

IFPRI AGM 2023

Ken Kamrin

Arjun Yennemadi

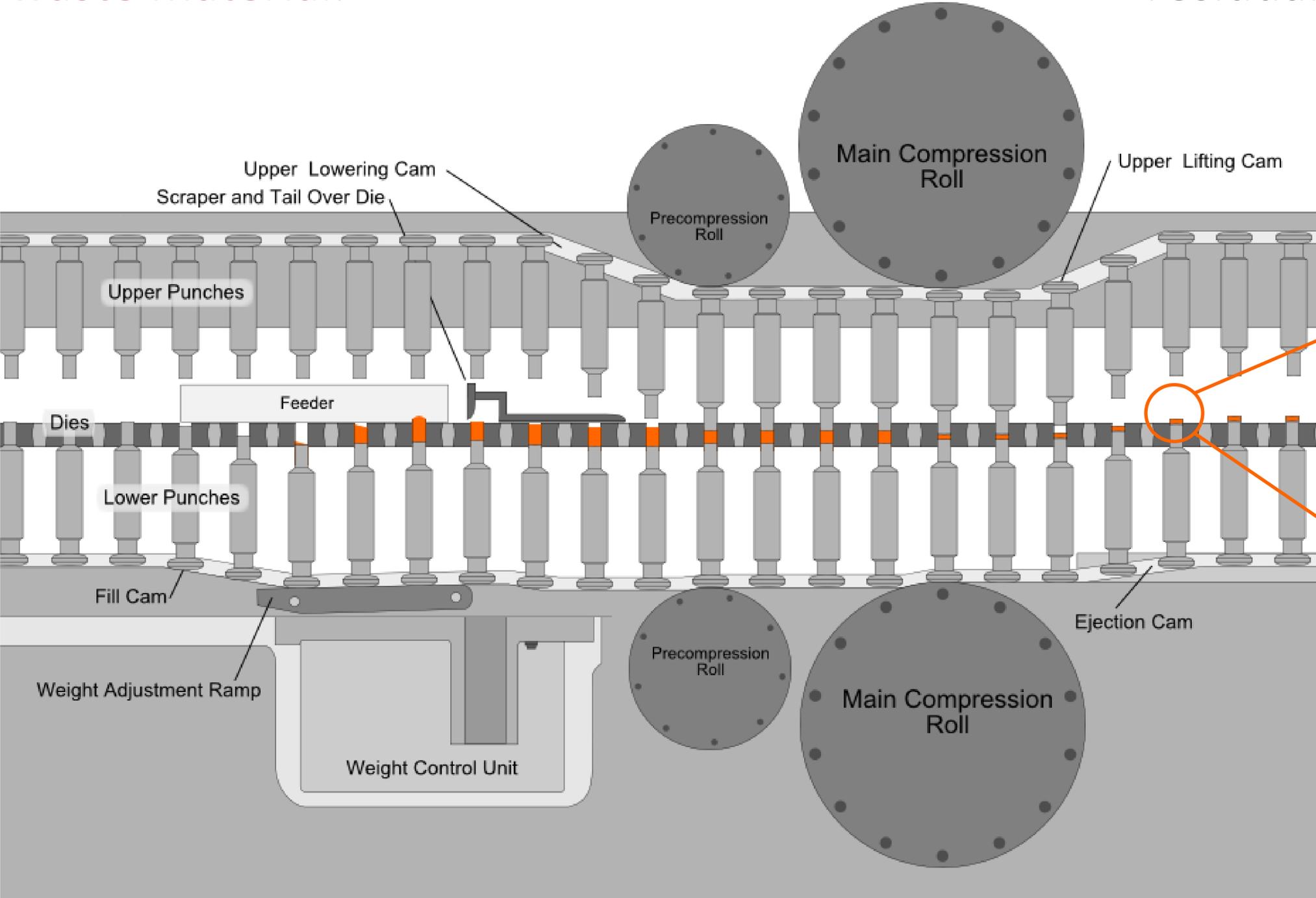
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Project Motivation

Use **mechanics-based approach** to **predict defect formation** to **reduce waste material**.

Question: What drives the fracturing process? Entrapped air? Inhomogeneous residual grain stress?

