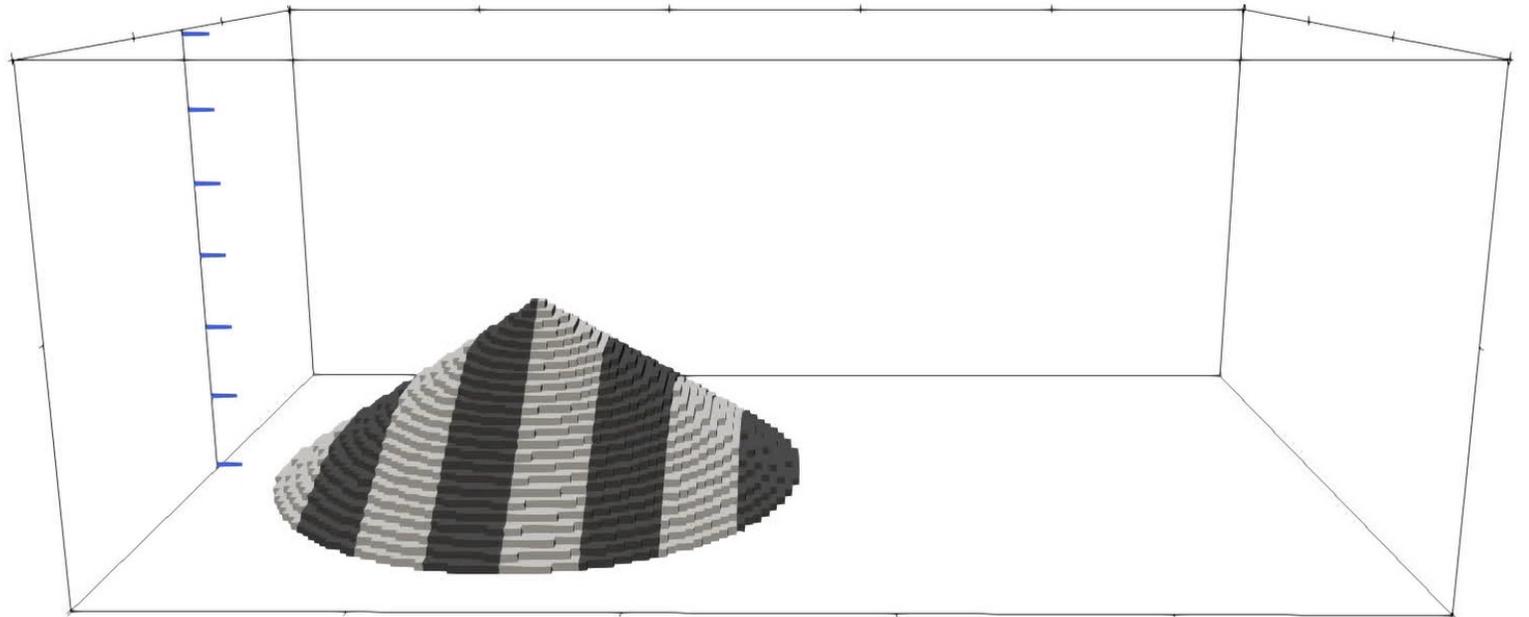# Teaser: FV-MPM for fluid-grain mixtures

Baumgarten et al, CMAME (2021)

Idea: Replace MPM representation of fluid phase
with a Finite-Volume Method representation.

Allows for a much better-behaved fluid
simulation and lets us simulate air!

Barchan dune:

b) Define Continuum Bodies   c) Discretize Continuum Bodies

$\mathcal{B}_f^t$

$||v_f||$

0.   5   10   15   20   25.

Time: 0.00s

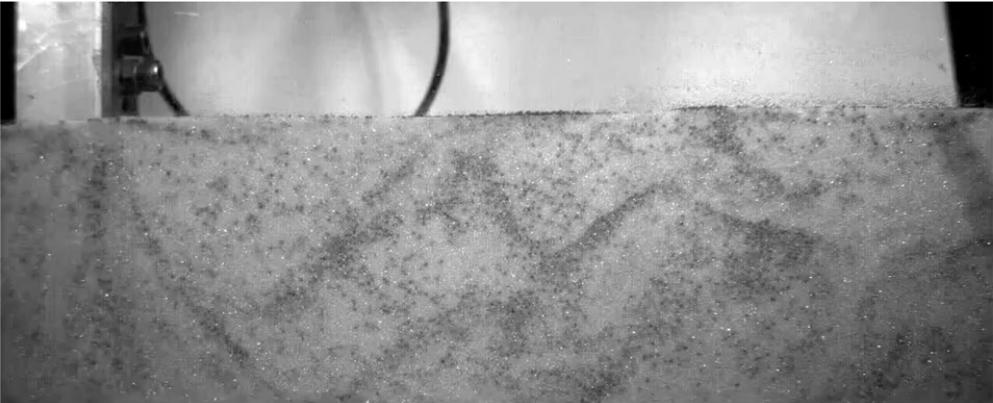# Example of stress-dilatancy model: Plowing

Agarwal et al, *Soft Matter*, (2021)

Experiments

Video credits: Nick Gravish, Daniel Goldman, Crab Lab,
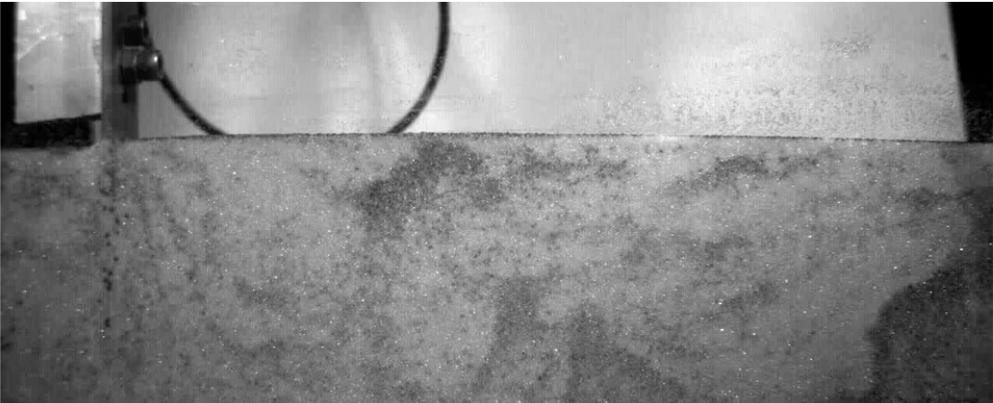Georgia Institute of Technology

