Effect of flexible walls on granular flows.

V. Kumaran,
Department of Chemical Engineering,
Indian Institute of Science,
Bangalore 560 012, India.

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1 Introduction

The flow and transport of granular materials constitute some of the most important components of unit operations in process industries. These include filling and discharge of containers, transport in chutes and hoppers and pneumatic transport. Models for powder flows have been developed using methods from solid plasticity and soil mechanics in the slow flow regime where particles are in continuous contact, and using kinetic theories for rapid flows where particles interact through instantaneous collisions. While these models have been successful for predictions in well-defined contexts, a general set of equations (similar to the Navier-Stokes equations for Newtonian fluids) is still lacking. Design of unit operations typically involves use of conservation laws with constitutive relations that are modification of yield criteria in soil mechanics and solid plasticity, and scale-up using models. An important design criterion for discharge from containers or hoppers is the flow regime, that is mass flow (where there is flow across the cross section) or channeling (where there is flow only in a narrow central region of the container)[1]. The design process is now well developed for containers with rigid walls. There have been relatively few studies on the flow in containers with walls that are compliant or deformable, and design criteria are currently not available. This is despite the importance of these flows in practical situations such as flexible intermediate bulk containers where the container or the discharge unit could have walls that are deformable. The present proposal aims to understand the mechanics of granular flows in conduits with flexible walls, and to provide some guidelines for designing the flow geometry and the wall elastic properties for efficient and safe flow and discharge.

1. **Shape deformation:** The shape change due to filling and discharge, and the effect of the modified container shape on discharge. This is primarily a problem of computing the shape of an elastic solid under loading, and its shape recovery when unloaded. Computationally, this involves formulating solid constitutive relations for the elastic or plastic solid, either 2D or 3D, and then solving these using moving and adaptive meshes. The criteria formulated by Jenicke[1], for example, can be applied to the deformed shape.

   An accurate formulation of the solid constitutive relation is required to predict deformation under loading. In this project, we propose to formulate appropriate constitutive relations for continuum 2D or 3D solids, and examine how these can be coupled to the stress balance conditions at the interface between the container and the granular material. The balance laws and the constitutive relations for the solid can be used to formulate a quasi-static model for the effect of loading on the shape. The solution of these equations for real-world problems is likely to require sophisticated computational techniques which is outside the scope of this projects. Here, we will try to obtain analytical results or one dimensional models for simple container shapes.

An alternate approach is to examine the scale-up process for solid storage and conveying in flexible containers. That is, to ask what are the appropriate elastic and viscous properties of the wall material for a scale model to ensure all dimensionless groups are the same as that for the real system. In this way, the ratio of the deformation and the characteristic scale will be the same for both, and this provides a way to scale-up the processes.

2. **Dynamical interaction:** The second issue is whether there could be a dynamical coupling between the granular flow and the stressed wall, leading to self-excited oscillations, and to shearing at the walls which would be absent for a rigid wall. This requires application boundary conditions at a moving wall, in addition to constitutive relations for the granular material and for the walls. These could be akin to the Schallamach waves[2] for the sliding of rubber-like materials, or dynamical flow instabilities such as those in the conveying of fluids in conduits with compliant walls. It is necessary to carefully define a base state in which the material is flowing past a static wall, to superpose disturbances on the base state and examine whether the disturbances decay or grow. A linear stability analysis[3] is the standard approach which has been widely used in fluid dynamics, and which could yield stability criteria for granular flows and the physical mechanism of the bifurcations. Analysis should be possible for simple flows, such as chute and vertical channel, but numerical computation is required for complicated shapes.

2 Methodology

Either a continuum or a discrete description can be used for each of the two constituents of the system, the granular material and the flexible wall. The discrete description is more useful in simulations where the particles are explicitly treated for systems of relatively small size. This approach is not tractable for large industrial systems, and an approximate continuum models are required. The discrete description contains particle parameters, whereas material parameters have to be specified in the continuum description. One of the important subjects of study is the relation between the parameters in the discrete and continuum models for both the particles and the flexible wall.

2.1 Granular material

The grains are treated as spherical particles in the discrete description, which is now well established. The particles interact through a contact model which consists of spring, damping and frictional forces which depend on the ‘overlap’ of the particles in directions parallel and perpendicular to the surface of contact. It is proposed to use this standard model, shown in figure 1, in our description.[4, 5, 6] Though the use of spherical particles appears a simplification, this model does result in the correct form of the constitutive relations at the large scale. Particles of realistic shapes that are non-spherical can be simulated, but these are significantly more complex and computationally intensive to simulate, and are not essential for model development. The discrete description can be used to model both flowing and static states.

