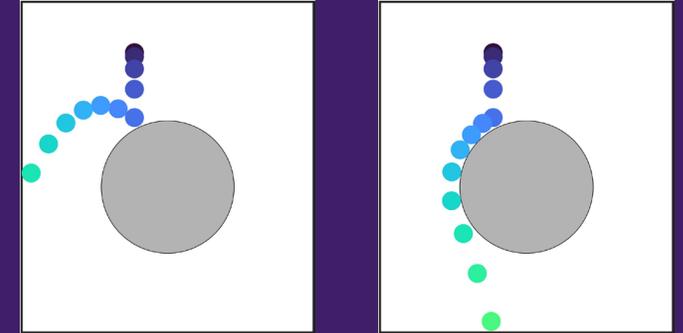


SEGREGATION OF COHESIVE PARTICLES IN GRANULAR FLOW



RICHARD M. LUEPTOW

with

PAUL B. UMBANHOWAR, JIZHI ZHANG, DING WANG, QIONG ZHANG

Funded by
International Fine Particle Research Institute
and
NSF CBET-2429545

Northwestern | McCORMICK SCHOOL OF
ENGINEERING

Segregation of Cohesive Particles in Granular Flow (IFPRI)

Segregation and Mixing of Cohesive Particles (NSF)

Research Goal: *To develop fundamental knowledge about cohesive particle segregation and mixing that will lead to an understanding of the key flow and particle parameters that influence segregation*

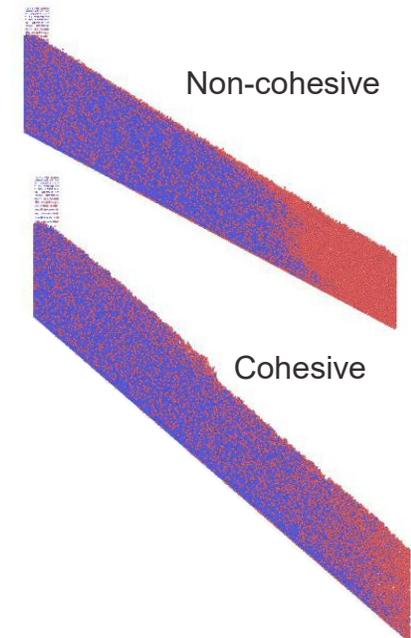
- *Insights into how to predict and control the segregation of cohesive particles (IFPRI)*
- *Methods to predict, model, and control the segregation of cohesive particles (NSF)*

Thrust 1: Cohesive particle DEM simulation development

Thrust 2: Validation experiments

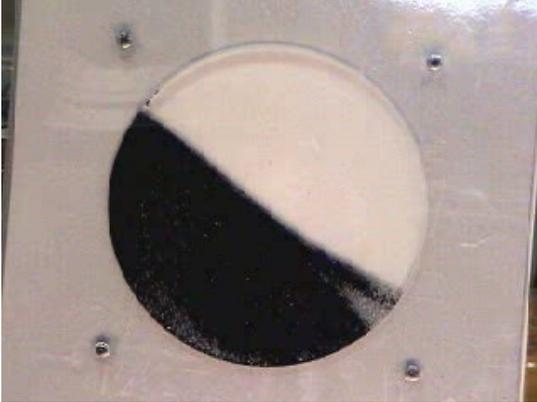
Thrust 3: DEM Simulations of cohesive particle segregation and mixing

Thrust 4: Underlying mechanisms and continuum model

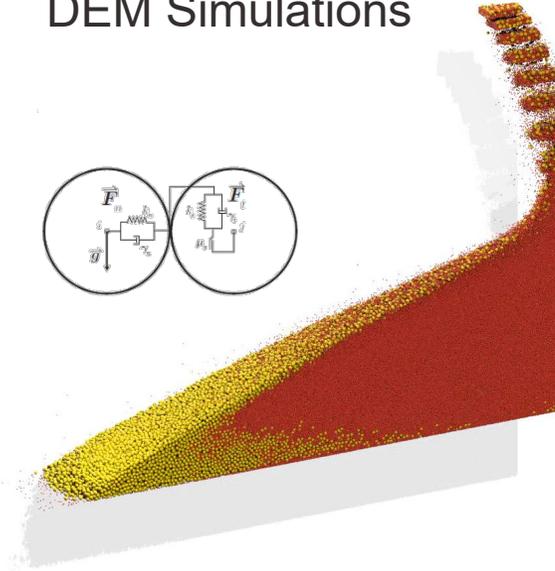


Tools to study cohesive particle de-mixing / segregation & questions

Experiments



DEM Simulations



Theoretical Models

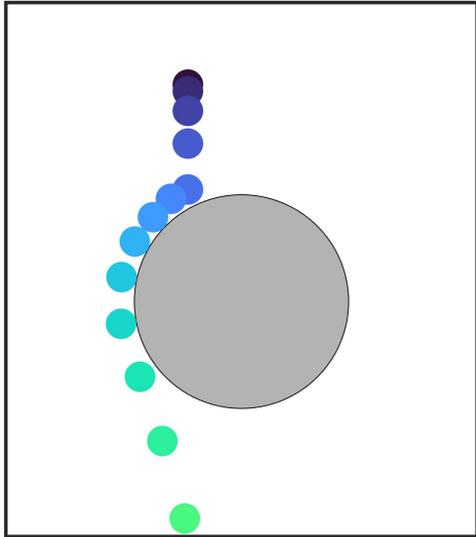
$$\frac{\partial c_i}{\partial t} + \underbrace{u \frac{\partial c_i}{\partial x} + w \frac{\partial c_i}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial}{\partial z} (w_{seg,i} c_i)}_{\text{Segregation}} = \underbrace{\nabla \cdot (D \nabla c_i)}_{\text{Diffusion}}$$

- Can we **understand** cohesive particle demixing/segregation?
- Can we **predict** cohesive particle demixing/segregation?
- Can we **prevent** cohesive particle demixing/segregation?

Assume that segregation/demixing is independent of details of cohesion

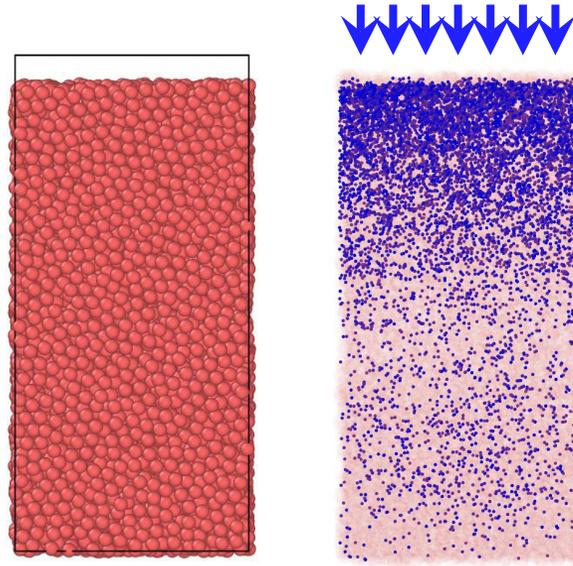
Three viewpoints

Two-particle interactions



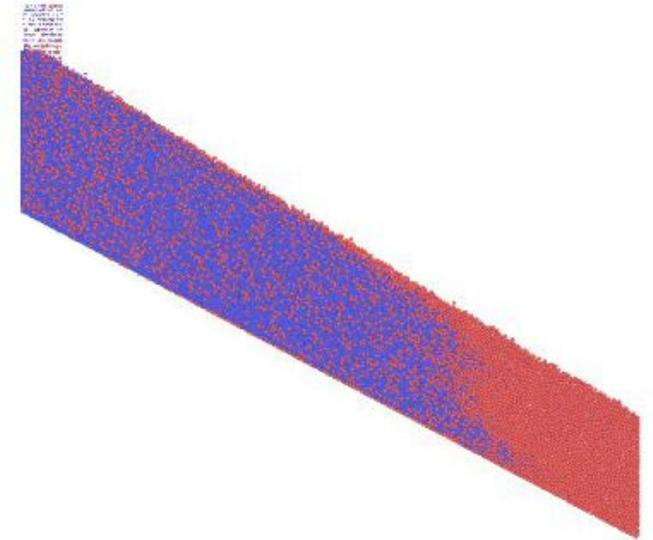
- Particle-particle interactions
- Influence of parameters