Continuum simulations

Eq. plastic strain 4×10^{-2} 10^{-1} 10^{0} 10^{1}
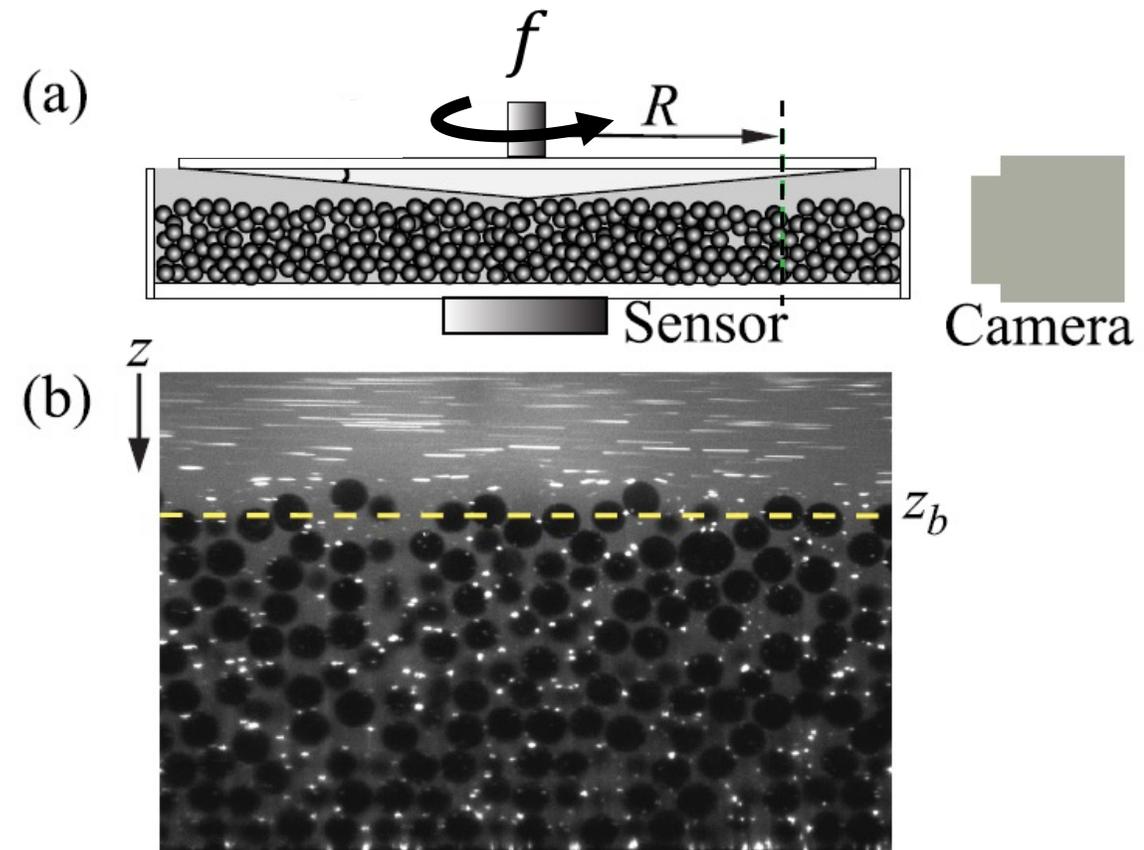
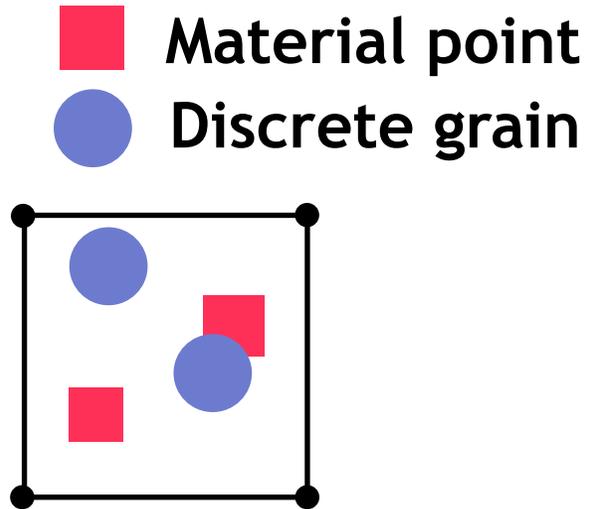Under compacted media ($\phi < \phi_c$)

Over compacted media ($\phi > \phi_c$)

Model applied to quasi-2D erosion flow

- Allen and Kudrolli (2017) drive fluid flow above a ~9mm bed of grains to analyze the quasi-2D behavior of the mixture

- Measure packing fraction and phase velocities for 1.05mm glass beads

- Observe sustained transport of granular materials at 'critical' driving frequency, f_c

Model captures most flow regimes

Time: 0.00s

ϕ

0.450   0.500   0.550   0.585

$f/f_c = 0.37$ $f/f_c = 1.04$ $f/f_c = 1.26$ $f/f_c = 1.33$

15.5 mm   20 elements

Baumgarten and Kamrin *JFM* 2019

Hybridization

Node based coupling:

■ Material point

● Discrete grain

$$W_c M_c \boldsymbol{v}_c^{n+1} + \lambda = W_c M_c \boldsymbol{v}_c^*,$$

$$W_d M_d \boldsymbol{v}_d^{n+1} - \lambda = W_d M_d \boldsymbol{v}_d^*,$$

$$\boldsymbol{v}_c^{n+1} = \boldsymbol{v}_d^{n+1}.$$

Solve:

$$\boldsymbol{v}_c^{n+1} = \boldsymbol{v}_d^{n+1} = (W_c M_c + W_d M_d)^{-1} \left(W_c M_c \boldsymbol{v}_c^* + W_d M_d \boldsymbol{v}_d^* \right)$$

Results: 2D Column Collapse
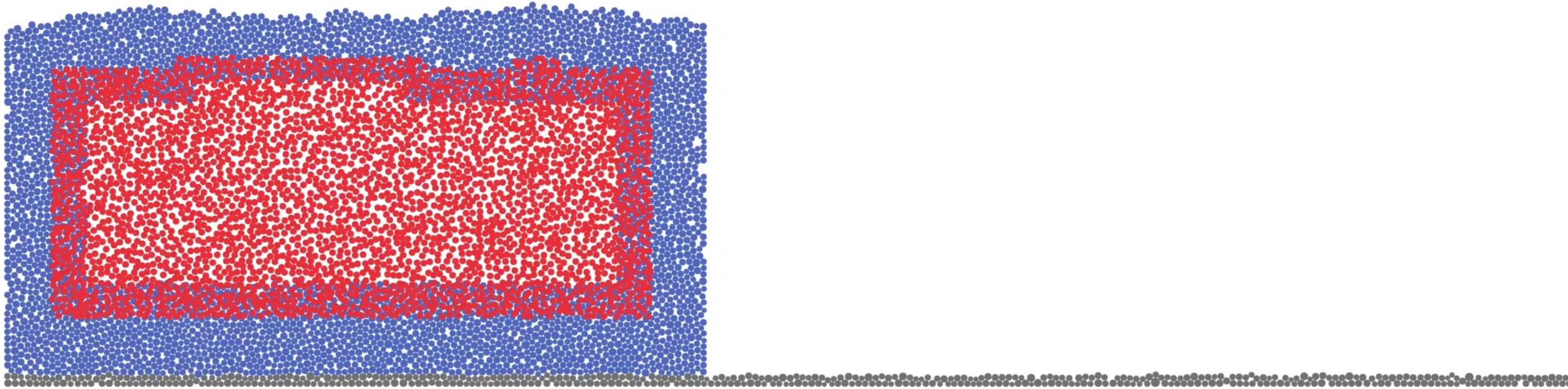
- **Discrete grains**
- **Material points**