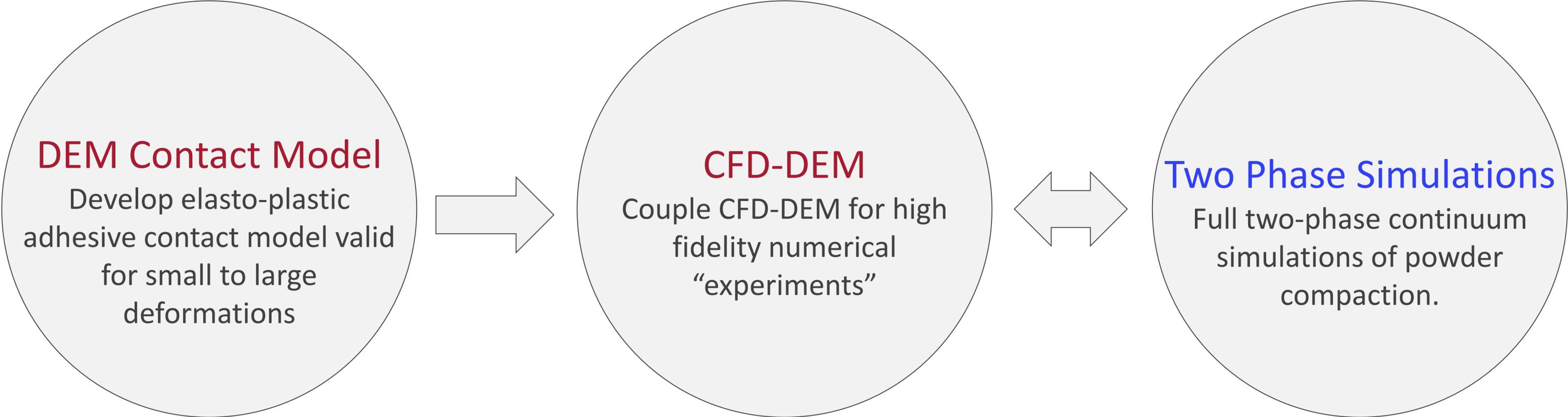


Capping/lamination



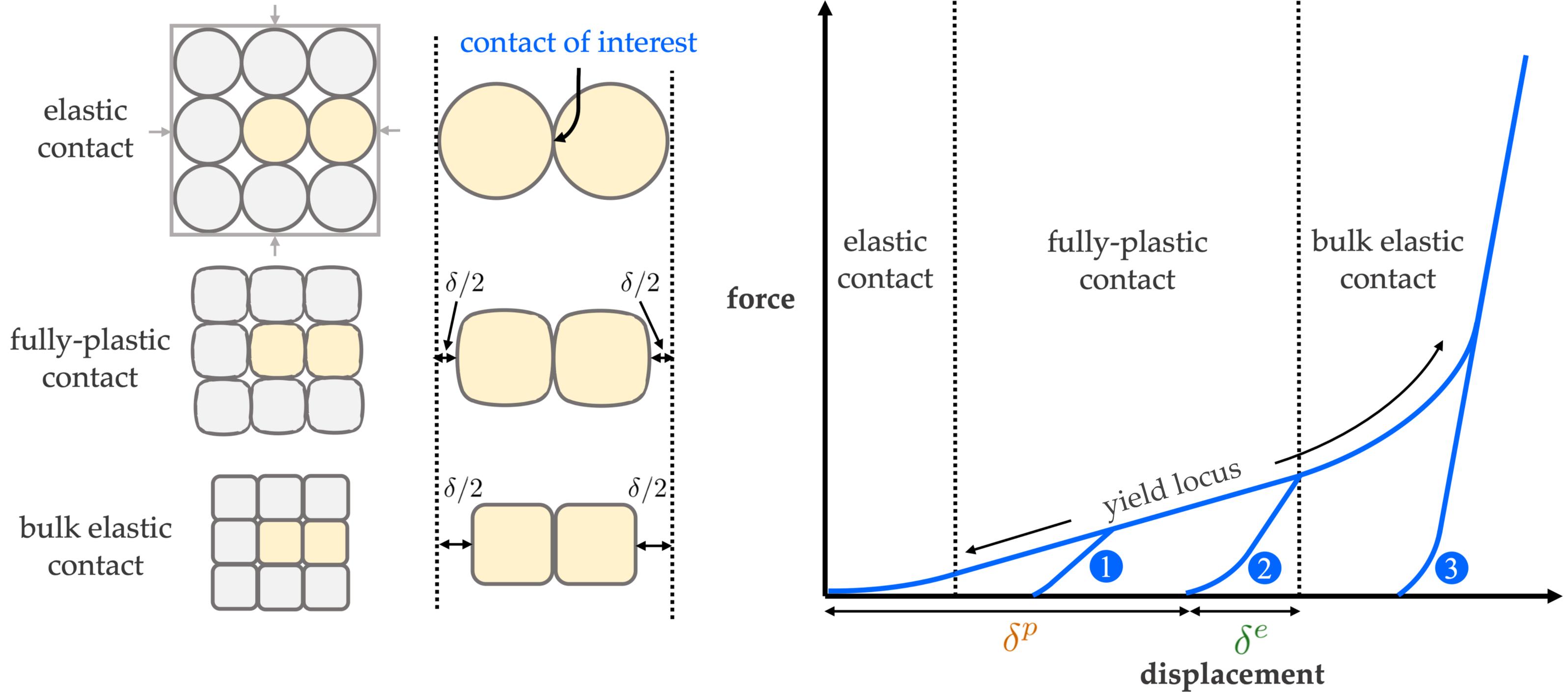
Cracking

Project Workflow



Discrete/Continuum
Fully Continuum

Elastic-plastic particle behavior during powder compaction

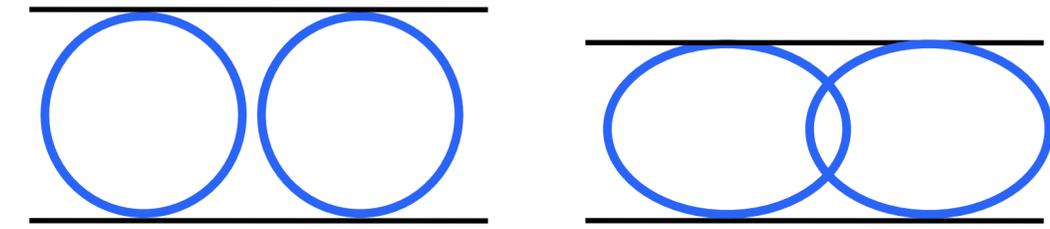


adhesion leads to tensile forces during unloading before separation

Previous contact models and their relevant features

Authors	Regimes	$\min(E/Y)$	Unloading	Adhesion	Nonlocalized
Chang et al., 1987	E, FP	1000	no	no	no
Storåkers et al., 1997	FP	rig. plas.	no	no	no
Mesarovic and Johnson, 2000	FP	10000	yes	yes	no
Zhao et al., 2000	E, FP	1000	no	no	no
Jackson and Green, 2003 and 2005	E, FP	1000	no	no	no
Etsion et al., 2005	E, FP	300	yes	no	no
Harthong et al., 2009	E, FP, BE	1000	no	no	yes
Zait et al., 2010	E, FP	500	yes	no	no
Brake, 2012	E, FP	300	yes	no	no
Gonzalez et al., 2012 and 2018	E	n.a.	yes	no	yes
Olsson and Larsson, 2013	E, FP	1000	yes	yes	no
Frenning, 2013 and 2015	E, FP, BE	50	no	no	yes
Brodu et al., 2015	E	n.a.	yes	no	yes
Rathbone et al., 2015	E, FP	160	yes	no	no
Garner et al., 2018	E, FP, BE	100	yes	yes	yes
Gonzalez, 2019	E, FP	100	yes	yes	no
Edmans and Sinka, 2020	E, FP	1	yes	no	no
Giannis et al., 2021	E	n.a.	yes	no	yes
Giannis et al., 2021	E, FP, BE	n.r.	yes	yes	yes

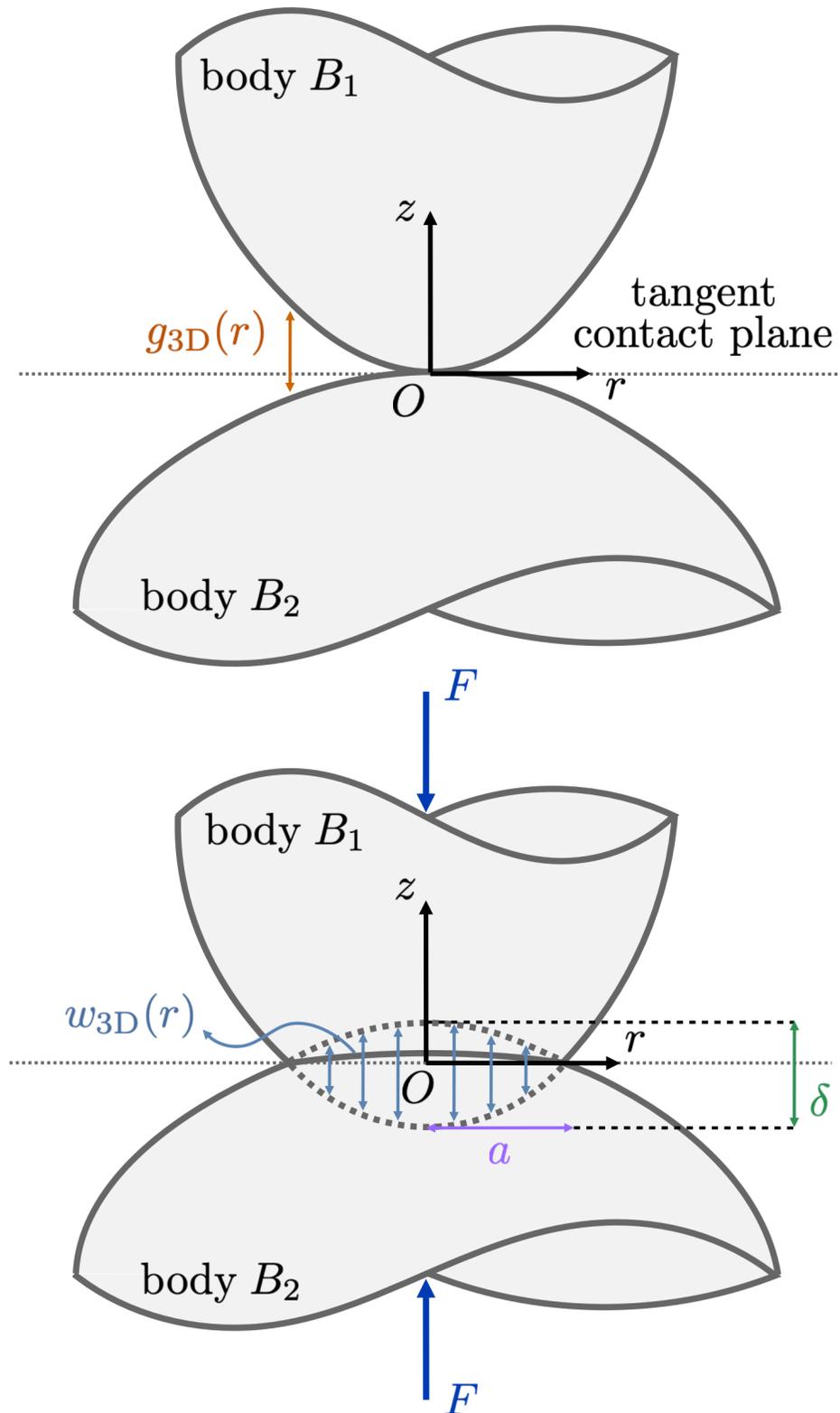
nonlocalized effects



- **very close to rigid plastic**
- **empirically fitted**, reduced use of mechanics principles to determine contact law
- **requires refitting** for each new material being modeled.
- **no information beyond the force-displacement** is provided.

The method of dimensionality reduction (MDR): Popov and Heß

3D elastic axisymmetric contact



total force, contact radius, and displacement are the same between 3D problem and 1D counterpart

