Evolution and Breakup of Polymeric Filament

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IFPRI
International Fine Particle Research Institute
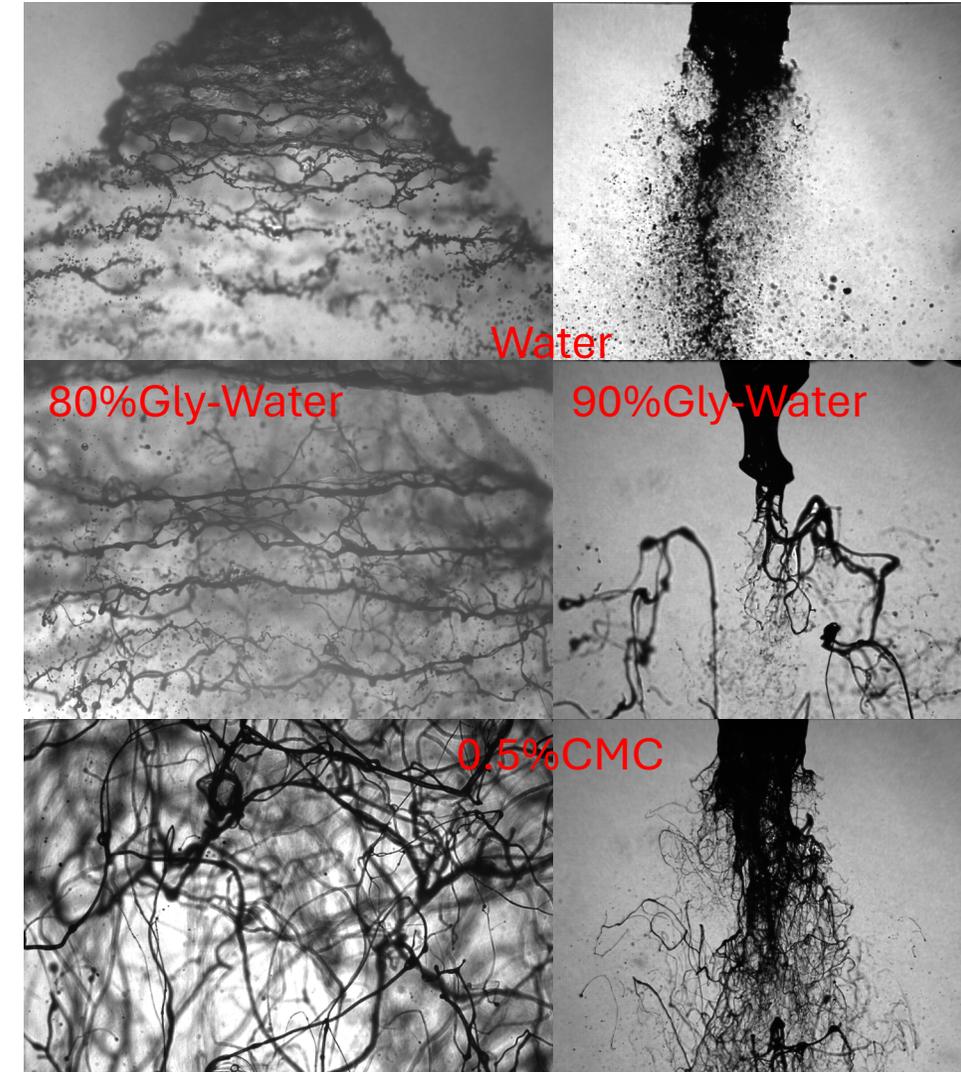
Atomization of Viscous and Polymeric Fluids

- Ligaments of viscous and polymerics fluids are stretched thinner before they break up into droplets

- Applying Rayleigh-Plateau instability on initial thickness will overestimate droplet size:

$$d_{32} = 1.882 d_{lig}(1 + 3Oh)^{\frac{1}{6}} \approx 2.2 d_{lig}$$

- This indicates that <u>thinning effect</u> on ligaments is crucial in determining droplet size


Swirl Nozzle   Twin Fluid Nozzle

Water

80%Gly-Water   90%Gly-Water

0.5%CMC

Objectives

1. Models for filament breakup of polymeric fluids and slurries.
2. Spray droplet size measurement of polymeric fluids and slurries.
3. Models for the atomization of high viscosity/polymeric fluids and slurries in swirl and twin fluid nozzles based on filament dynamics.