Static bed percolation with $R > 7$



- Addresses fine particle cohesion
- Cohesion with no shear
- Influence of parameters

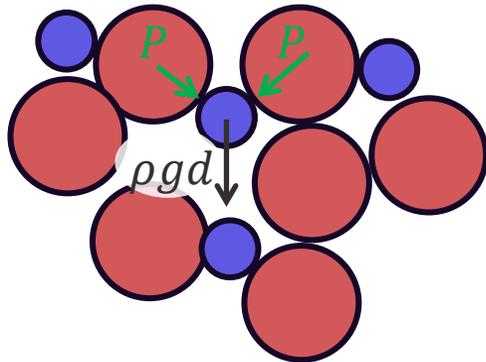
Heap flow with $R < 3$



- Practical applications
- Effect of shear
- Compare to non-cohesive models

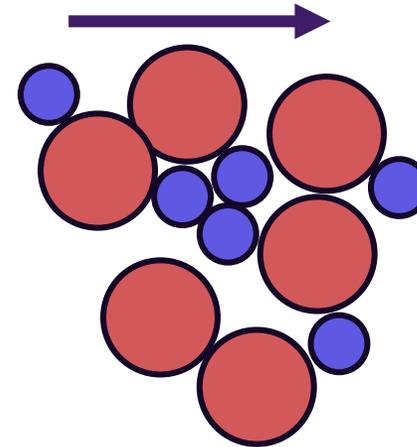
Dimensionless numbers in terms of stresses

- Normal stress (percolation)
 - Weight: $\rho g d$ (stress scale) or Pressure: ΔP
 - ΔP & $\rho g d$ interchangeable in free surface flows.



$$Bo \sim \frac{\text{cohesion}}{\text{weight or pressure}} = \frac{\sigma/d}{\rho g d \text{ or } \Delta P}$$

- Shear stress (mixing, clump-breaking, etc.)
 - Shear rate: $\rho \dot{\gamma}^2 d^2$ or Shear stress: τ
 - $\rho \dot{\gamma}^2 d^2$ & τ inter-related via rheology.



$$Co \sim \frac{\text{cohesion}}{\text{shear}} = \frac{\sigma/d}{\rho \dot{\gamma}^2 d^2} \quad \text{Collision cohesion number}$$

Relation to Inertial number (shear vs normal stress):

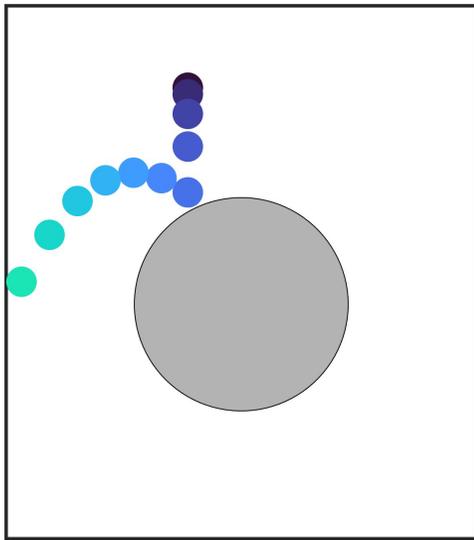
$$Bo/Co = \frac{\dot{\gamma}^2 d^2}{\Delta P/\rho} = I^2$$

Starting point: Interrelation of different parameters

- Additional trapping mechanisms with cohesion
- Cohesion vs restitution vs rolling resistance

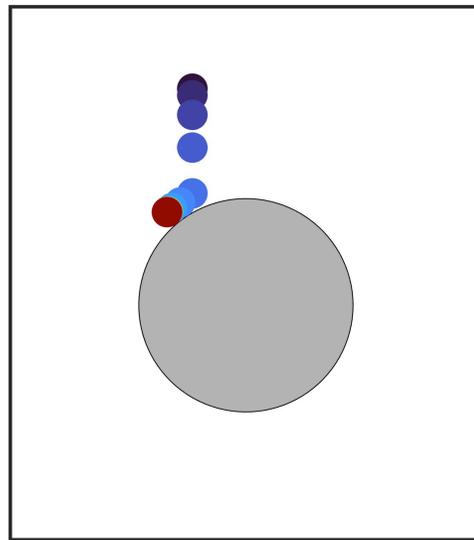
$$R = 7, v_0 = 0.2 \text{ m/s}, \mu = 0.4$$

Bouncing



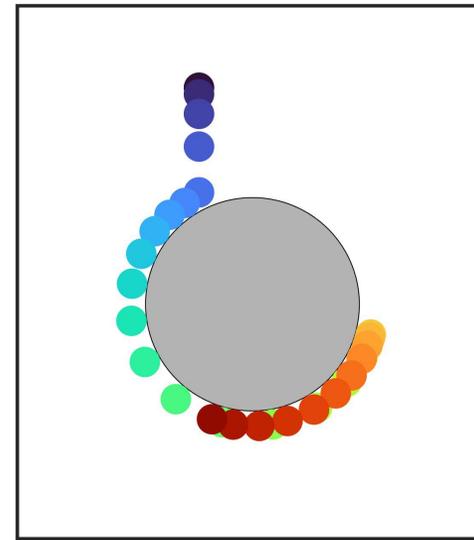
$$Bo = 6, e = \underline{0.8}, \mu_r = 0.03$$

Sticking



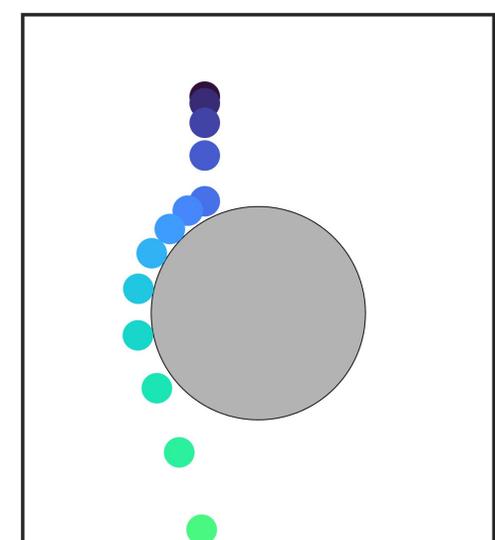
$$Bo = 6, e = \underline{0.3}, \mu_r = 0.03$$