$$F = \int_{-a}^a q_{1D}(x) dx$$

$$= 2\pi \int_0^a p_{3D}(r) r dr$$

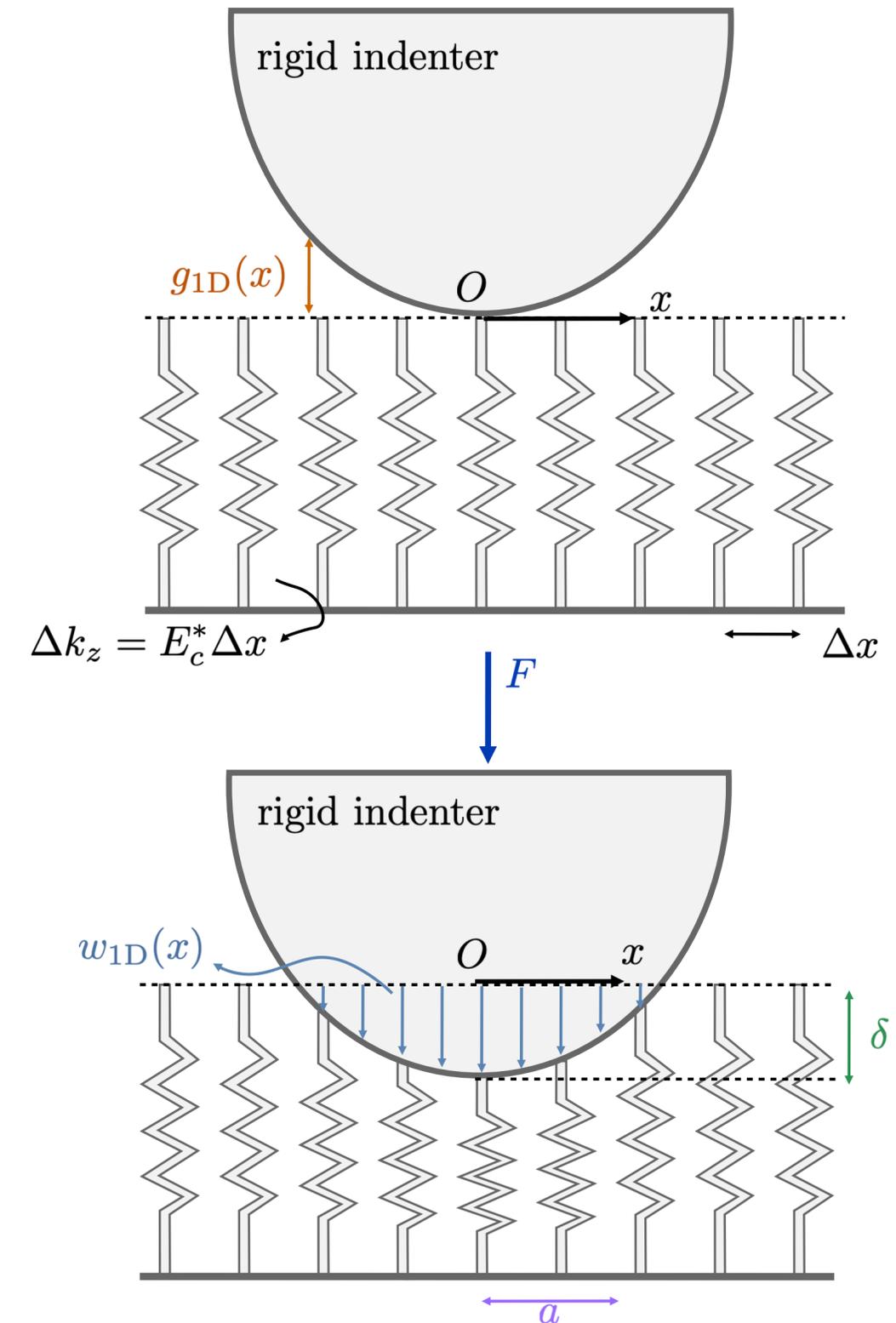
integral transforms

$g_{3D}(r)$ $\xleftrightarrow{\text{gap function}}$ $g_{1D}(x)$

$w_{3D}(r)$ $\xleftrightarrow{\text{normal disp.}}$ $w_{1D}(x)$

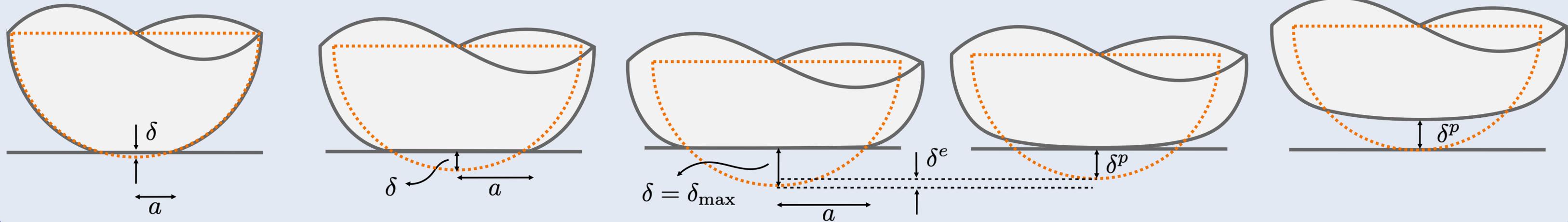
$p_{3D}(r)$ $\xleftrightarrow{\text{pressure}}$ $q_{1D}(x)$

1D Winkler foundation



Single elastic-plastic contact—3D and 1D evolution

3D



purely elastic

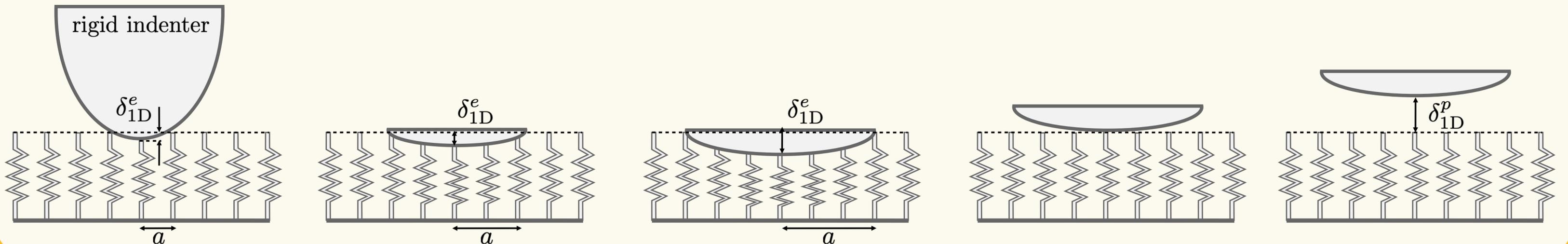
fully-plastic begins, yield criterion met

plastic deformation accumulates

unloading is an elastic process

blunted profile

1D



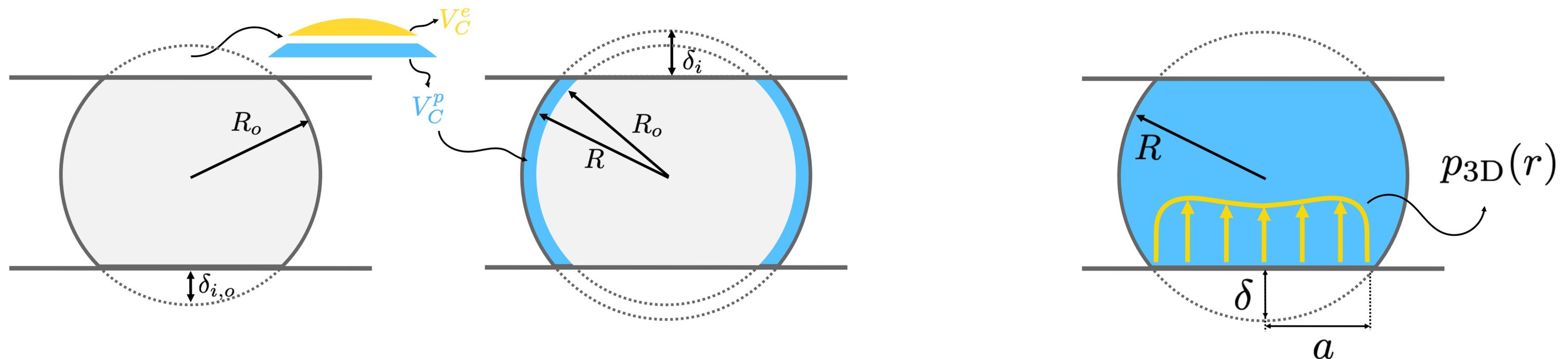
Like a sequence of elastic contacts with evolving reference configuration