Experiment Setup
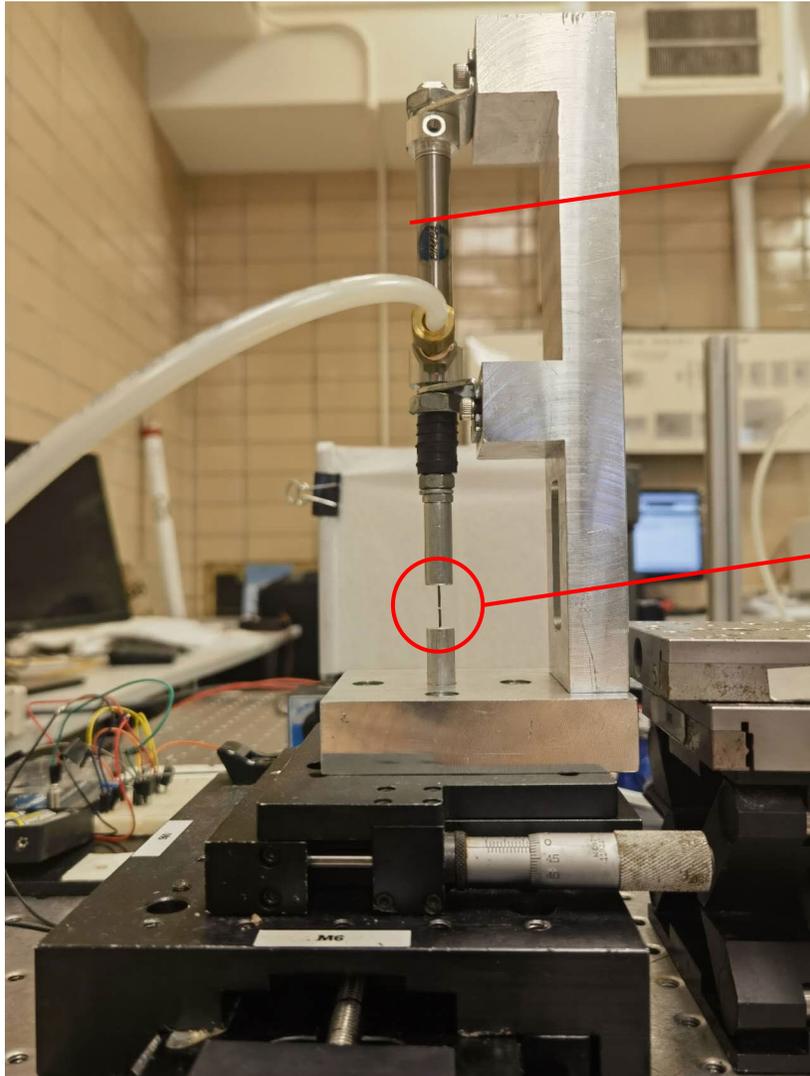
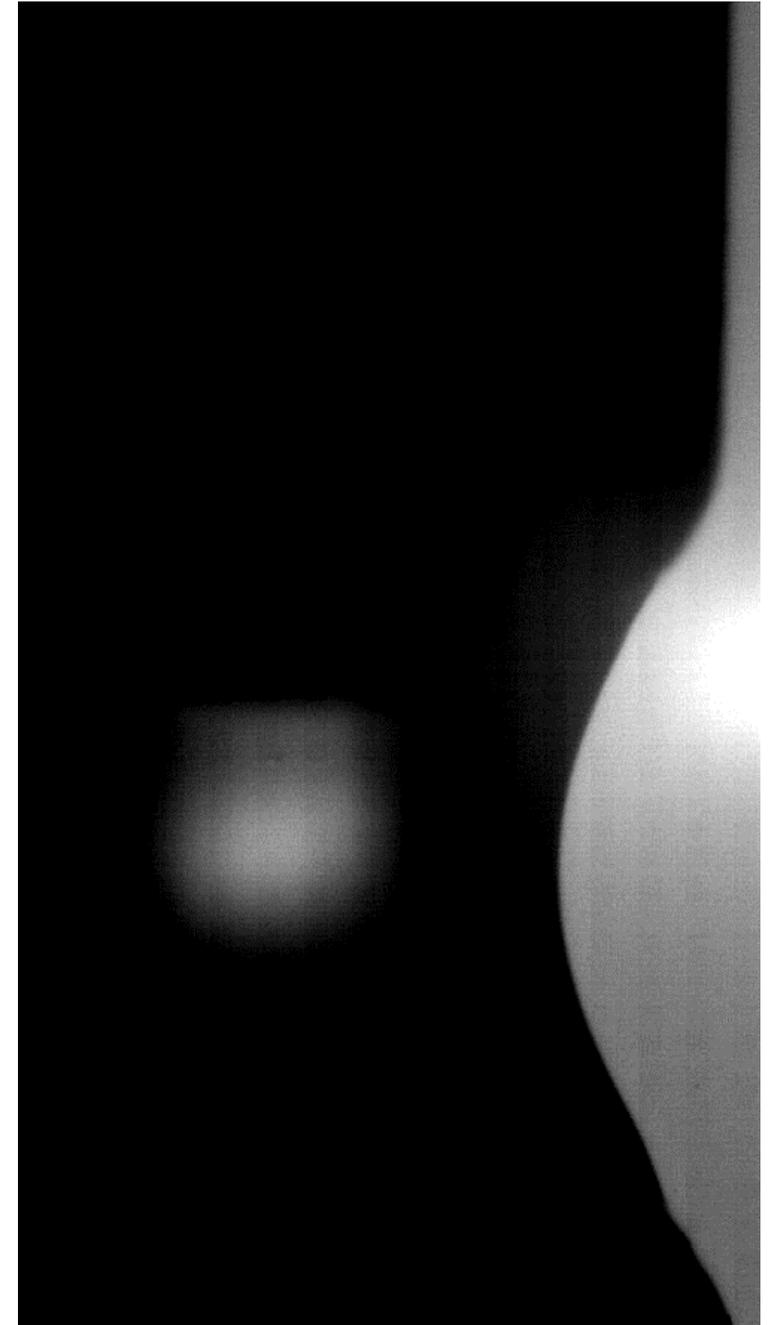
Air cylinder retractor:
retracting velocity at $0.1 m/s$
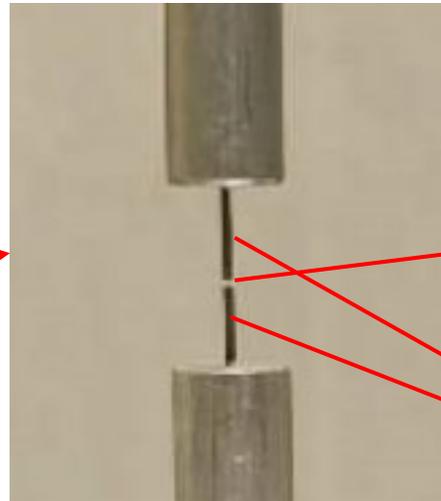
