Segregation of cohesive granular materials

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Summary

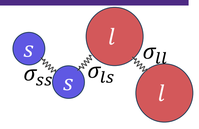
Goal: To develop a physics-based approach to predicting and modeling cohesive particle segregation and mixing

Key points:

- Discrete element simulations of cohesive size-bidisperse mixtures at large and small size ratios $R = \{7, 2\}$
- Low cohesion does not affect segregation; high cohesion reduces segregation and promotes mixing
- Cohesion changes flow dynamics (continuous → intermittent)
- Developing particles with controlled cohesion for experimental validation

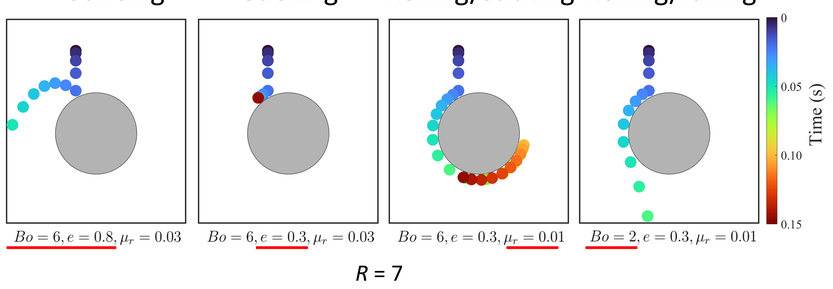
Dimensionless numbers

ρ: density
d_s, d_l: diameters
σ: surface energy density
g: gravity
ΔP: pressure
$\dot{\gamma}$: shear rate



- **Bond number:** $Bo \sim \dfrac{\text{cohesion}}{\text{weight or pressure}} \propto \dfrac{\sigma/d}{\rho g d \text{ or } \Delta P}$
  - **Size-bidisperse systems**: $\{Bo_{ll}, Bo_{ls}, Bo_{ss}\}$ (Li & McCarthy, 2005)
  - $Bo_{ls} > Bo_{ss} > Bo_{ll}$ for identical σ
- **Cohesion collision number**: $Co \sim \dfrac{\text{cohesion}}{\text{shear}} \propto \dfrac{\sigma/d}{\rho \dot{\gamma}^2 d^2}$
  - **Size-bidisperse systems**: $\{Co_{ll}, Co_{ls}, Co_{ss}\}$ (Li & McCarthy, 2006)
- **Relation to Inertial number:** $Bo/Co \sim \dfrac{\dot{\gamma}^2 d^2}{\Delta P/\rho} = I^2$

Two-particle interactions



References

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H Li & JJ McCarthy, *Powder Technology* 164.1 (2006): 58-64. A
Jarray et al., *Scientific reports* 9.1 (2019): 13480.
A Gans et al., *Physical Review E* 101.3 (2020): 032904.
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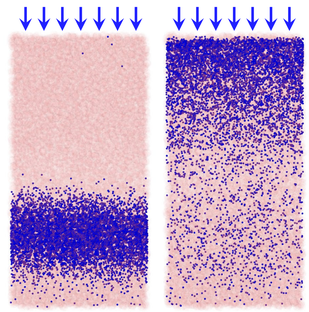
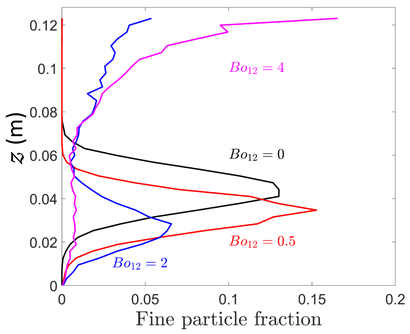
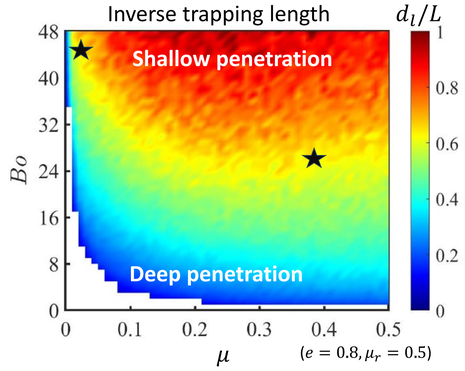
Percolation in static beds ($R = 7$)

Fine particle interactions turned off → dilute limit
(individual fine particles percolate through the bed)