Fundamental solution pathway

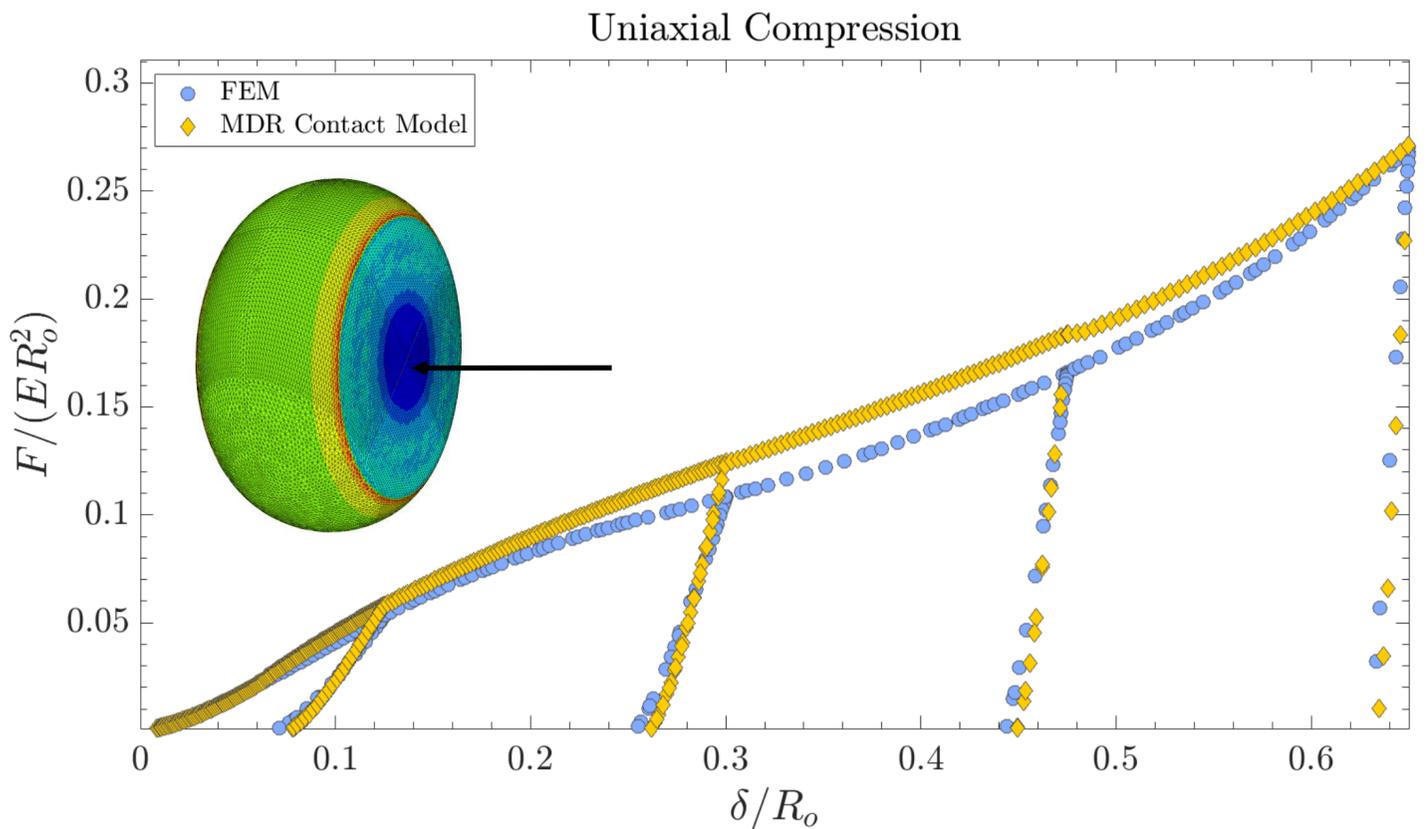
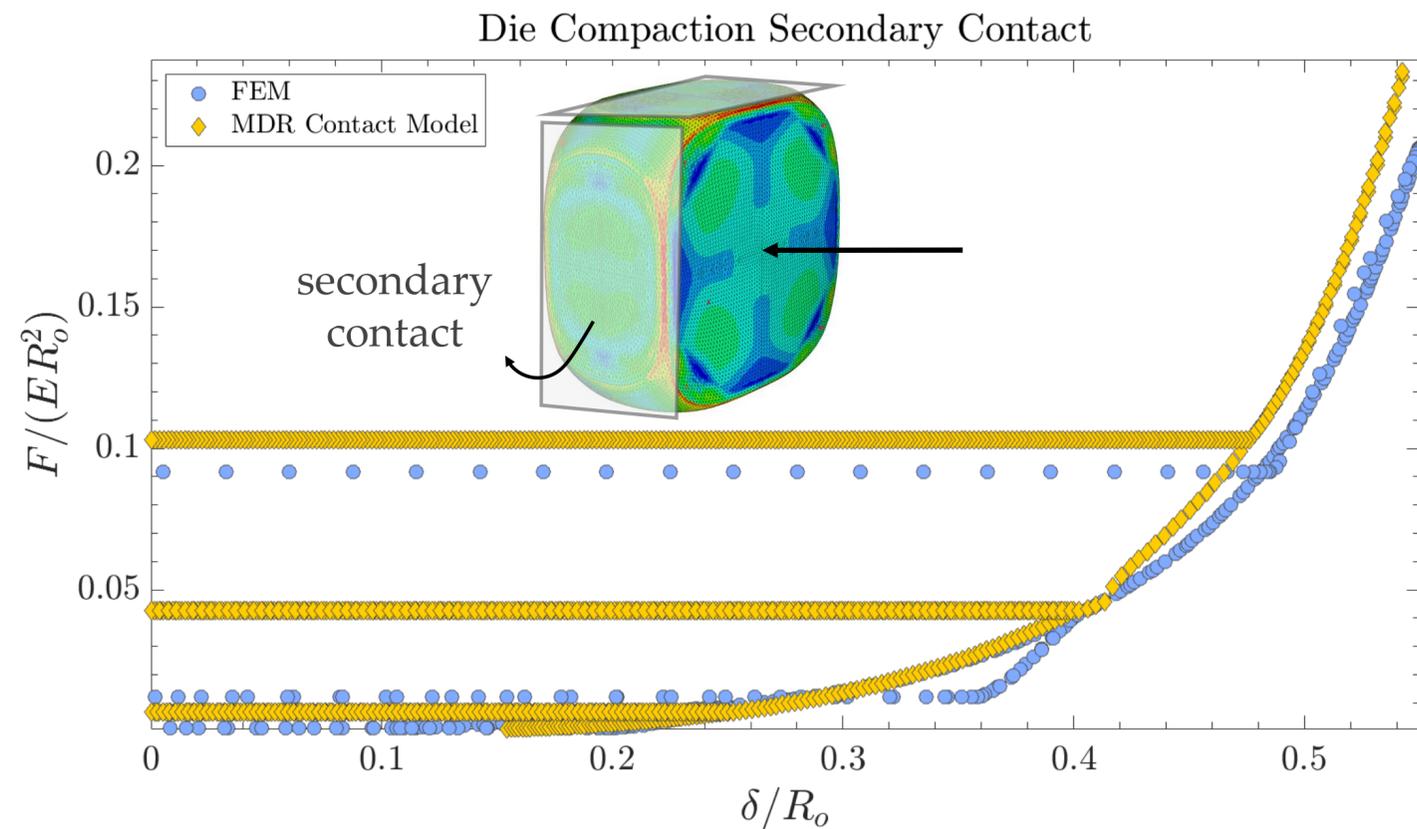
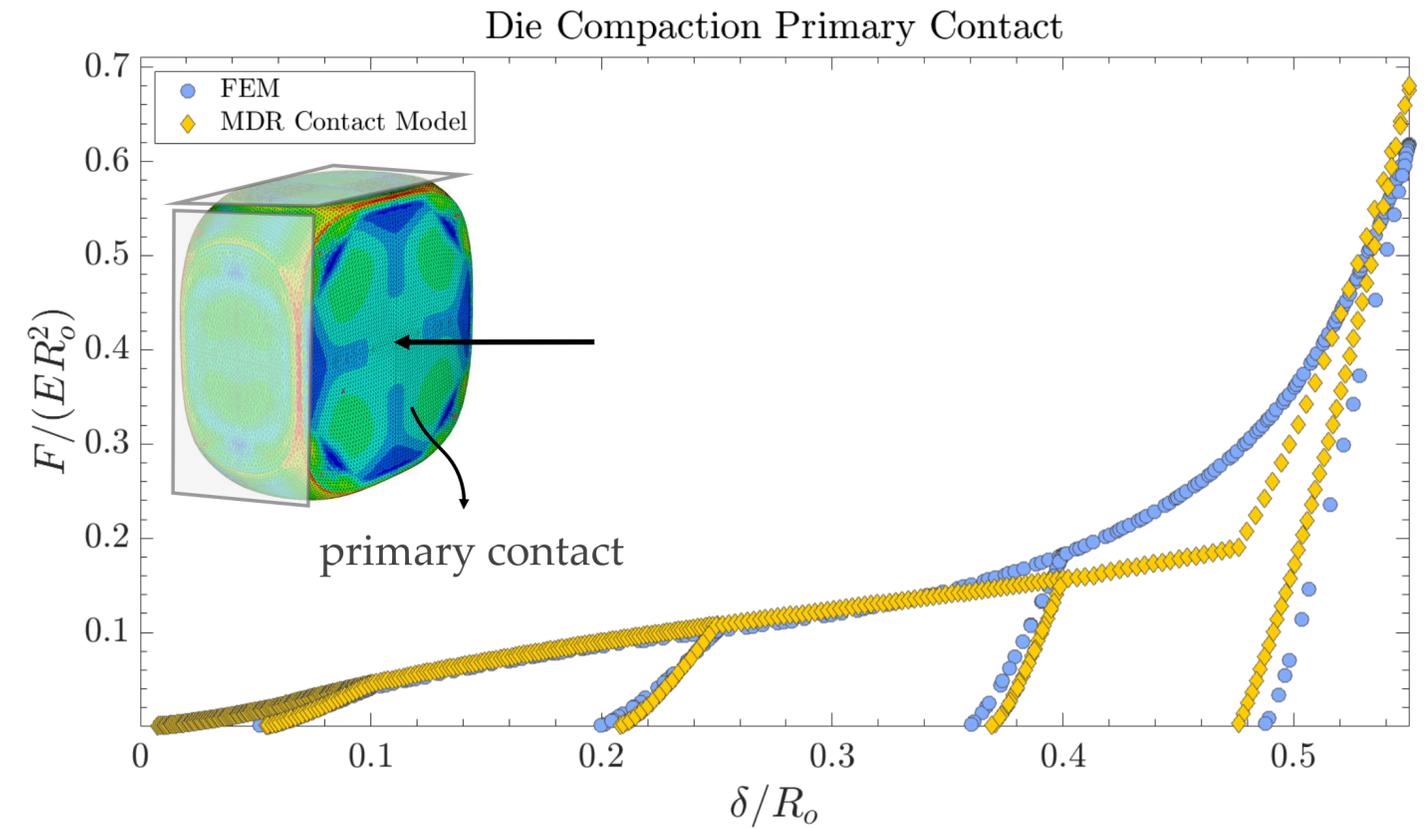
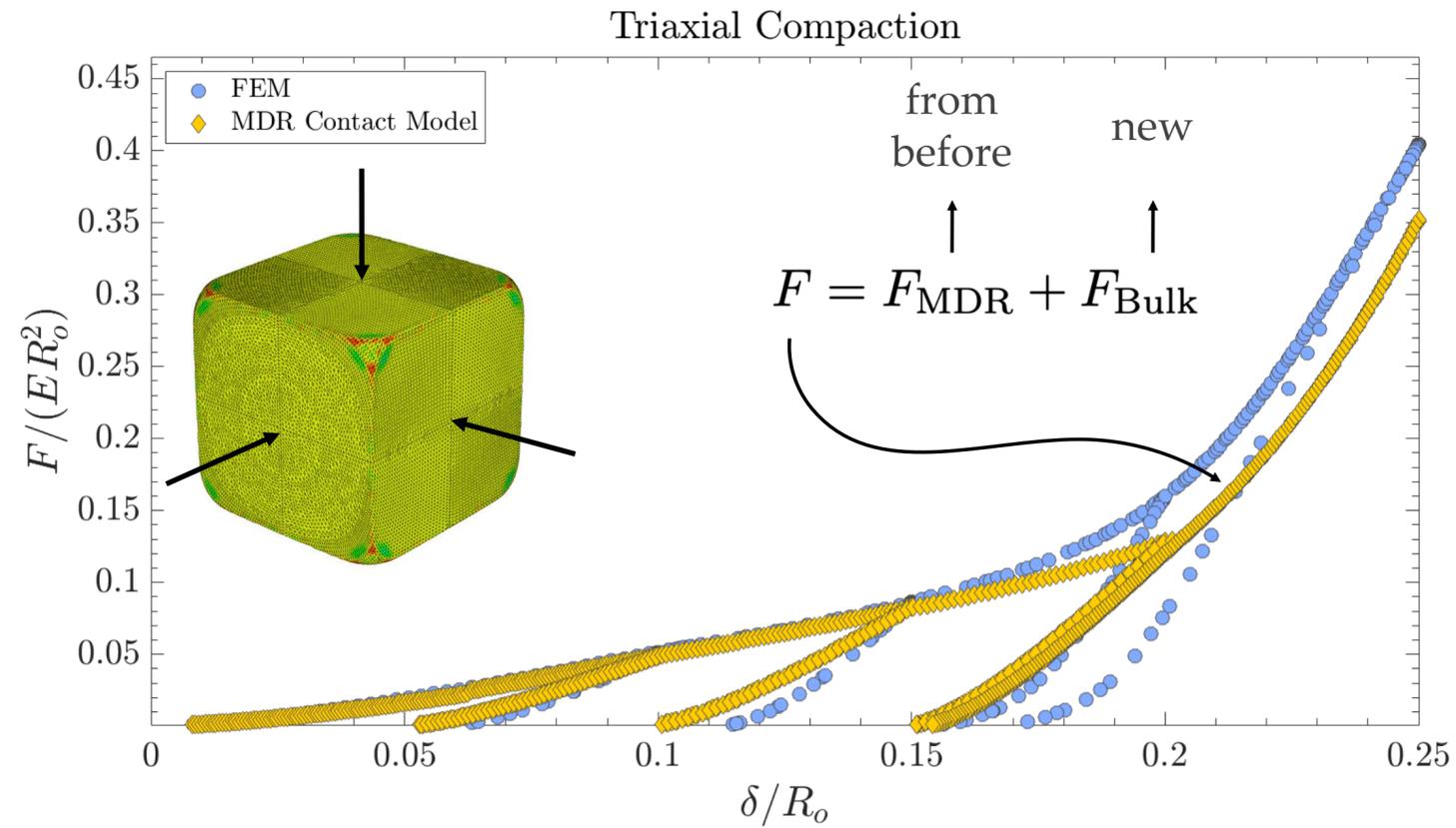
Idea: Determine corresponding 1D indenter shape at each instant: $g_{1D} = \hat{g}_{1D}(\delta^p \text{ or } \delta_{\max})$