Gap for fluid:
0.25% aqueous Polyethylene
oxide (PEO), semidilute solution
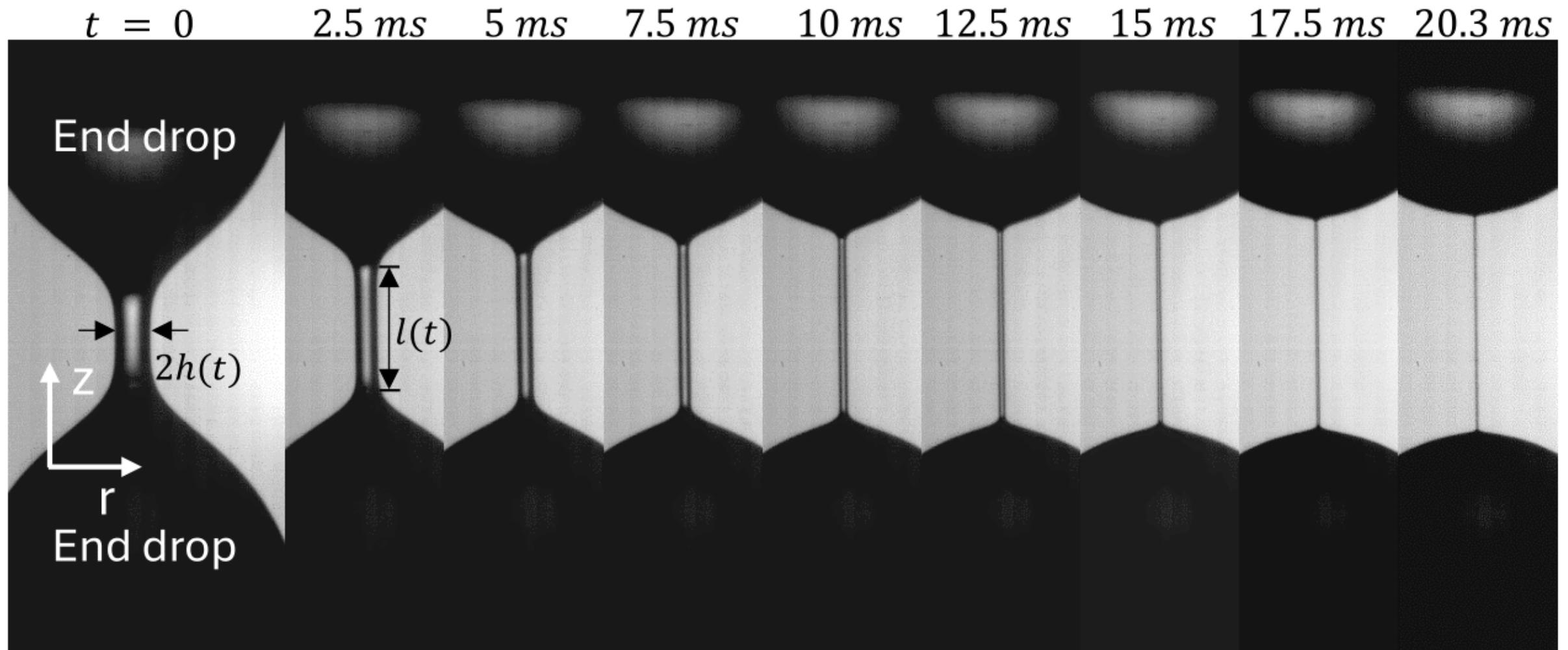$M_v \sim 2 \times 10^6 \, g/mol$
$\mu_0 = 7 mPa \cdot s$