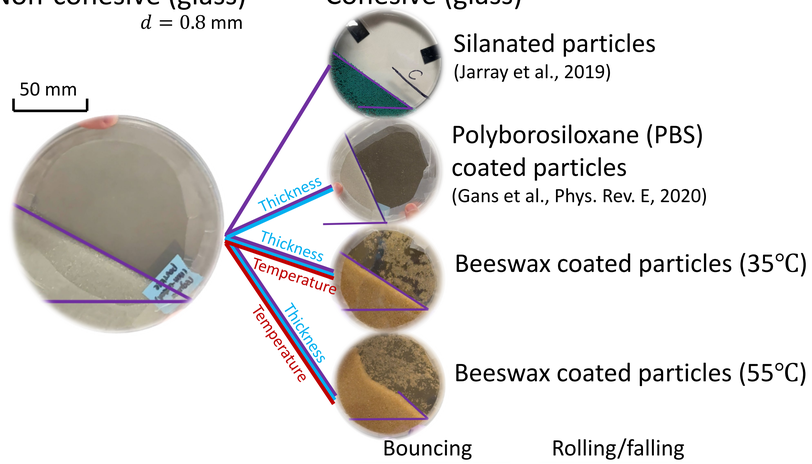
Different parameter combinations (*Bo*, *e*, μ, μ_r) yield similar results

e: restitution
μ: sliding friction
μ_r: rolling friction

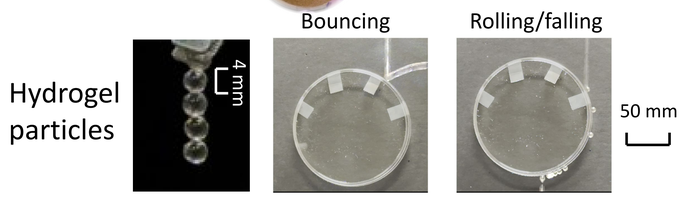
Cohesive-particle experiments

Non-cohesive (glass)
$d = 0.8$ mm

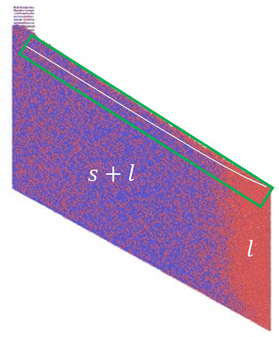
Cohesive (glass)



Hydrogel particles



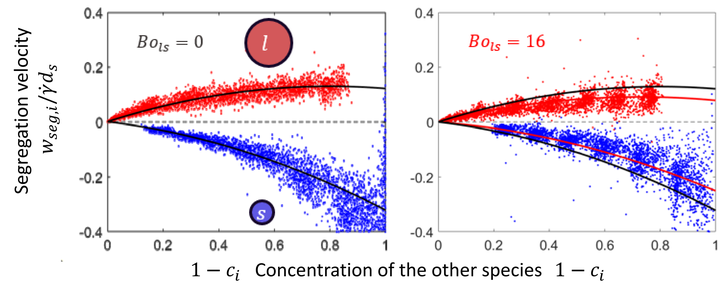
Bounded heap flow ($R = 2$)

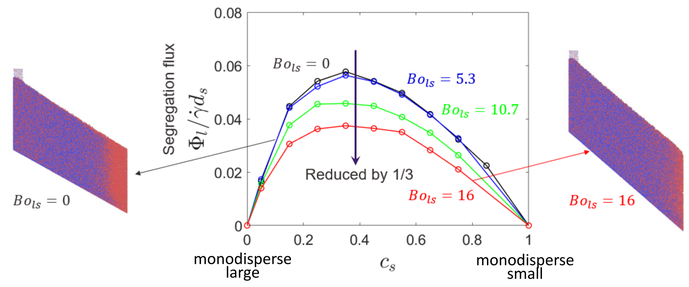


JKR model: $F = \underbrace{\frac{4Ea^3}{3r_e}}_{\text{stiffness}} - \underbrace{2\pi a^2 \sqrt{\frac{4\sigma E}{\pi a}}}_{\text{cohesion}}$

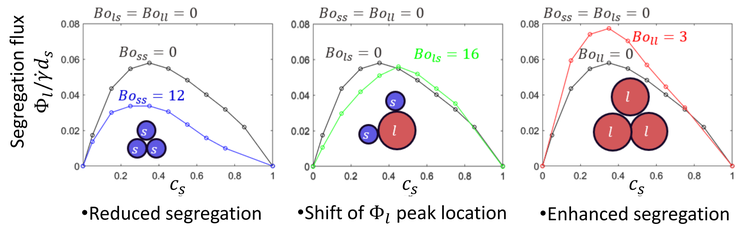
E: Young's modulus
a: radius of contact patch
r_e: effective radius $\frac{r_1 r_2}{r_1 + r_2}$
δ: overlap $\delta = \frac{a^2}{r_e} - 2\sqrt{\frac{\pi\sigma a}{E}}$

Cohesion reduces segregation (homogeneous σ)





Pairwise cohesion (heterogeneous σ)



•Reduced segregation •Shift of Φ_l peak location •Enhanced segregation

Explanation: $R = \frac{d_l}{d_{s,cluster}} \downarrow$ Large particles carry smalls $R = \frac{d_{l,cluster}}{d_s} \uparrow$