Rolling/sticking

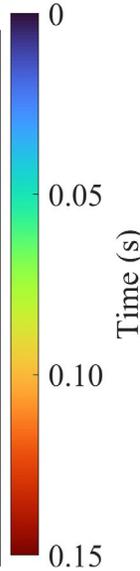


$$Bo = 6, e = 0.3, \underline{\mu_r} = 0.01$$

Rolling/falling

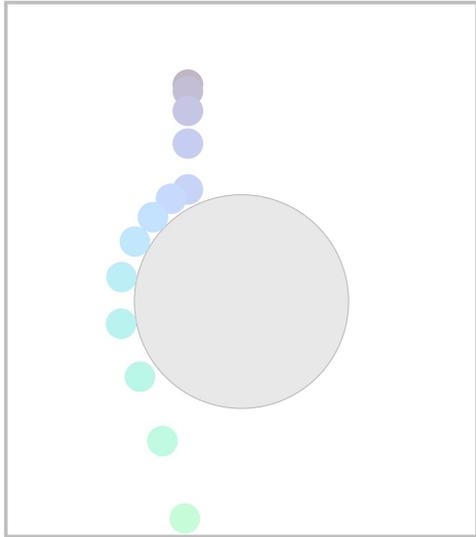


$$\underline{Bo} = 2, e = 0.3, \mu_r = 0.01$$



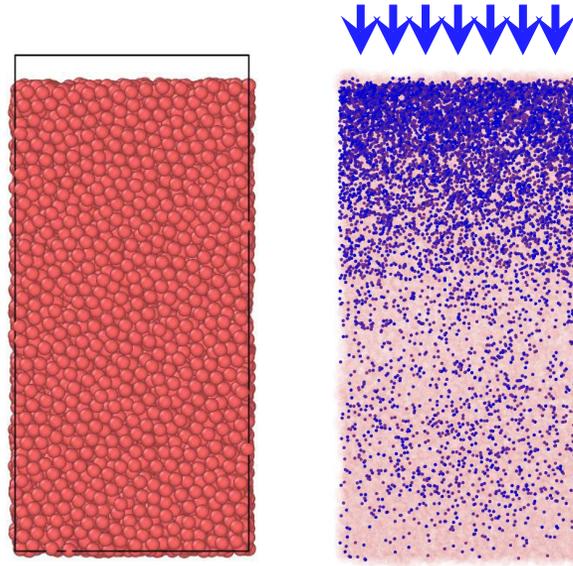
Three viewpoints

Two-particle interactions



- Particle-particle interactions
- Influence of parameters

Static bed percolation with $R > 7$



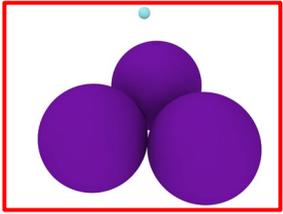
- Addresses fine particle cohesion
- Cohesion with no shear
- Influence of parameters

Heap flow with $R < 3$

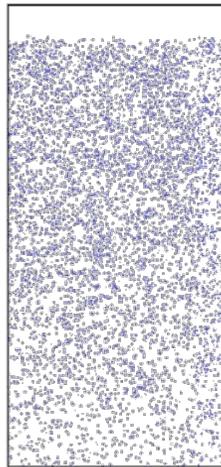


- Practical applications
- Effect of shear
- Compare to non-cohesive models

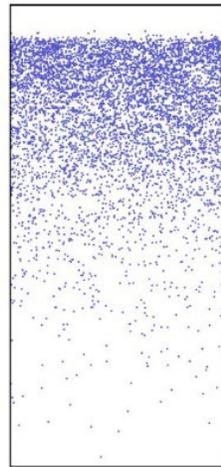
Static bed percolation simulation with cohesive particles



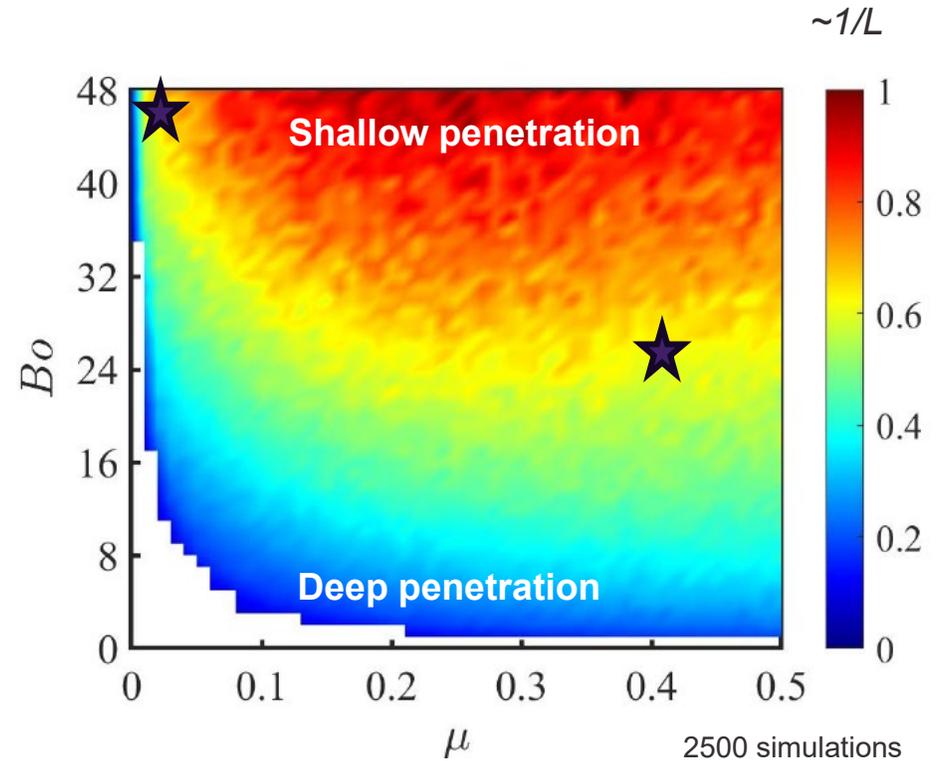
- $R = 7, d_p = 4$ mm
- $e = 0.8, \mu = 0.01, \mu_r = 10^{-7}$
- DMT in LAMMPS



$Bo = 16$



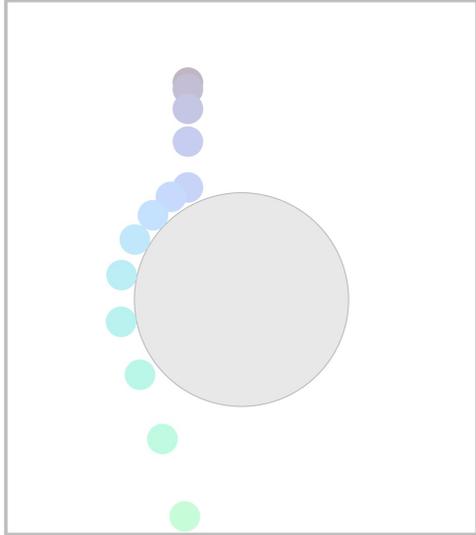
$Bo = 48$



Different parameter combinations (Bo, e, μ, μ_r) yield similar results

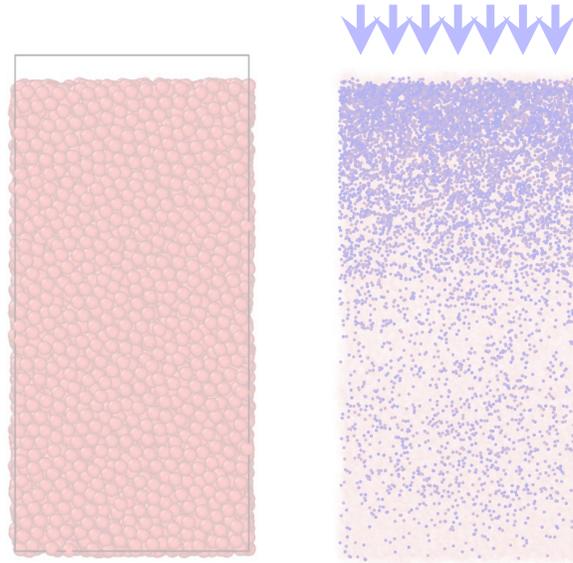
Three viewpoints

Two-particle interactions



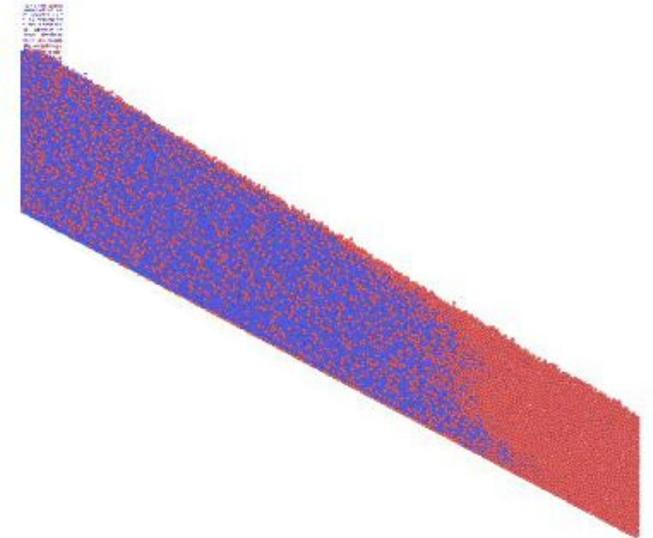
- Particle-particle interactions
- Influence of parameters