Results: 2D Column Collapse
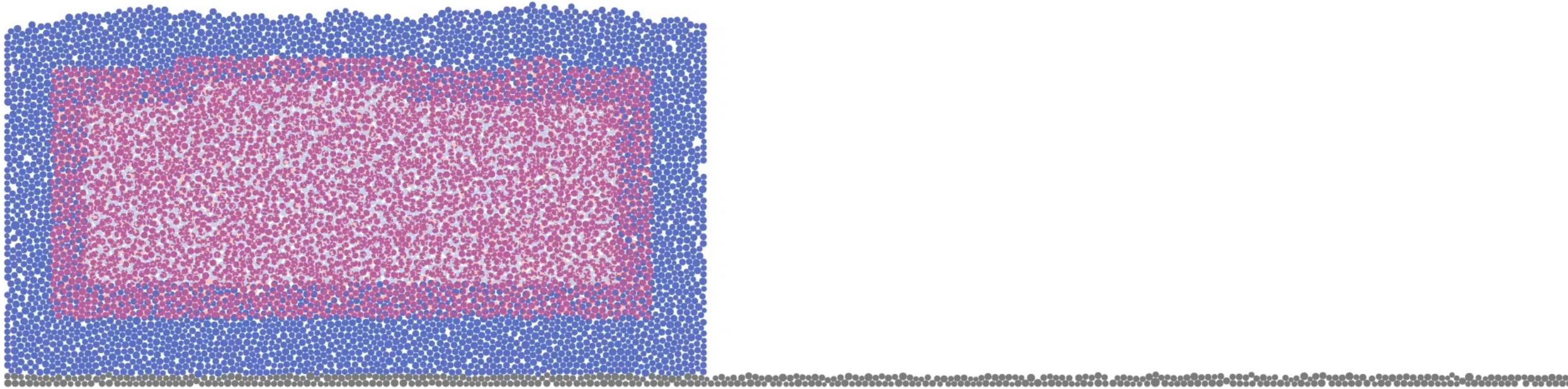
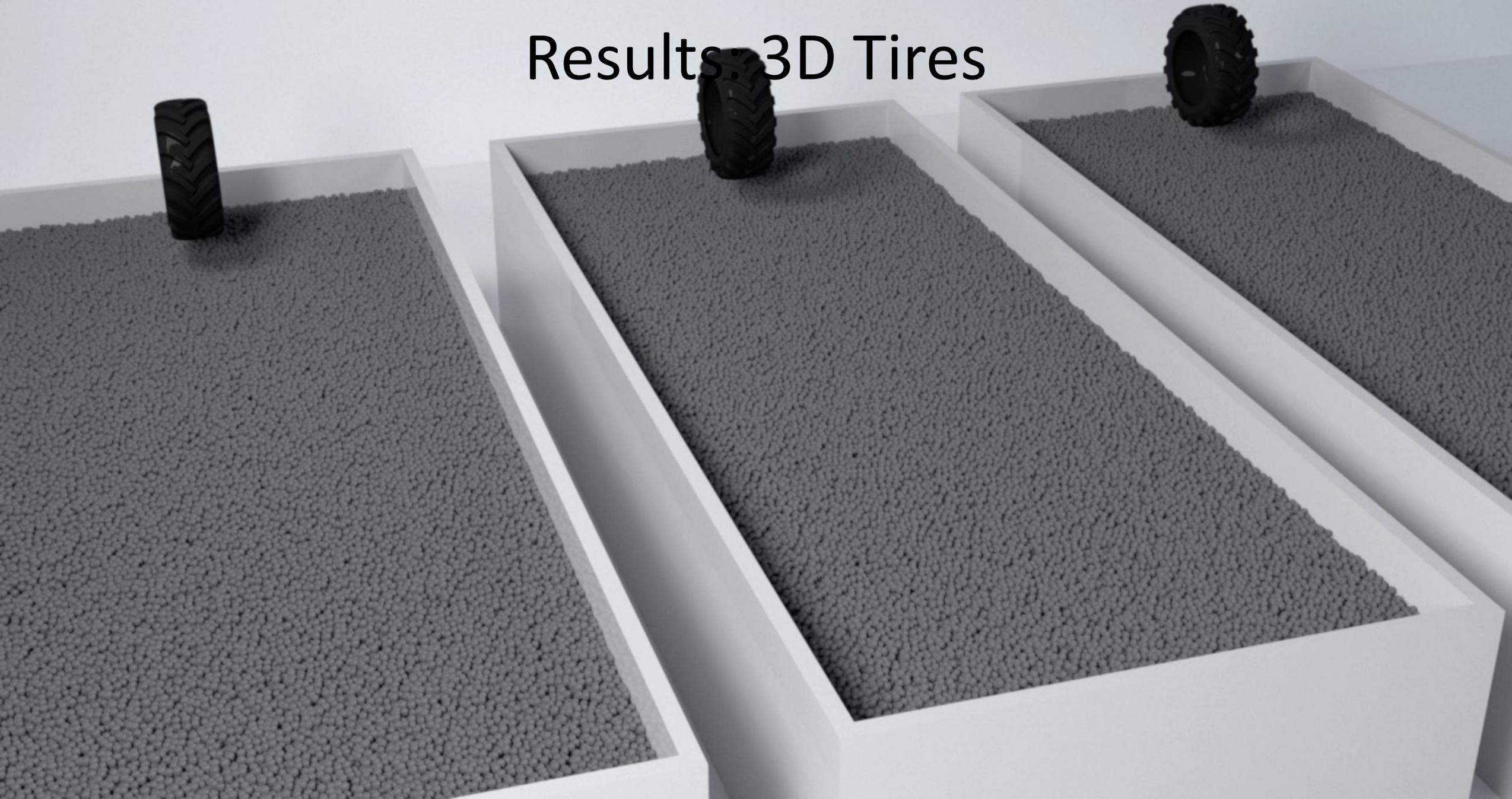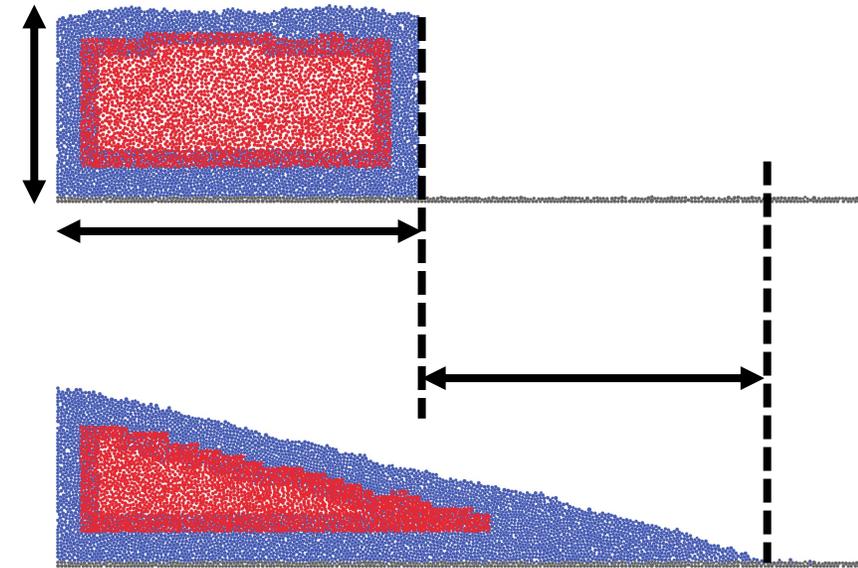- Discrete grains
- Material points