The continuum models are of two types, those that predict yielding depending on the stress distribution and those for the velocity profiles in flowing granular materials. The yield criteria for granular flows have been modeled extensively; for containers with compliant walls, it is necessary to determine how the wall compliance affects the stress distribution, and then to use the modified stress distribution for determining the yield criterion.

Two types of continuum models have been used for flow prediction—the extensions of the frictional models for yielding to incorporate the rate of deformation[7, 8, 9], and the kinetic models based on hard-particle interactions[10, 11, 12]. Though there has been a lot of work in each of these areas, neither provides a complete description of a dense flowing granular material, especially in complicated geometries. The initial focus of our work will be on simple geometries, such as the flow down a chute or in a vertical hopper, where several models have already been evaluated for flow between rigid boundaries. The effect of coupling with a flexible boundary on the flow dynamics will be evaluated using these models.

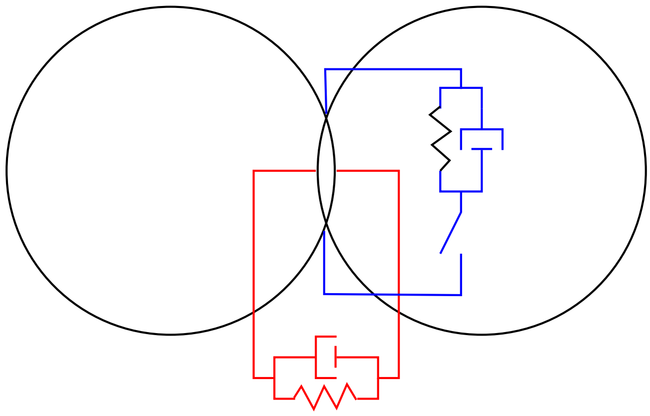

Figure 1: The contact model for particles that 'overlap' consists of a spring-dashpot force due to the deformation normal to the surfaces at contact (red) and a spring-dashpot-slider force due to the deformation tangential to the surfaces at contact.

2.2 Compliant wall

Compliant walls could be modeled as either two-dimensional deformable sheets (membranes) or as three-dimensional visco-elastic continua. Some models used for flow in compliant conduits are the following.

1. Membranes are continuous surfaces with a specified area density (mass per unit area) when there is no stress. The deformation due to stress is described using a displacement field along the surface, which is the displacement of material points from their equilibrium positions, as shown in figure 2 (a). When a membrane is deformed, there is a force exerted normal to the surface due to curvature and bending, and a force tangential to the surface due to displacement along the surface.

2. A three-dimensional model of a compliant wall is a viscoelastic continuum is shown in figure 2 (b). Here, the dynamical variable is the displacement of material points from their equilibrium positions. The constitutive relation for the stress contains an elastic term due to the strain, a viscous term due to the strain rate and possibly a pressure to enforce incompressibility.

3. A simpler model for a compliant wall is the spring-dashpot model, shown in figure 2 (c), where the wall is considered as a thin deformable plate that is attached to a rigid substrate through spring and damping elements. The normal stress at the surface is related to the displacement of the deformable plate.

It should be noted that the displacement and strain fields have to be calculated in a 'frame-invariant' manner. Consequently, the relation between the strain and the displacement field is inherently non-linear and it is difficult solve for large displacements. Linear approximations, which are valid for small deformations, are easier to solve, and they can be used for examining instabilities due to a dynamic coupling between the granular flow and the wall.

Discrete versions of continuum wall models may be more compatible with discrete particle simulations in some cases. An example is shown in figure 2 (d), where the spring-backed wall has been approximated as a series of spheres connected to springs. In a similar manner, it is possible to approximate a membrane by hexagonally packed spheres connected by springs to their nearest neighbours. These are advantageous for discrete particle simulations, since the same interaction details are required for particle-particle and particle-wall interactions.