Static bed percolation with $R > 7$



- Addresses fine particle cohesion
- Cohesion with no shear
- Influence of parameters

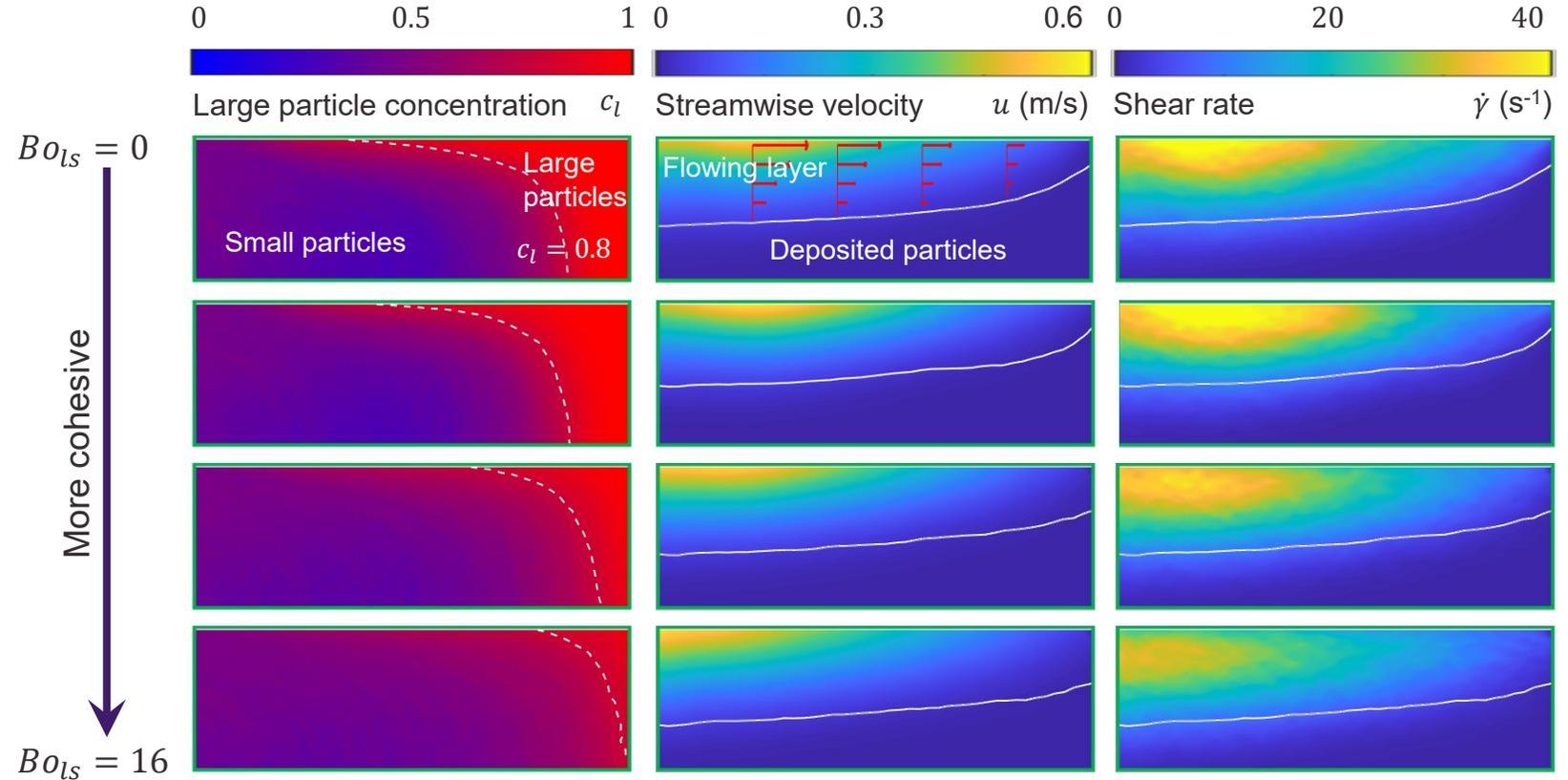
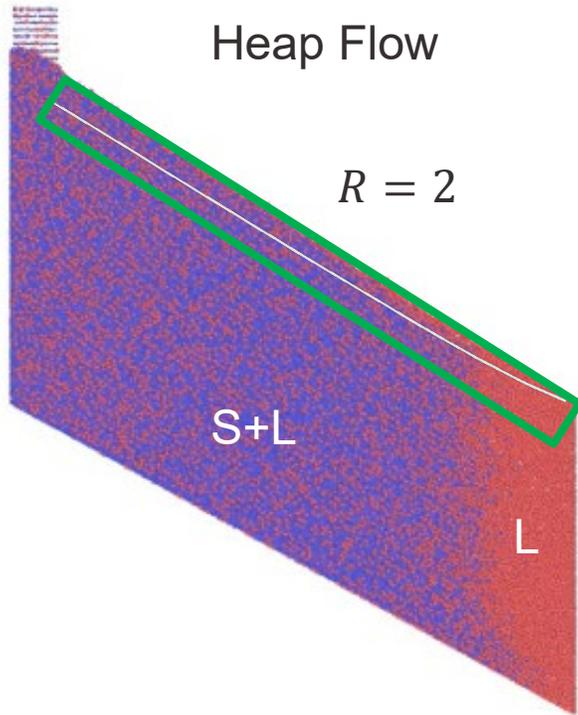
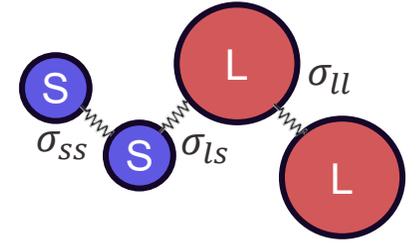
Heap flow with $R < 3$



- Practical applications
- Effect of shear
- Compare to non-cohesive models

Influence of cohesion in heap flow

Bond number measures degree of cohesion: $Bo_{ls} = \frac{\text{pulloff force}}{\text{particle weight}} = \frac{3\pi\sigma r_{eff}}{m_s g}$



- Increased cohesion leads to:
- Reduced segregation
 - Similar velocity field
 - Deeper flowing layer
 - Reduced shear