Pathway: Use reverse contact solution methodology supplied by the MDR: $\{p_{3D}(r), a\} \rightarrow g_{1D}(x)$

- exploit three simple properties of elastic-plastic spherical contact
 1. pressure is approximately uniform in fully plastic regime
 2. area is purely geometric in fully plastic regime
 3. plastic deformation is incompressible



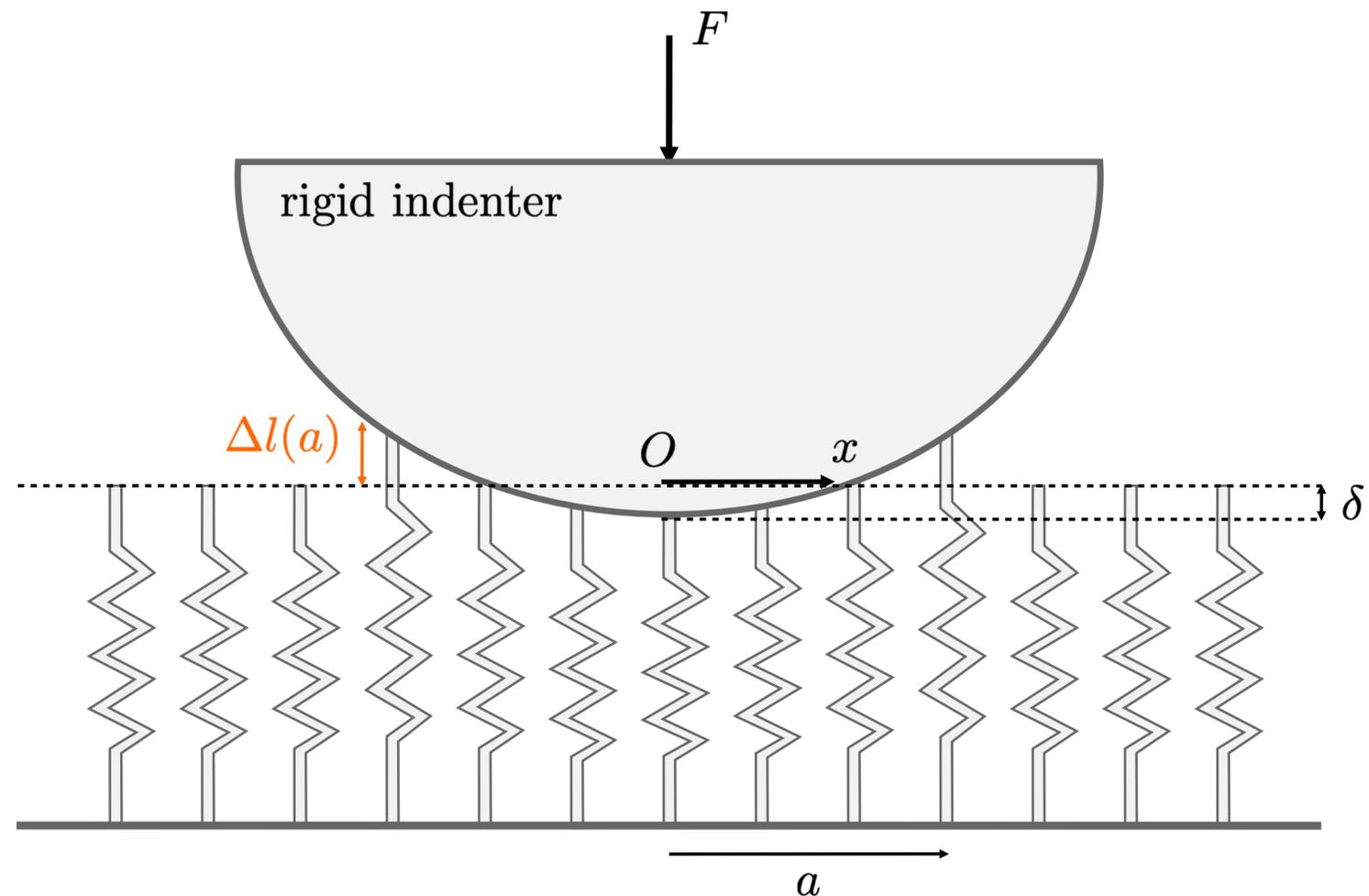
Some force-displacement comparison with finite element simulations



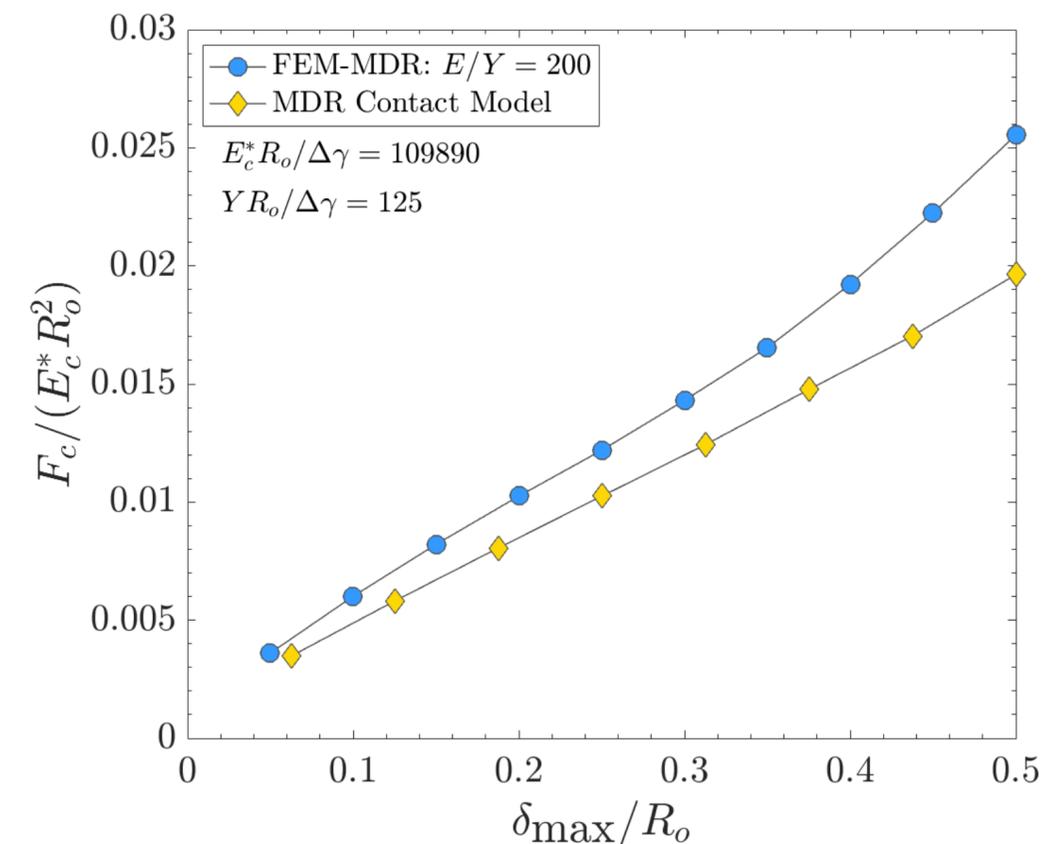
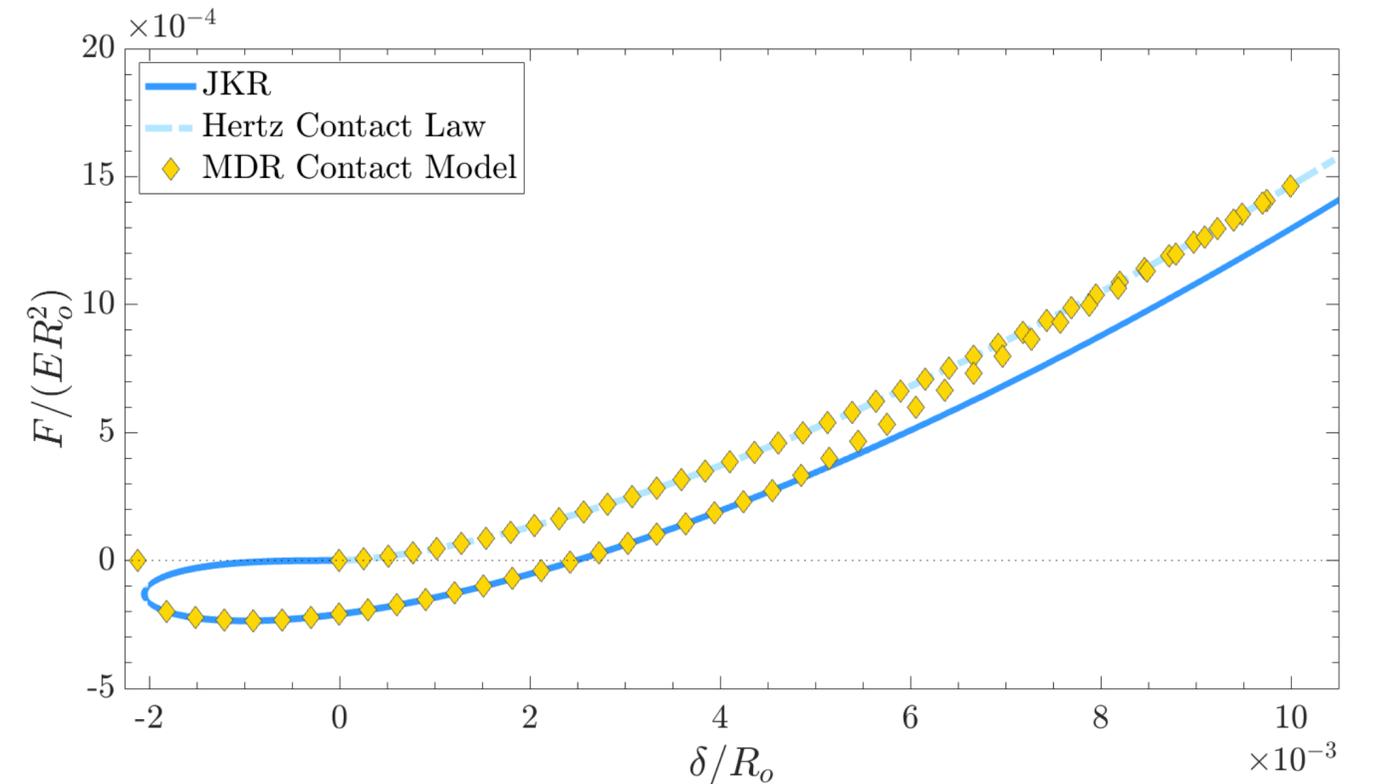
Adhesion — an easy addition in MDR framework