Results: 2D Penetrometer

discrete

hybrid

55
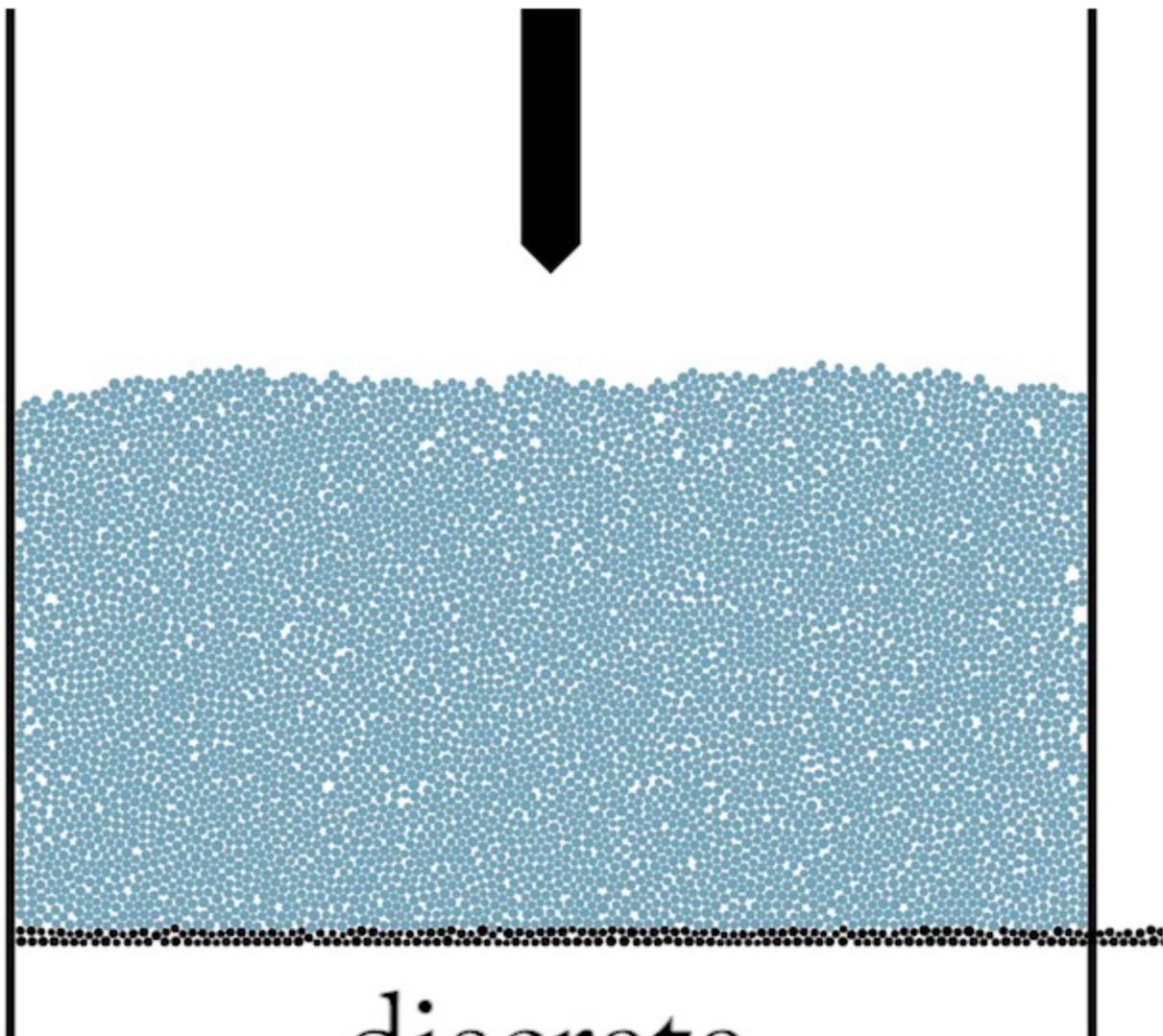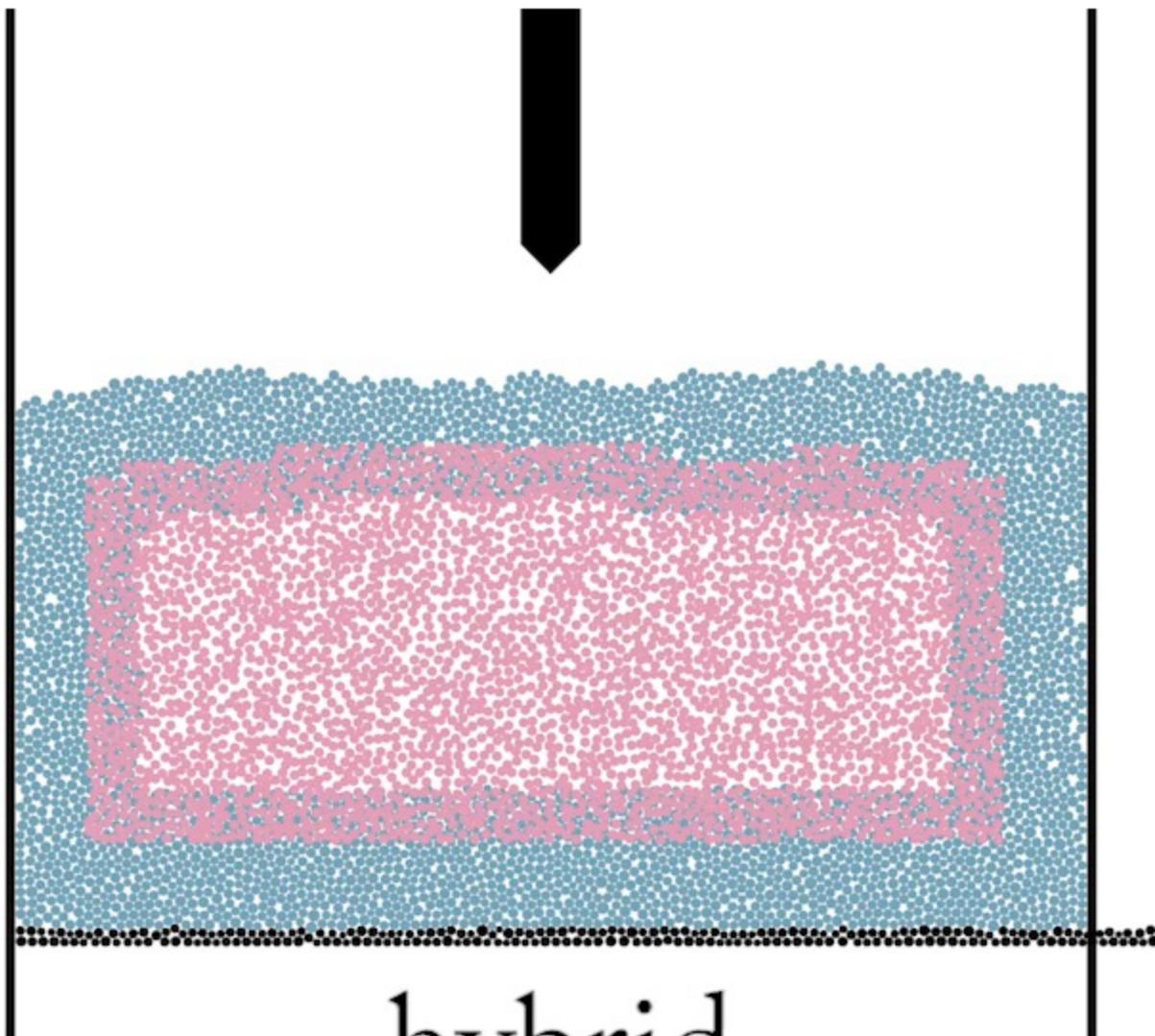
Results: 3D Tires