Influence of cohesion on segregation velocity

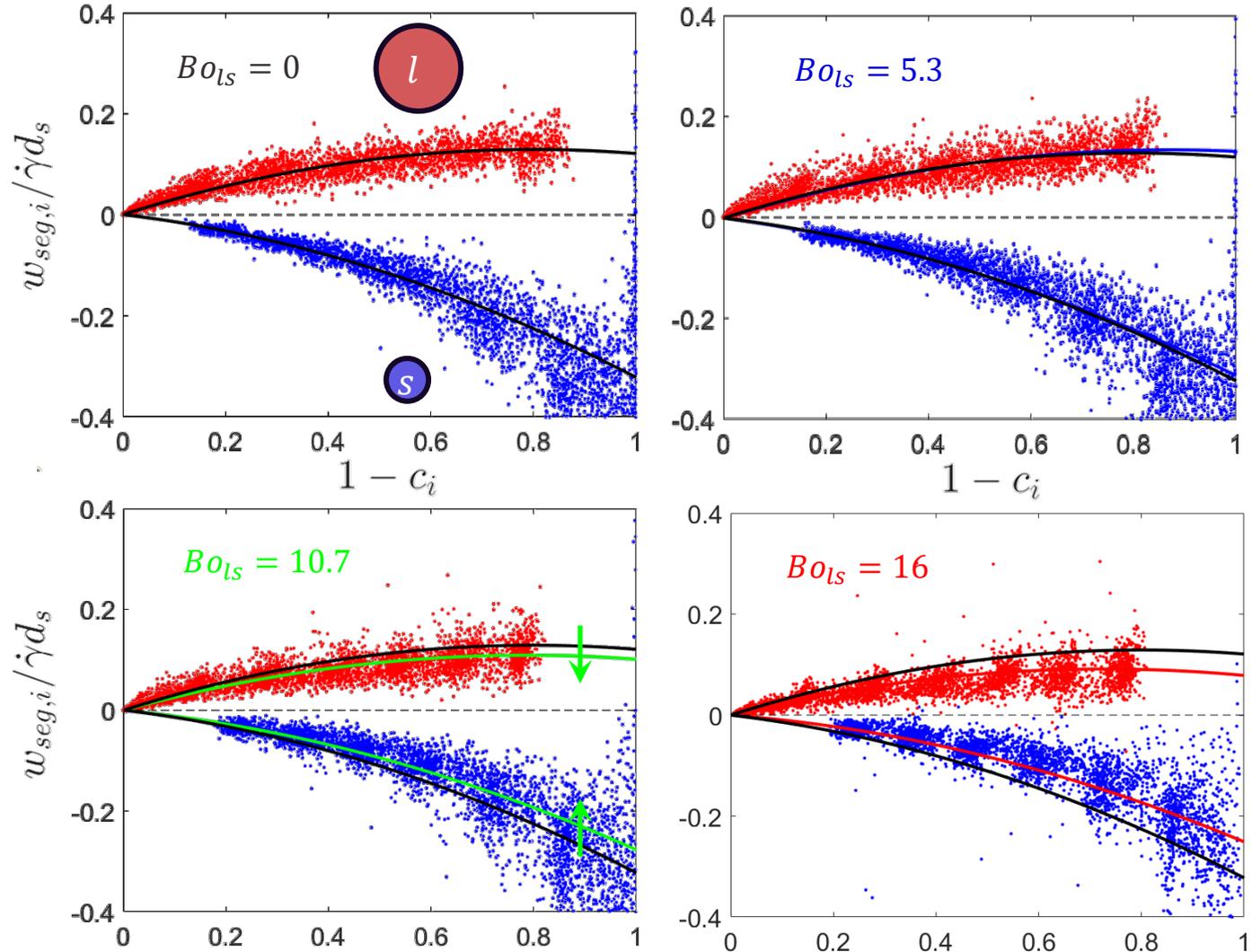
- Segregation velocity model

$$\frac{w_{seg,i}}{\dot{\gamma}d_s} = f(c_i)$$

- Pairwise Bond numbers

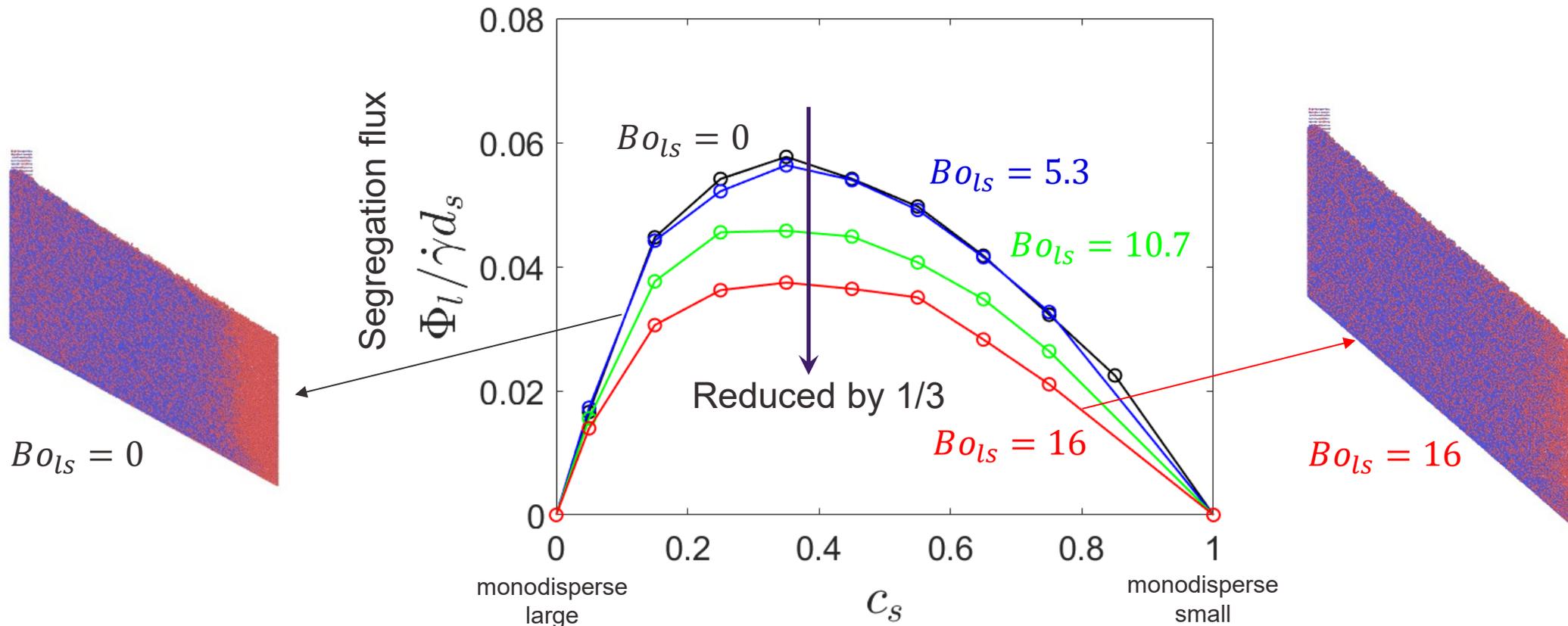
$$Bo_{ls} : Bo_{ss} : Bo_{ll} = 5.3 : 4 : 1$$