Johnson, Kendall, Roberts (JKR) theory of adhesion in the MDR

sticky springs with critical extensional length



$$\Delta l(a) = \sqrt{\frac{2\pi a \Delta \gamma}{E_c^*}}$$

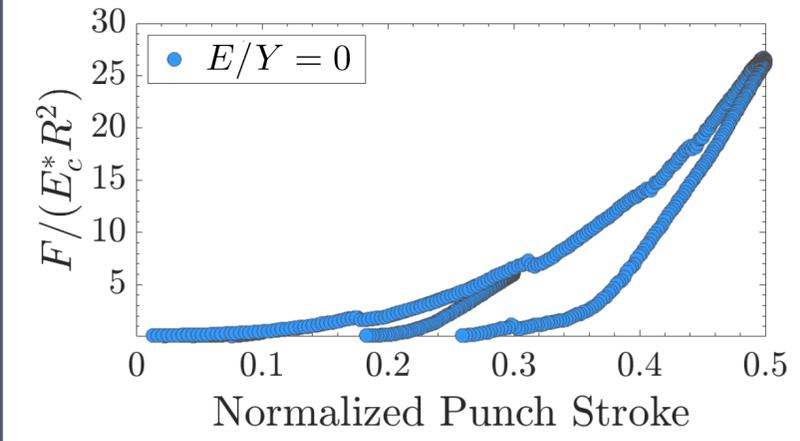
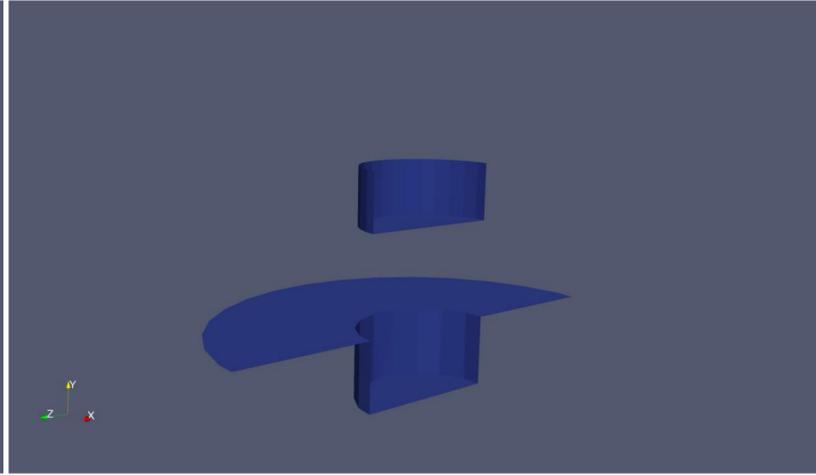
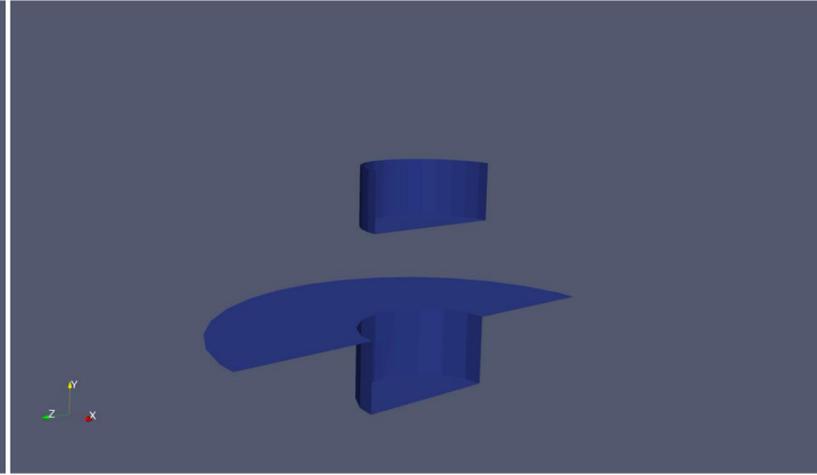
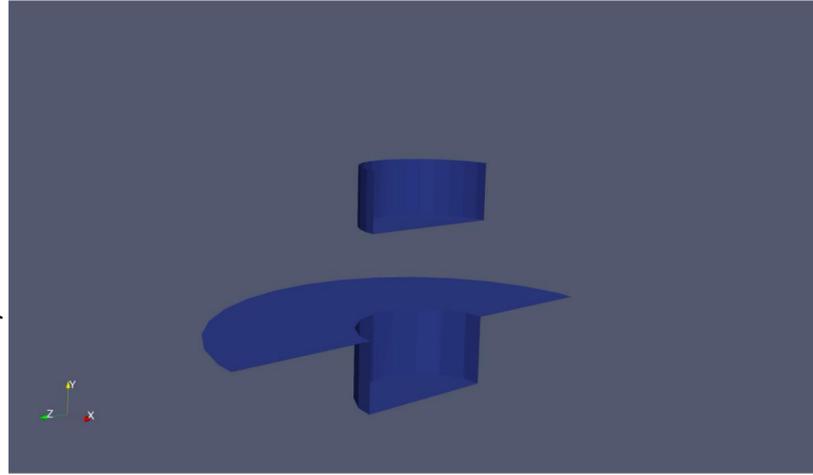


10% compression

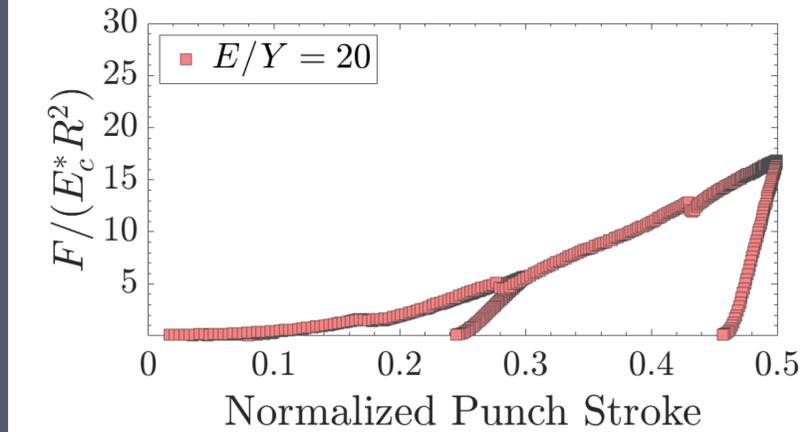
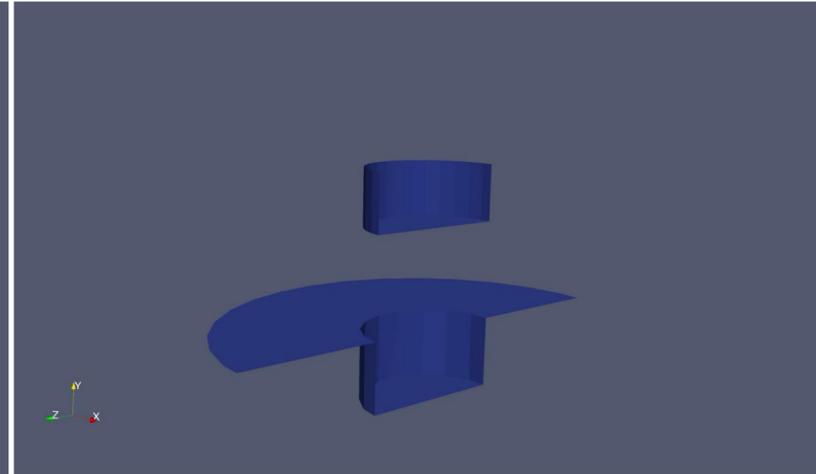
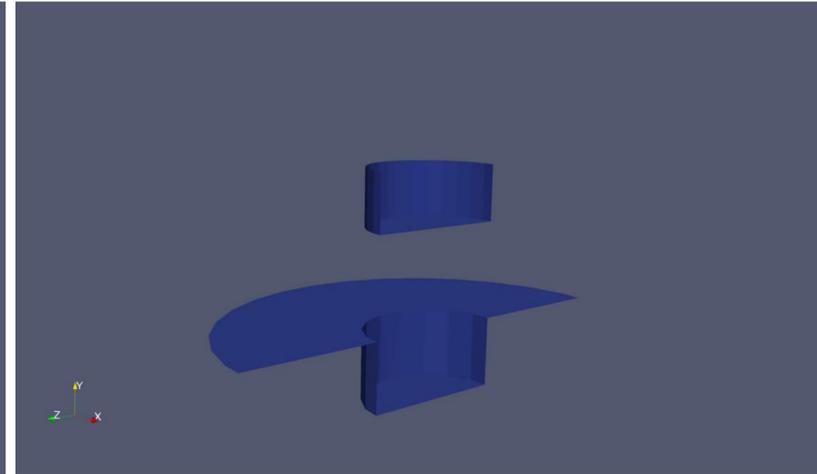
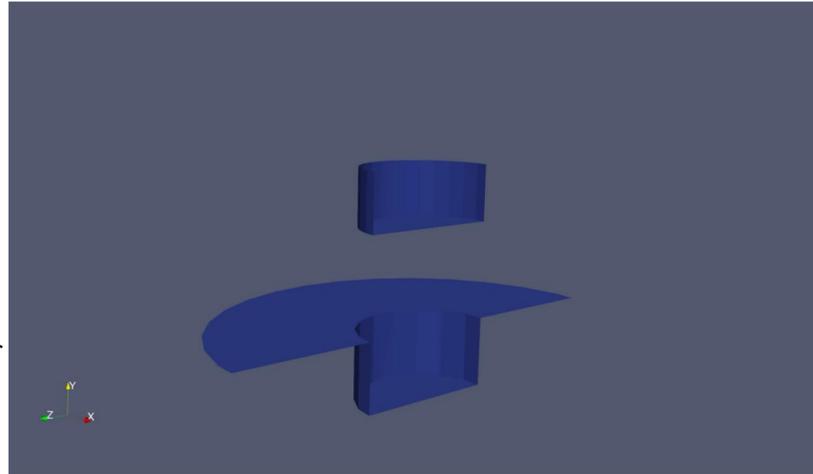
30% compression