Aspect Ratio vs Run-out Distance
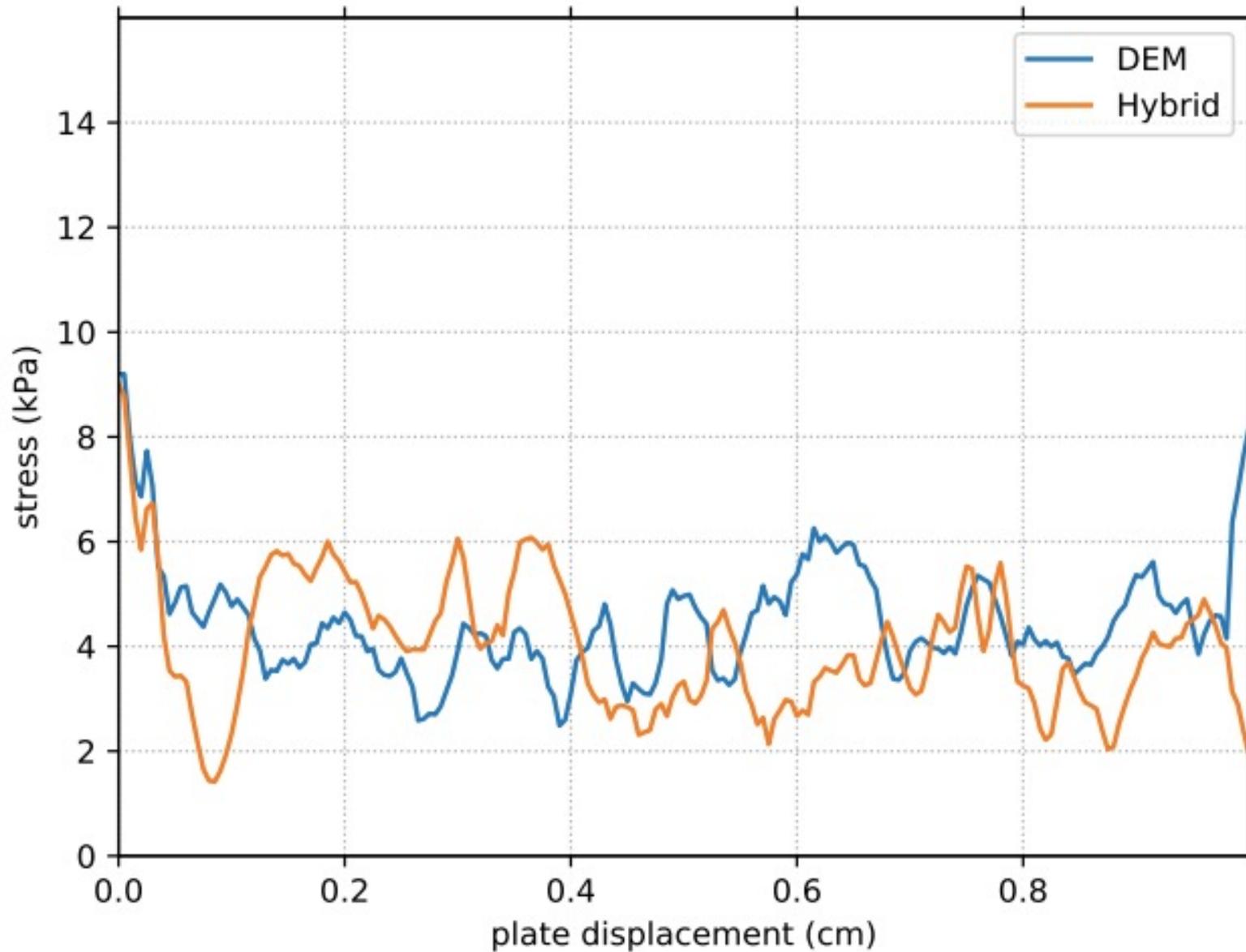

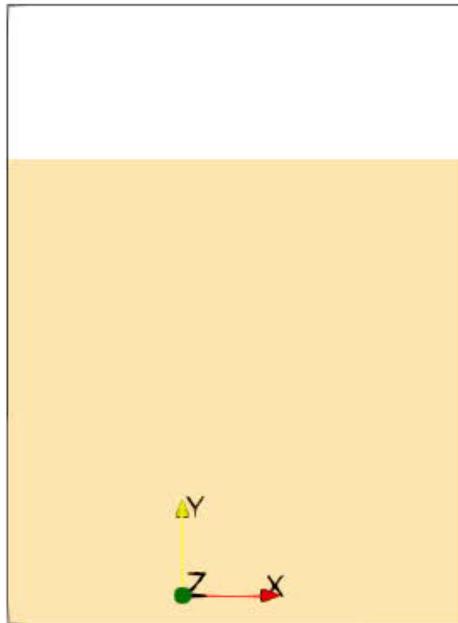~ Power law [Lube+ 2005]