- Cohesion reduces segregation



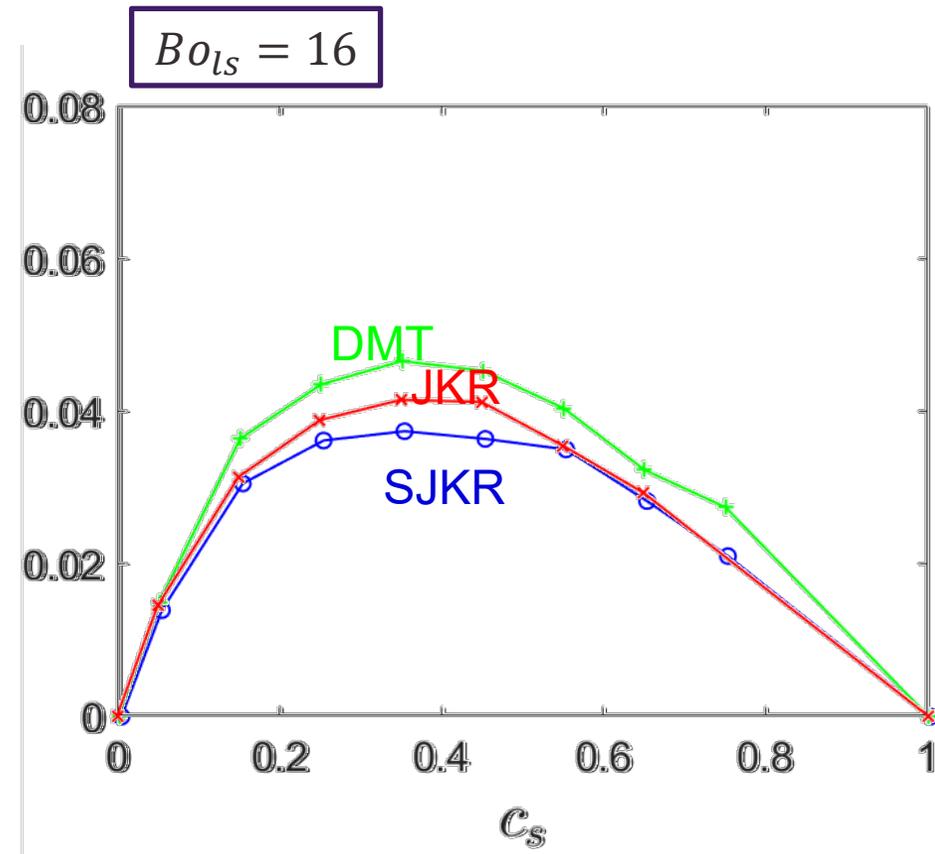
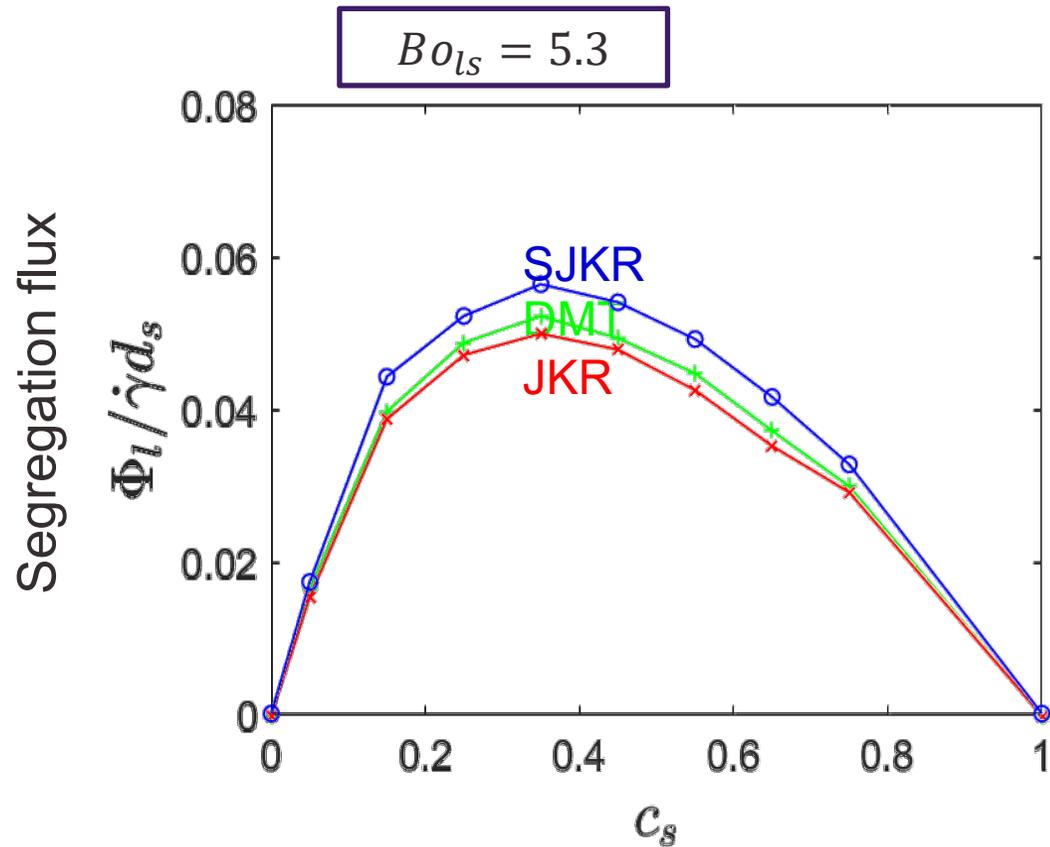
Segregation flux

- Definition: $\Phi_1 = w_{seg,l} c_l$
- Small particle flux downward = large particle flux upward
- Higher cohesion $Bo_{ls} > 10$: cohesion reduces segregation

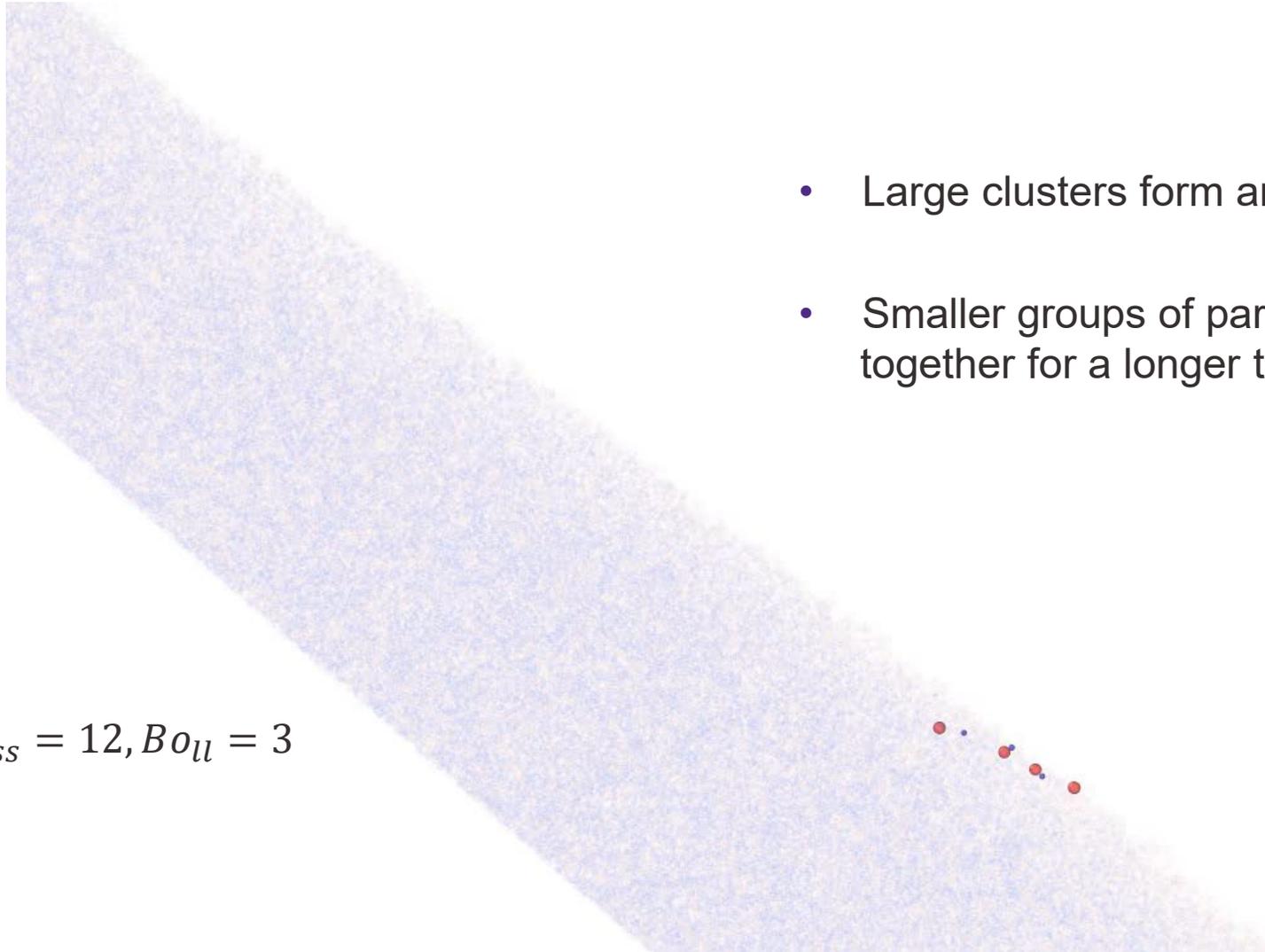


Segregation flux (SJKR vs DMT vs JKR)

- Models agree to first order -- cohesion reduces segregation.
- But details are different