50% compression

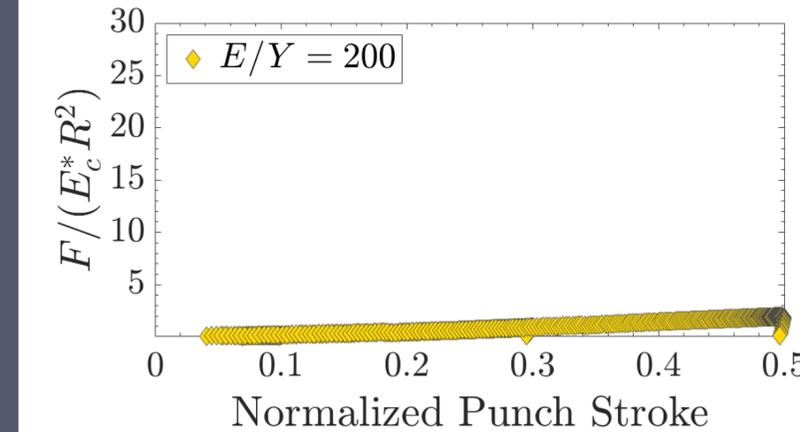
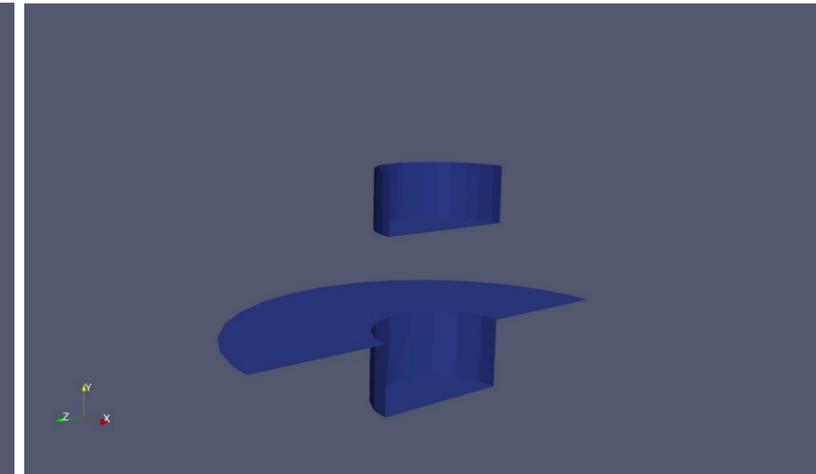
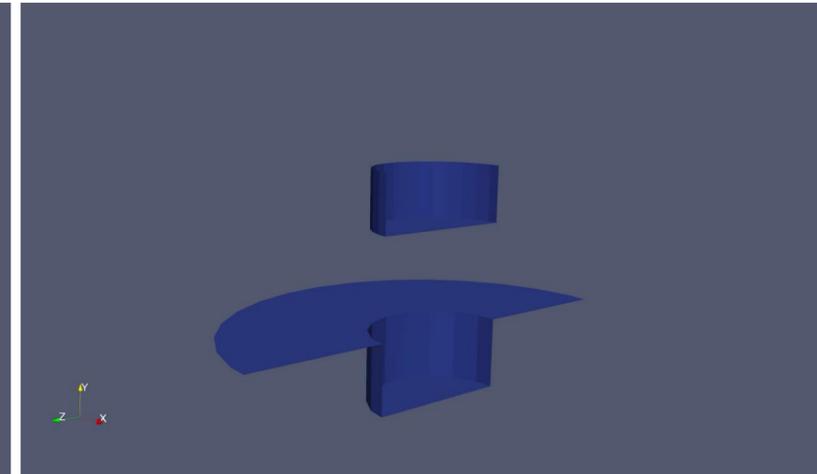
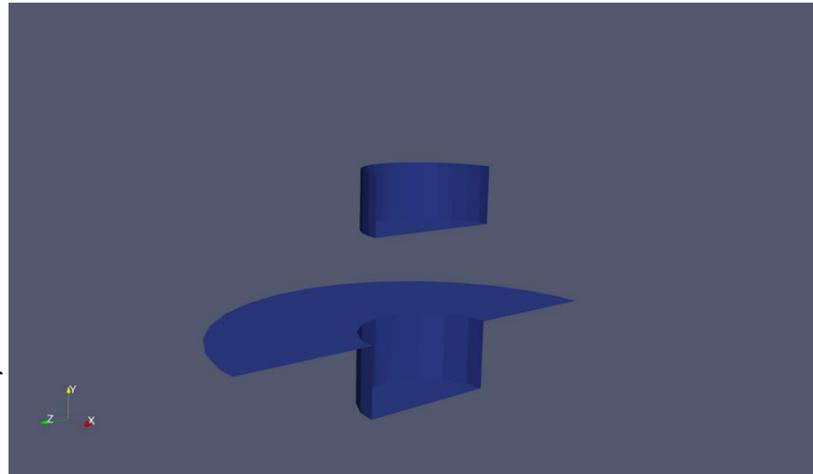
$E/Y = 0$



$E/Y = 20$



$E/Y = 200$

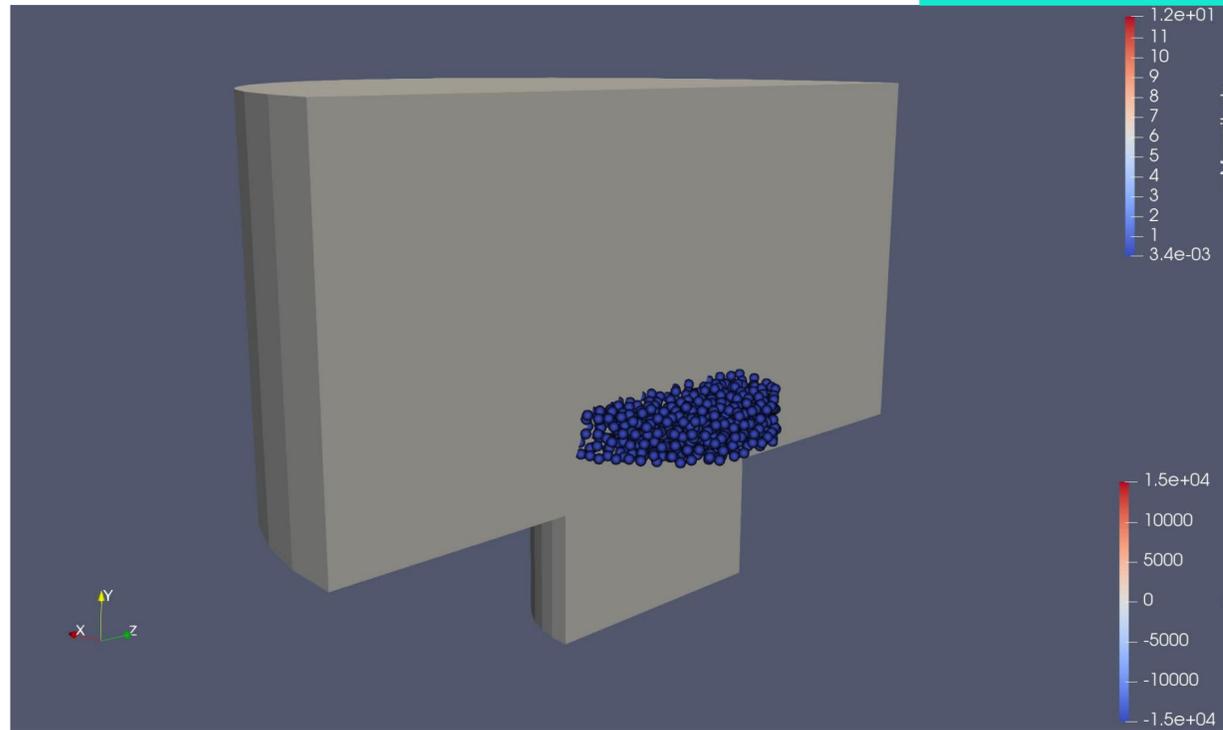
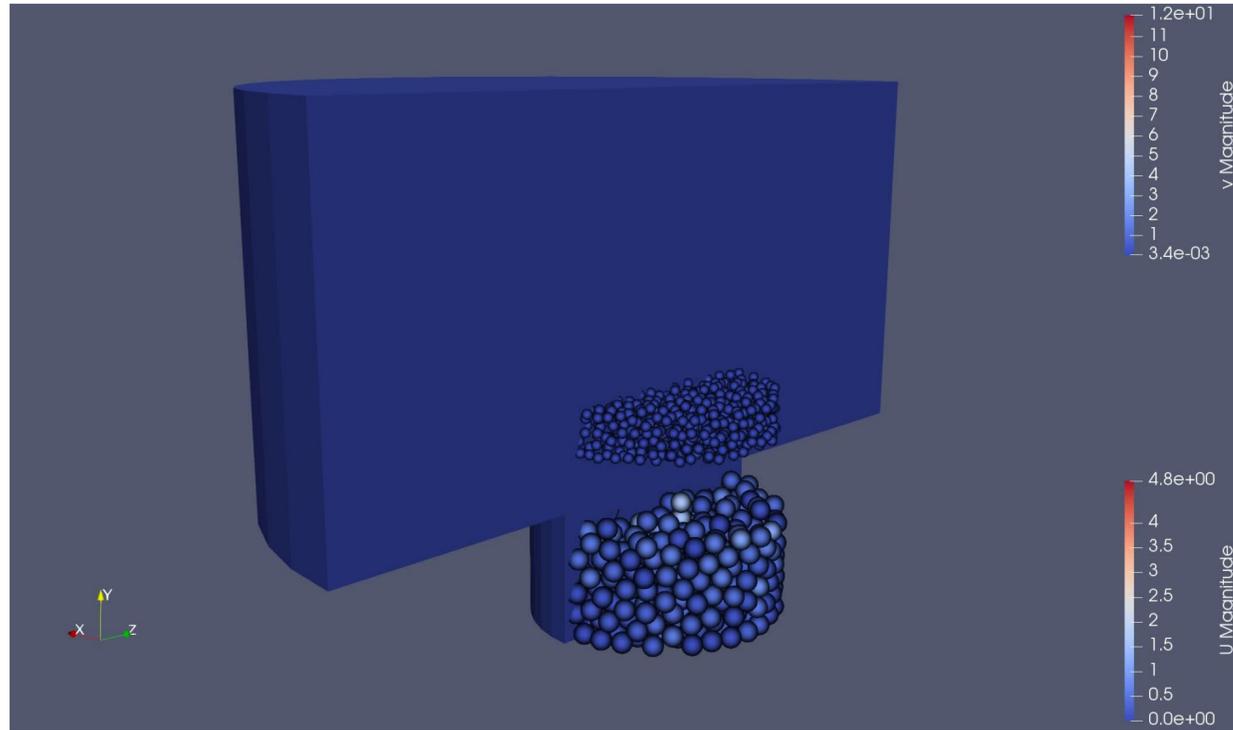