3 Proposed work

The focus of the work for the initial three years is proposed in three areas. These include discrete particle simulations, linear stability analysis and design of experiments to identify and validate dimensionless groups of

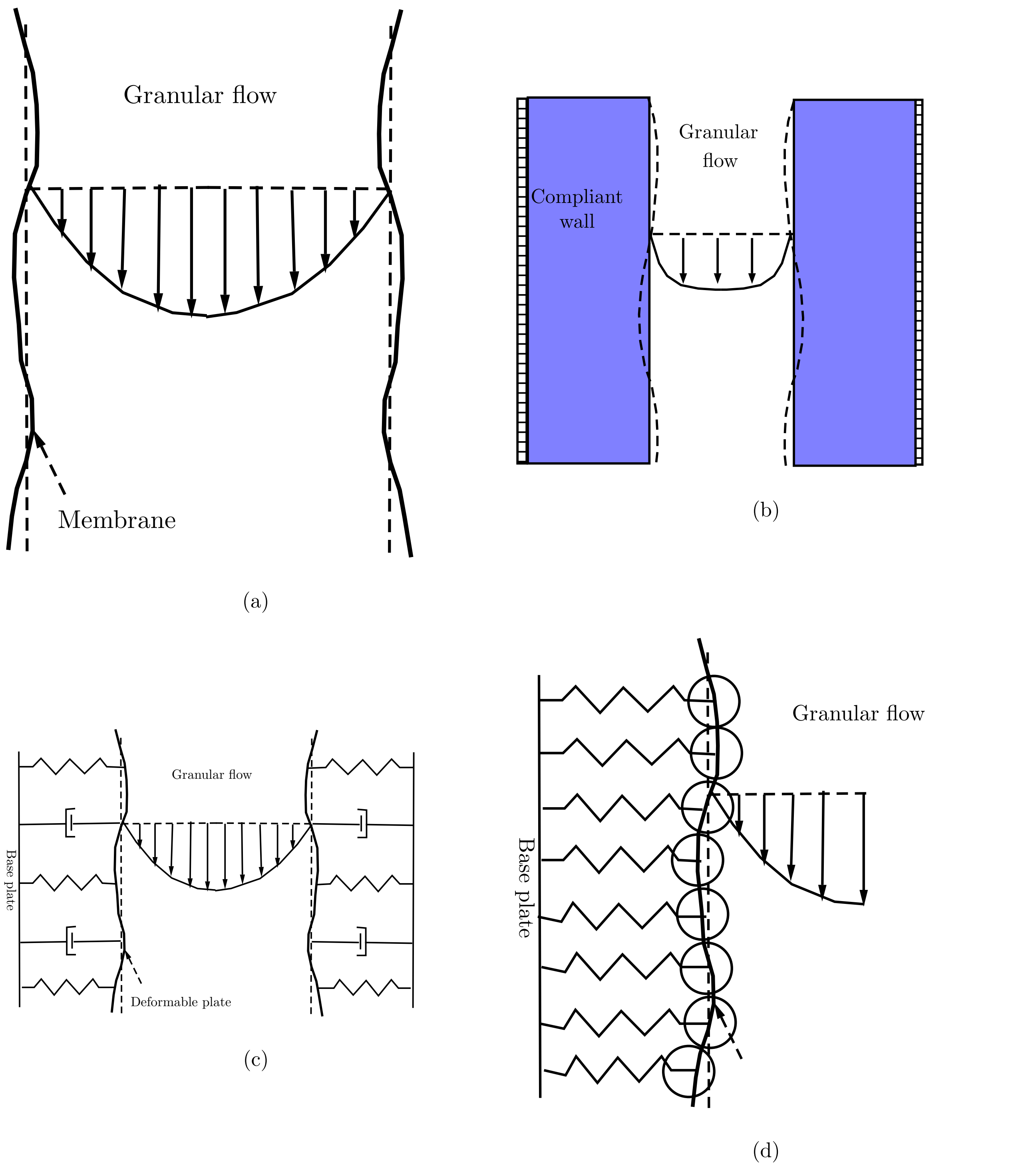

Figure 2: Different wall models for the granular flow past a compliant wall, (a) membrane, (b) viscoelastic continuum, (c) spring-backed wall and (d) discrete spring-backed wall.

importance in the design of compliant containers.These are all in simple geometries using small deformation regimes where possible, in order to examine build up the methodology that can be extended to more complex geometries and finite deformation regimes. The modeling of deformation of the bottom wall of containers is the fourth area that is proposed; this will involve dimensional analysis to characterise the parameters that determine deformation, and this will require collaboration with IFPRI members working in this area.

3.1 Stability of the granular flow past a compliant surface

The continuum models for different types of surfaces will be coupled with kinetic models for static and flowing granular configurations in order to examine the growth or decay of perturbations at the surface in a linear stability analysis. This will be carried out for simple geometries, a vertical channel an inclined plane, where reasonably accurate models are available for the granular flow.