Formation and breakup of cohesive clusters: example

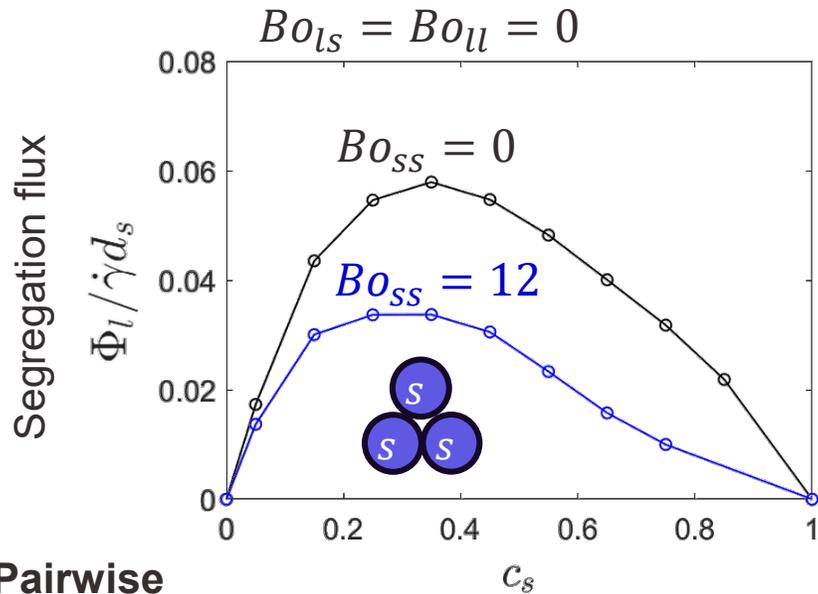


- Large clusters form and break
- Smaller groups of particles stay together for a longer time

$$Bo_{ls} = 16, Bo_{ss} = 12, Bo_{ll} = 3$$
$$R = 2$$

Varying pairwise cohesion independently ($R = 2$)

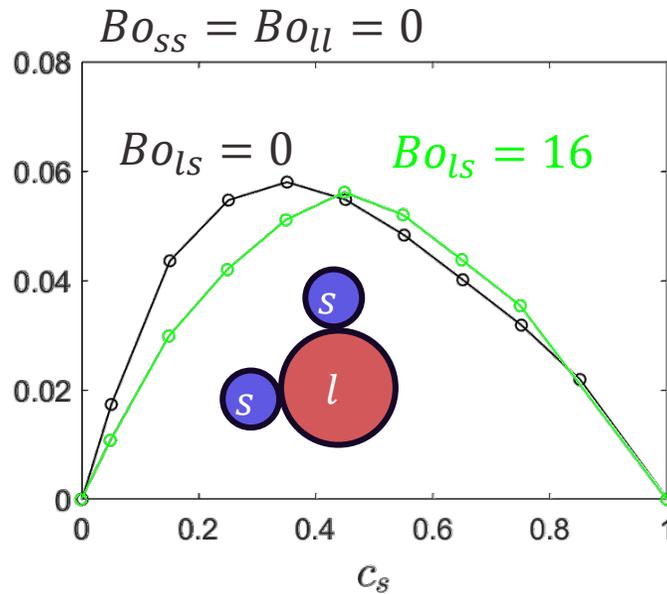
- What if σ_{SS} , σ_{LS} , and σ_{LL} are not identical?
 - Due to different materials, coatings, textures, or other surface properties
- Set one of $\{Bo_{SS}, Bo_{LS}, Bo_{LL}\}$ to nonzero, one at a time



Pairwise cohesion leads to:

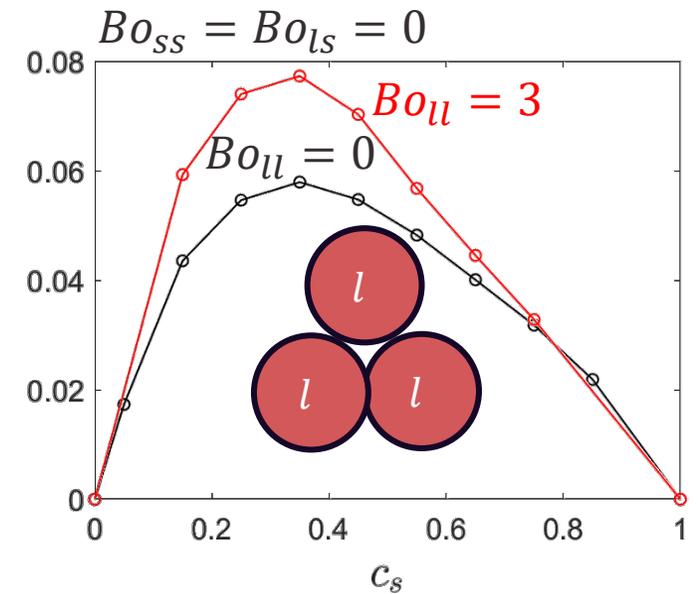
- Reduced segregation

Explanation: $R = \frac{d_l}{d_{s,cluster}} \downarrow$



- Shift of Φ_l peak location

Large particles carry smalls

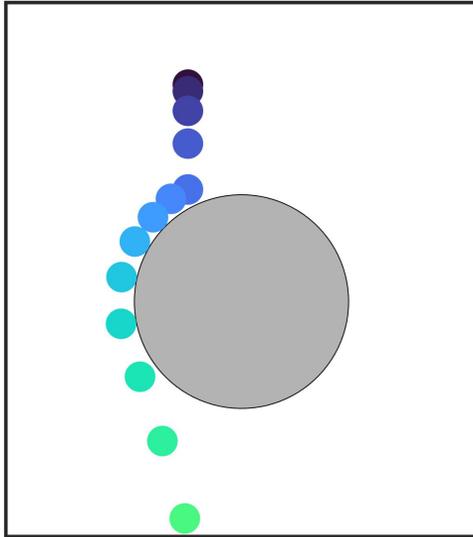


- Enhanced segregation

$R = \frac{d_{l,cluster}}{d_s} \uparrow$

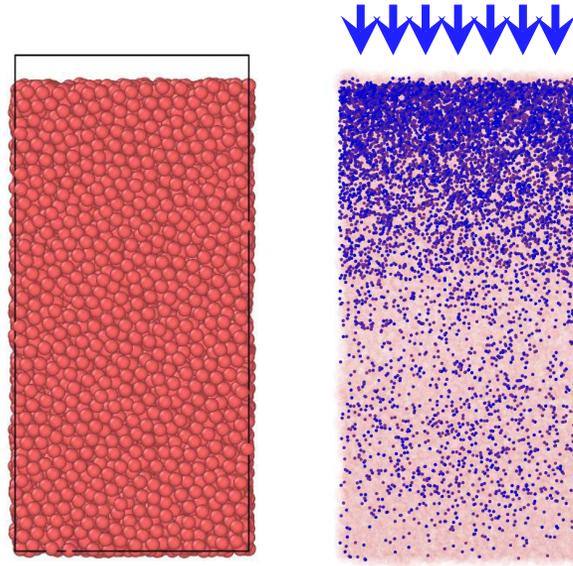
Three viewpoints

Two-particle interactions



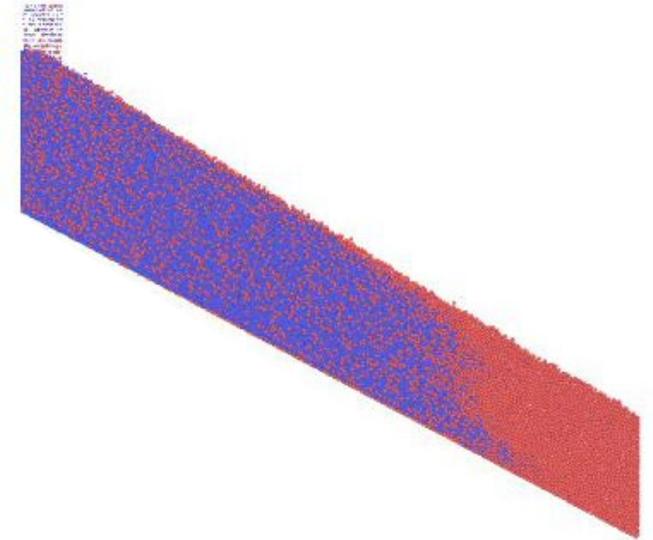
- Particle-particle interactions
- Influence of parameters