Linear regime [Lube+ 2005]
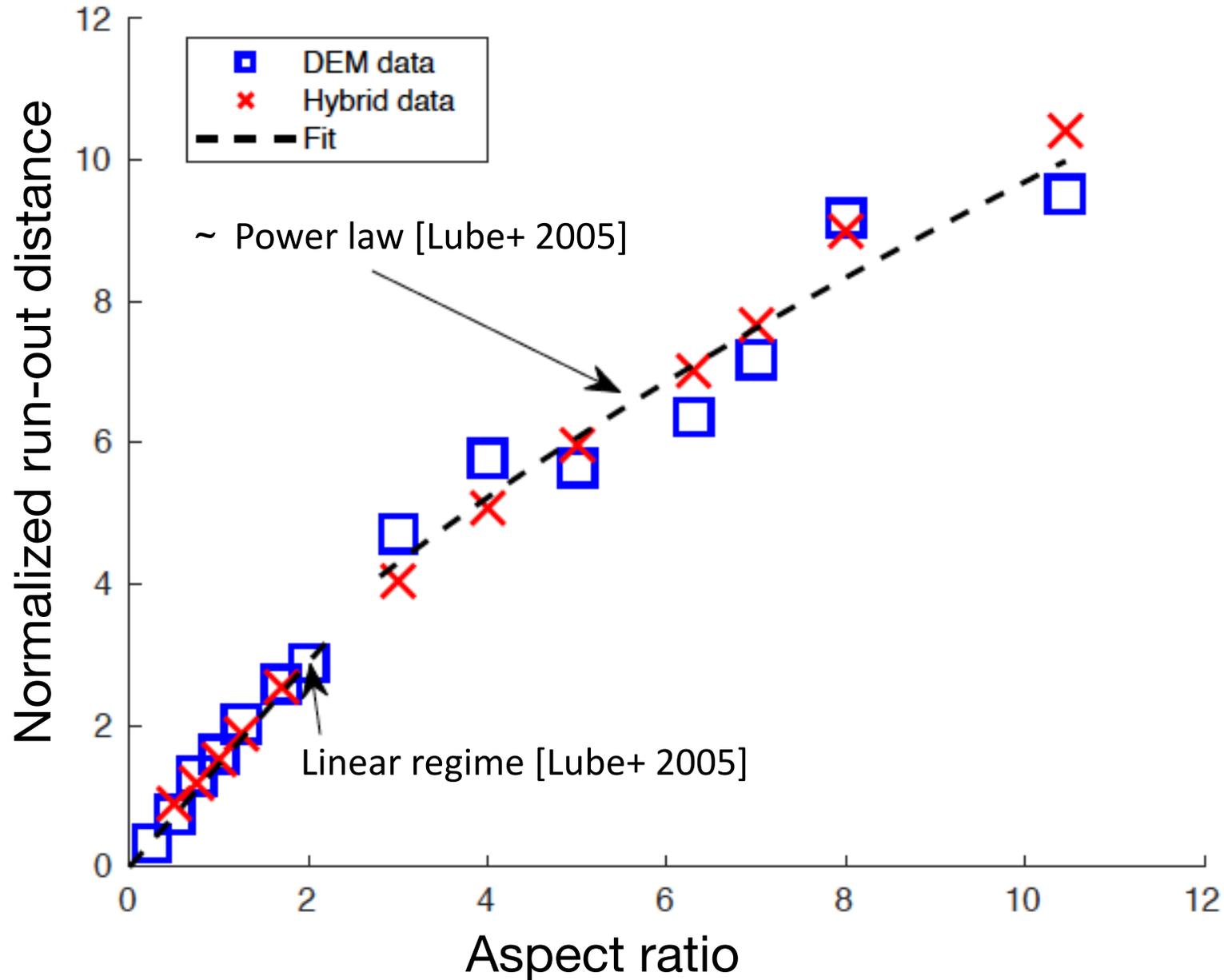
c.f. **Collapses of two-dimensional granular columns**:
G. Lube, H. E. Huppert, R. S. J. Sparks, A. Freundt.
Phys. Rev. E 72 (Oct 2005), 10. Issue 4
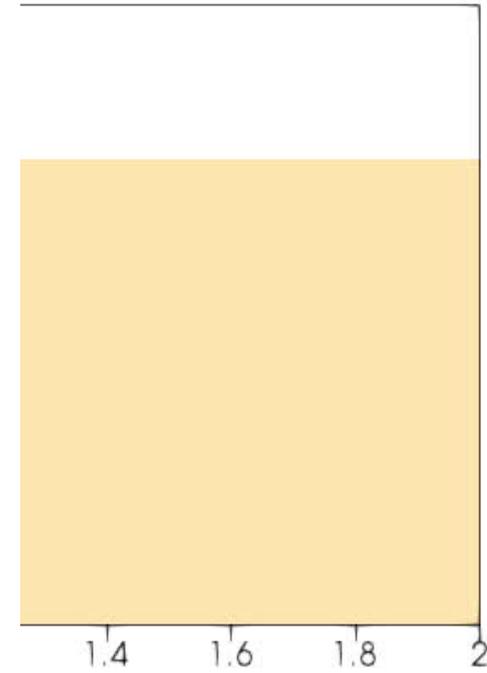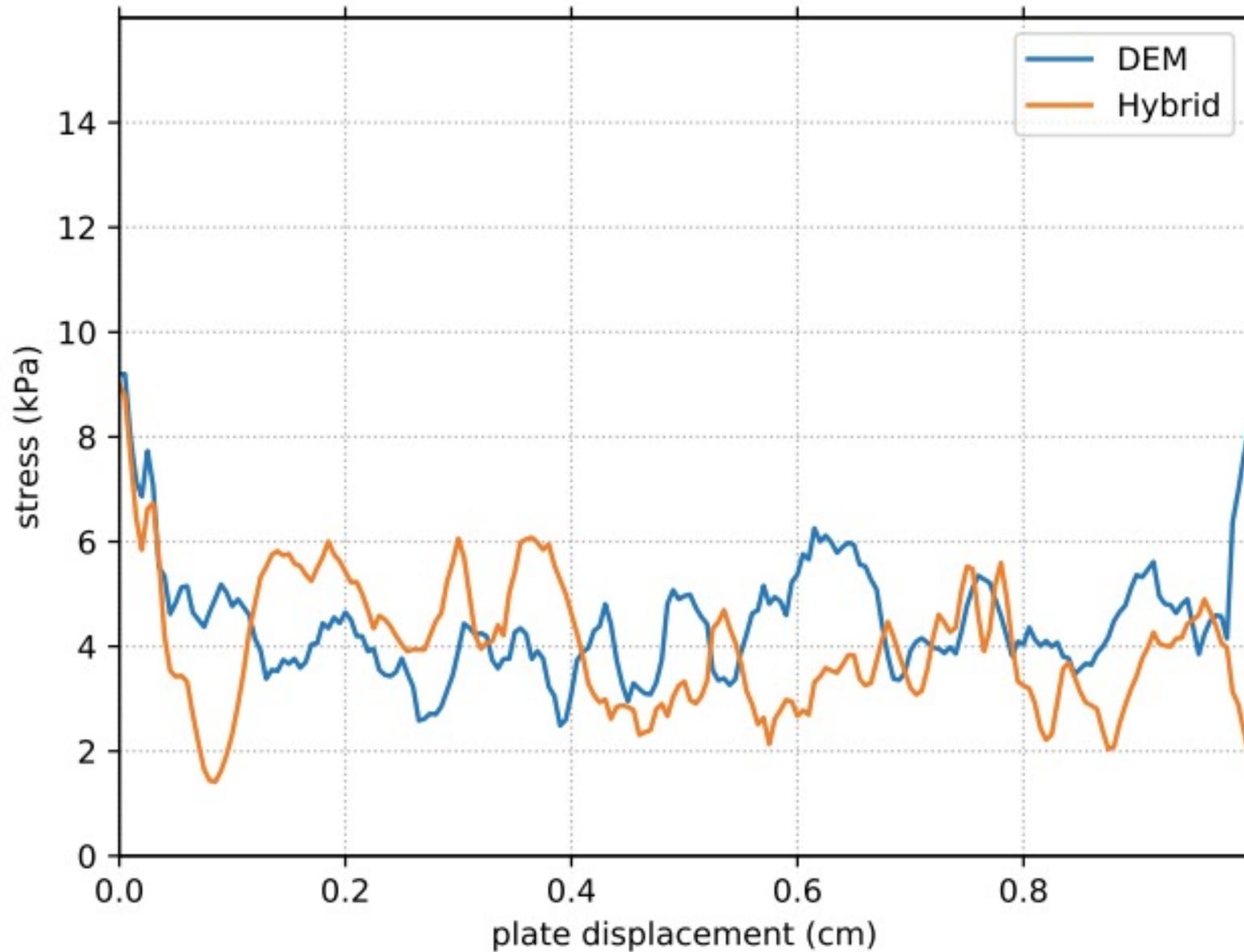