In these calculations, the base state will be a static or flowing granular medium bounded by a compliant wall that is stationary, but is under stress due to the granular flow. Small perturbations in the form of Fourier waves will be applied on this base state. The kinetic equations will be solved in the granular domain in order to determine the perturbations to the volume fraction and velocity fields, and those for the compliant wall will be solved to obtain the perturbations in the displacement fields. These will be coupled through the boundary conditions, and solved to determine the growth rate and frequency of the disturbances.

The stability analysis only provides the bounds on the stability of the base state in terms of a dimensionless group which is a function of the wall and granular properties. In future work, this will be extended to mass flows in bins and hoppers, in order to provide parameter regimes where dynamical coupling is expected between the flow and the wall motion.

Based on the theoretical and simulation results, we will examine the feasibility of setting up experiments in a vertical channel with flexible walls. There are two aspects that have to be resolved. The first is the identification of suitable materials with elastic properties matching those used in simulations. The second is the detection of wall oscillations. The latter is likely to require sophisticated experiments, since the oscillations will have relatively small amplitude and large frequency. Assistance will be sought from IFPRI members for this.

This work builds up on the expertise in our group in the fluid flow past compliant surfaces, as well as the formulation of kinetic models for granular flows. We will need to interact with IFPRI members for guidance on realistic models for compliant materials, and realistic parameter values in these models, in order to calculate the relevant dimensionless groups.

3.2 Discrete simulations of granular flows

Discrete simulations will be used to model granular flow in compliant-walled channels and chutes, where the particles are treated as discrete spheres. The simulations will be used to verify the predictions of the linear stability analysis with regard to the dominant modes of excitation of the coupled system, and to examine the flow regime after transition.

Discrete particle simulations have been carried out in our group for a long time for granular flows down chutes and hoppers using the Discrete Element Method (DEM)[13] and for flow adjacent to walls of complicated shape[14, 15]. The additional development required is the description of the compliant wall. The choice of the wall model, continuum or discrete, will be made based on computational complexity of the model and the compatibility of the model with the discrete particle simulations. The objective is to examine whether there are self-induced oscillations due to the interaction between the particles and the wall, and the dependence of the frequency response on the wall model and the wall elasticity. The results will be compared with the theoretical predictions for the stability of the base state, and the flow modification due to the instability.

3.3 Effect of wall flexibility on discharge

There have been several studies on discharge in hoppers, and the Beverloo correlation[16] for the discharge rate, and the Jenicke criteria[1] for mass and funnel flows, have been proposed. Here, the effect or wall flexibility on the discharge of the material from the bottom of a container will be studied, using the spring-backed wall model for the bottom wall. It is known that the flow of material is often hindered by arching at the bottom, where an arch of particles forms and transmits the overburden to the bottom of the container. If the wall is deformable, deformation may either enhance or inhibit the deformation, leading to a modification of the discharge rates. This will be simulated using the discrete particle model and either the continuum or the discrete wall model depending on the computational complexity.

Simulations on filling and discharge using the discrete particle model in rigid containers are feasible for fairly large system sizes, and these have been used in our group as well. The wall description in simulations has to be formulated; and this will be one of the major tasks in our simulation study.

Based on the simulation results, we will attempt to set up an experiment on the discharge from a vertical channel. For this, it is necessary to identify materials that have the same properties as those used in the simulation parameters, and fabricate the exit slot of the channel with these materials. This feasibility study, and the experiments if suitable materials can be identified, will be carried out after the simulations.

3.4 Wall deformation

The effect of wall deformation on the principal stresses in a rectangular/cylindrical container with a flexible bottom will be analysed, in order to determine how deformation affects the principal stresses during the filling process. As a first step, the bottom surface will be modeled using the principal curvatures in a manner similar to the deflection of beams or shells, and the dynamics of the filling process will be analysed as a function of the shell stiffness.

While simulations of the deformation of the bottom surface can be set up, the validation of these will require significant collaboration with IFPRI members. Assistance will be necessary for measuring the elastic properties of wall materials, and comparing the experimentally observed results with the continuum model and with simulations. Collaboration will also be important for studying scale-up, that is, what are the dimensionless groups that have to be maintained a constant between the model and the real system in order to correctly reproduce the scaled shape deformation and force distributions.

4 Time schedule

Activity	First Year	Second Year	Third Year
Continuum wall parameterisation			
Formulating discrete wall model			
Linear stability analysis			
Simulation of inclined plane and vertical channel			
Simulations on effect of wall flexibility on discharge			
Feasibility of experiments on flow in a channel with flexible surfaces			
Feasibility of experiments on discharge through a flexible orifice			

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