Static bed percolation with $R > 7$



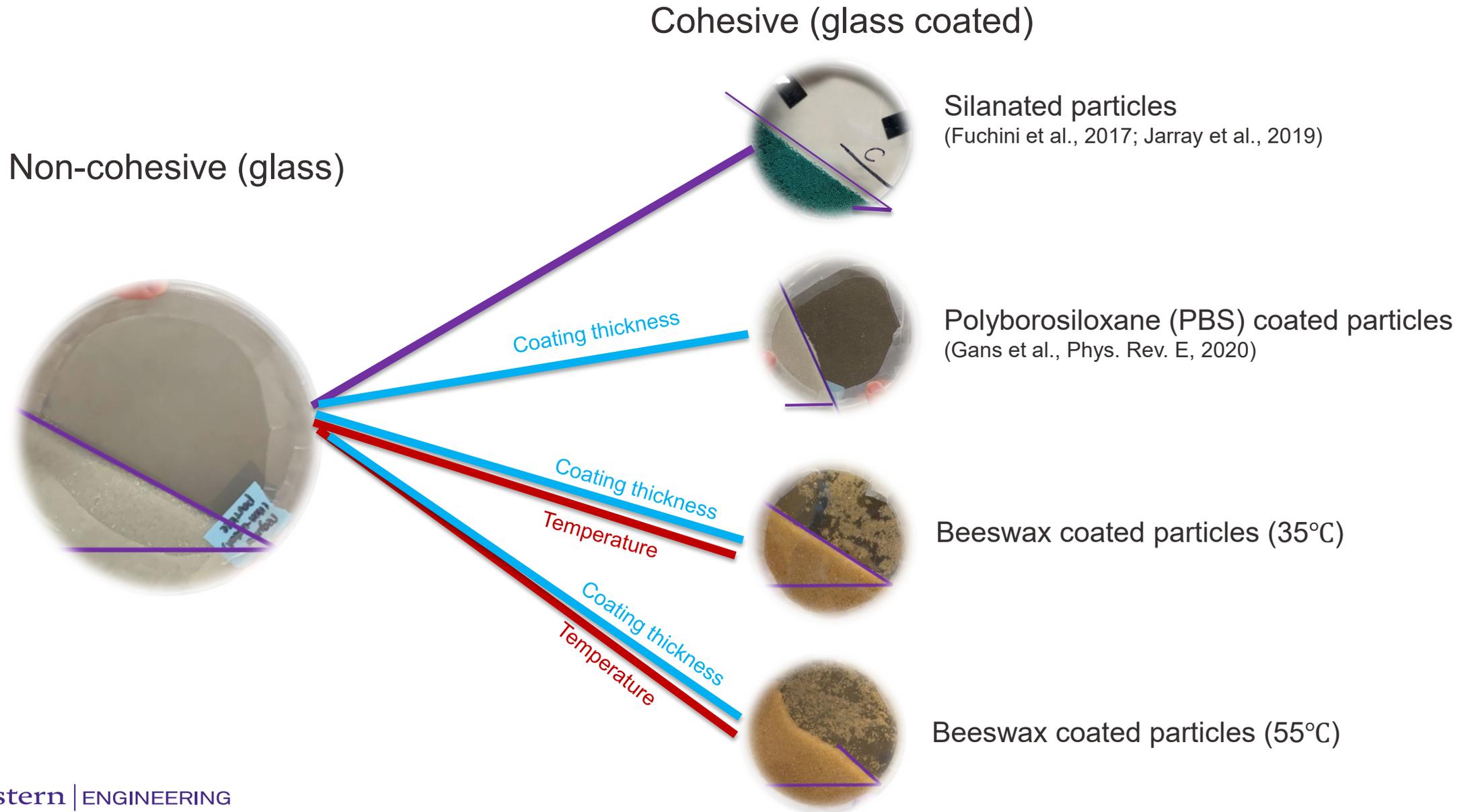
- Addresses fine particle cohesion
- Cohesion with no shear
- Influence of parameters

Heap flow with $R < 3$



- Practical applications
- Effect of shear
- Compare to non-cohesive models

Experiments to validate simulations

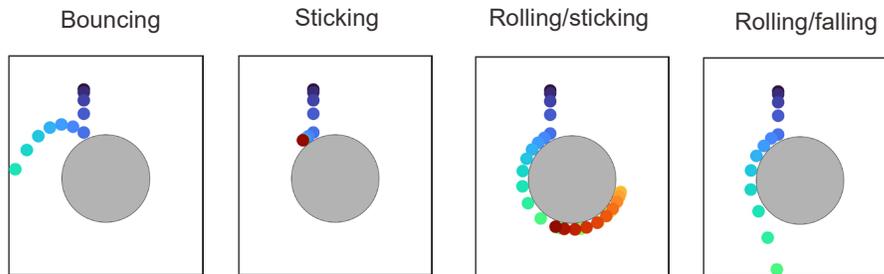


Summary

- **Scaling relation for segregation in bounded heap flow**

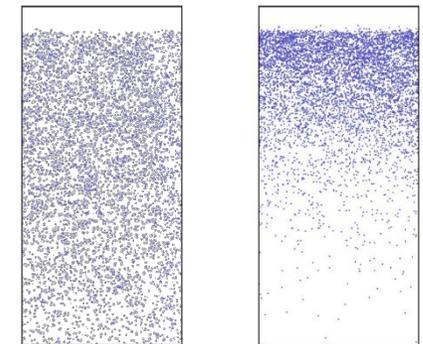
- Bond number: $Bo \sim \frac{\text{cohesion}}{\text{weight}} = \frac{\sigma}{\rho g d^2}$
 - Collision cohesion number: $Co \sim \frac{\text{cohesion}}{\text{shear}} = \frac{\sigma/d}{\rho \dot{\gamma}^2 d^2}$
- $$\left. \begin{array}{l} Bo \\ Co \end{array} \right\} Bo/Co = \frac{\dot{\gamma}^2 d^2}{P/\rho} = I^2$$

- **Two-particle interactions**



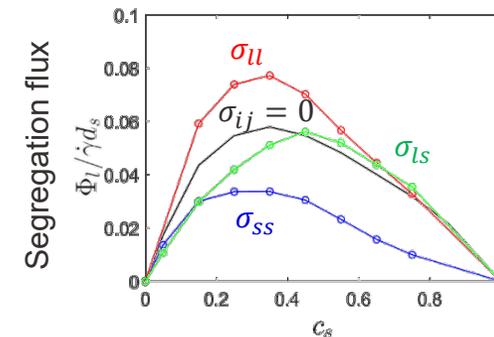
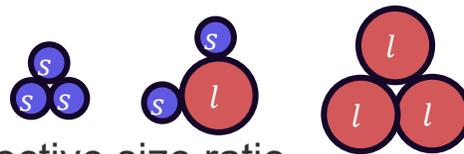
- **Fine particle percolation in static bed**

- Cohesion, restitution, friction, & rolling friction influence trapping and percolation



- **Role of pairwise cohesion: large-large vs large-small vs small-small**

- σ_{ss} reduces segregation
- σ_{ls} shifts Φ_l peak location
- σ_{ll} enhances segregation
- “selective clustering” changes the effective size ratio



SEGREGATION OF COHESIVE PARTICLES IN GRANULAR FLOW

RICHARD M. LUEPTOW

with

PAUL B. UMBANHOWAR, JIZHI ZHANG, DING WANG, QIONG ZHANG

Funded by
International Fine Particle Research Institute
and
NSF CBET-2429545

Northwestern | McCORMICK SCHOOL OF
ENGINEERING