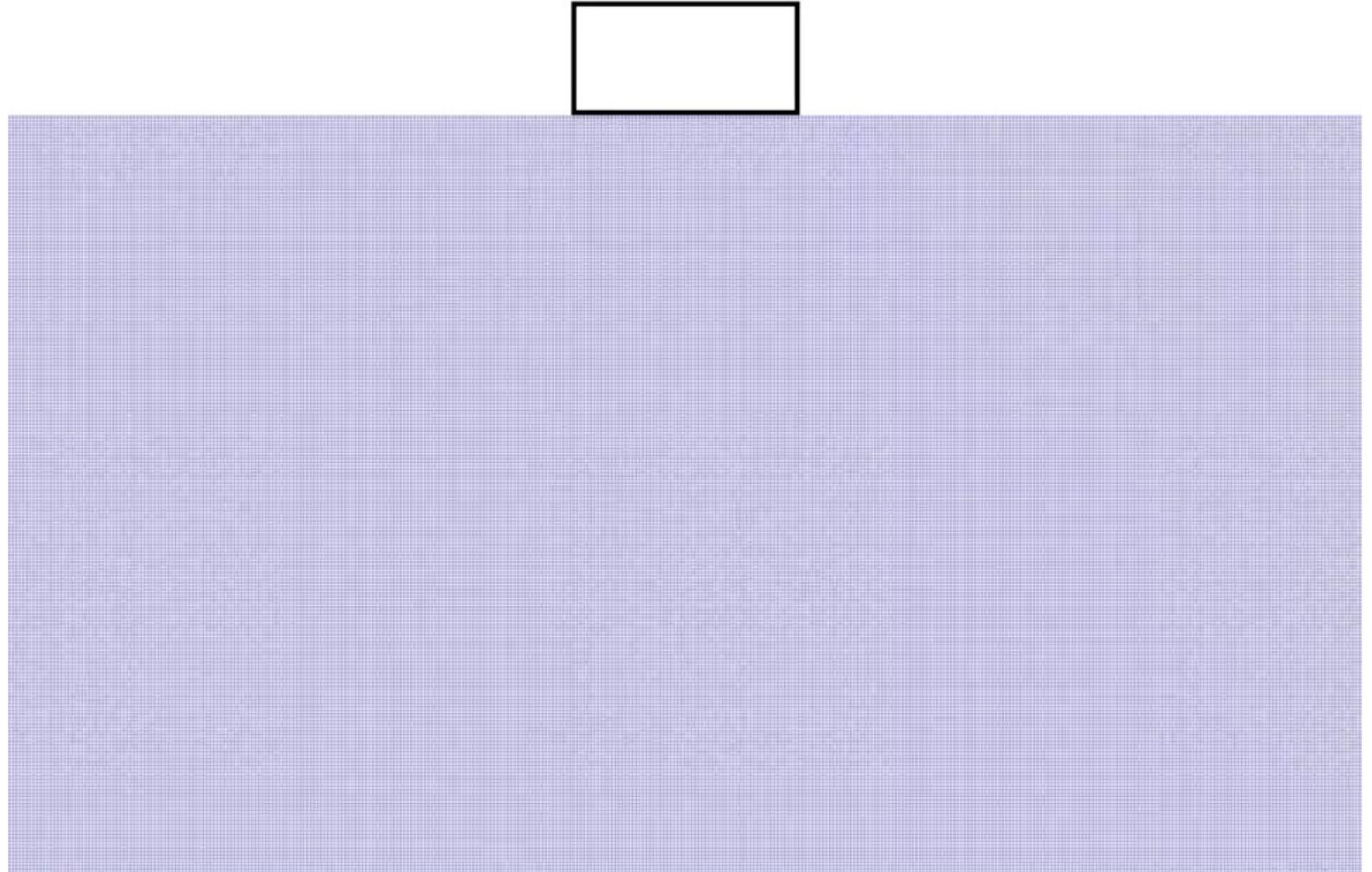# Irregular Shear Bands: Plate Indentation

Irregular Shear Bands: Plate Indentation

)x faster)

Ex: >100x speed-up over pure DEM

Grains 72 times
smaller than
intruder width.

Model applied to circular intruder

Results: 2D