Implemented in LIGGGHTS
+ OpenFOAM (CFDEM)
[Without grain plasticity yet!]

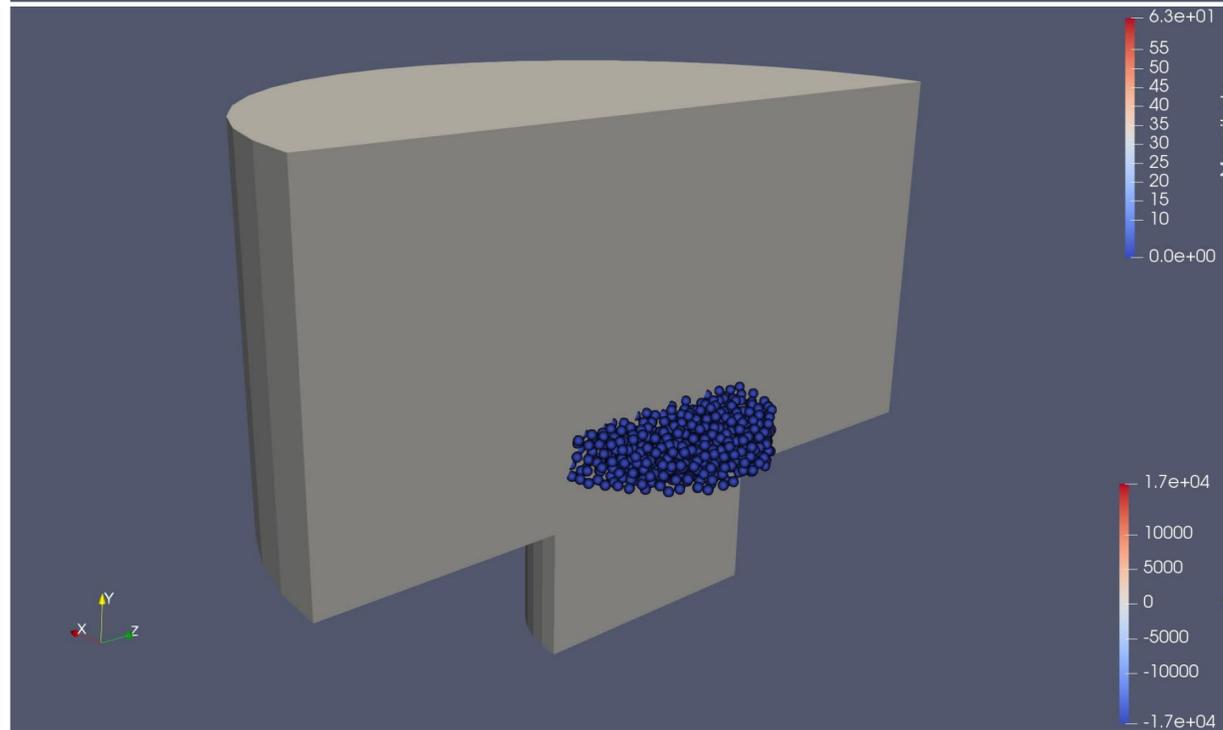
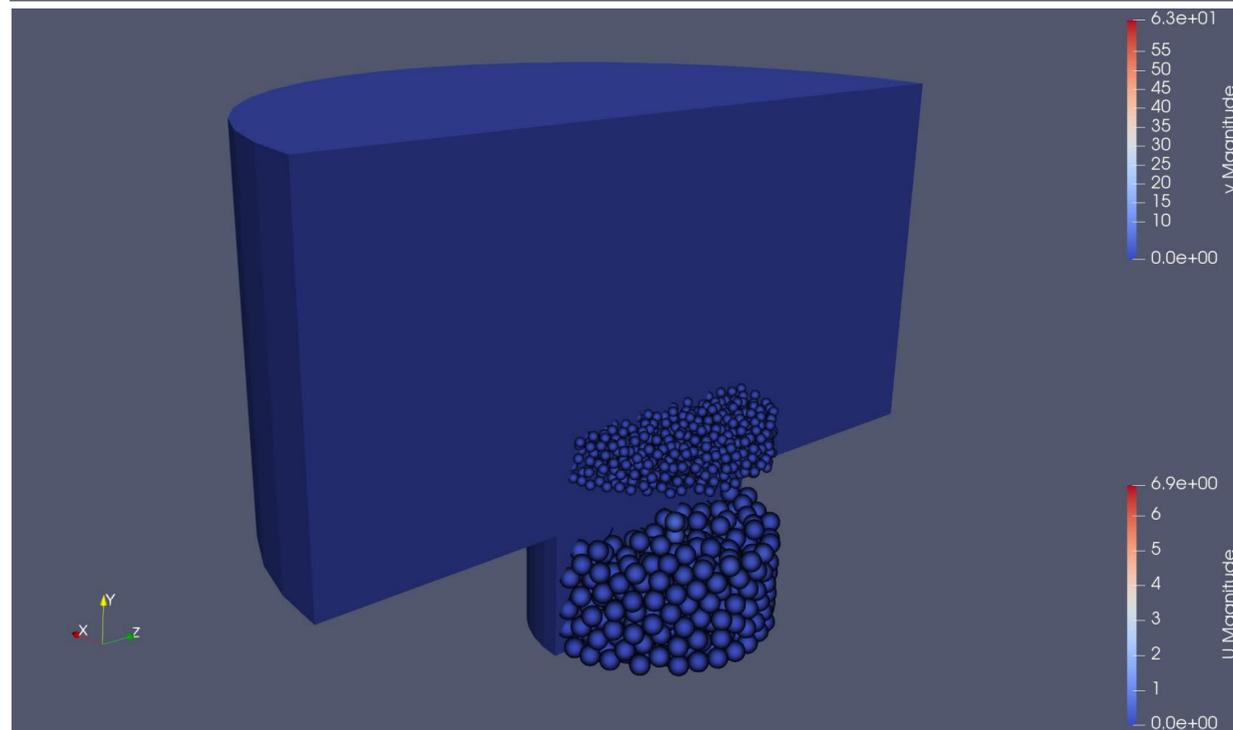
Velocity

Air
Pressure

10% compression



30% compression



Contact model capabilities — a summary and future outlook

Features of contact model

1. Analytical mechanics based formulation.
2. Model inputs: Young's modulus, Poisson ratio, yield stress, and effective surface energy.
3. Treats all three regimes: elastic, fully plastic, and bulk elastic.
4. Unloading is "built-in".
5. Treats adhesive contact in a manner consistent with the JKR theory of adhesion.
6. Accounts for nonlocalized effects: increased force and formation of new contacts.
7. Tracks plastic deformation, contact area, and contact pressures.
8. Handles large E/Y variations (i.e. no assumption of nearly rigid-plastic).

Future outlook

1. Account for hardening effects.
2. Implement into widespread DEM code for open source use (work in progress).
3. Verify against experiments (promising results already with Portela group data).
4. Predict sidewall pressure during die compaction, Vertex employees identified this as important problem.
5. Use CFD-DEM to predict interesting phenomena such as defect formation during tableting.
6. Use to simulate cold isostatic compaction of metal parts.