Rod: $d = 0.8mm$

Evolution of Polymeric Filament

- The evolution of polymeric filament can be divided into 3 stages:
  - Initial thinning
  - Formation of beads-on-string structure
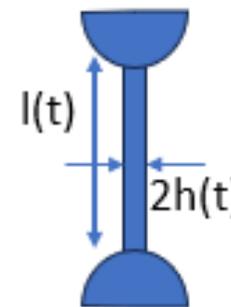 - Coalesce and breakup of beads-on-string structure

1. Initial Thinning

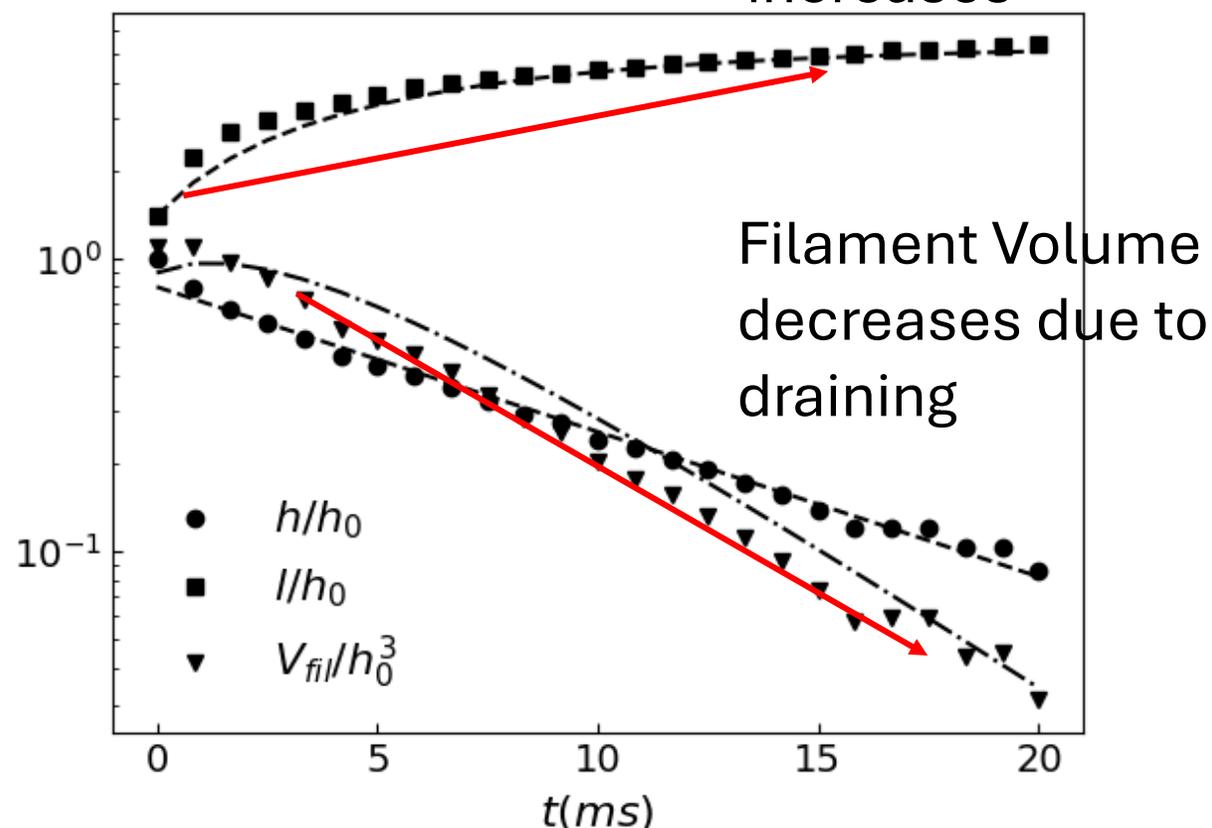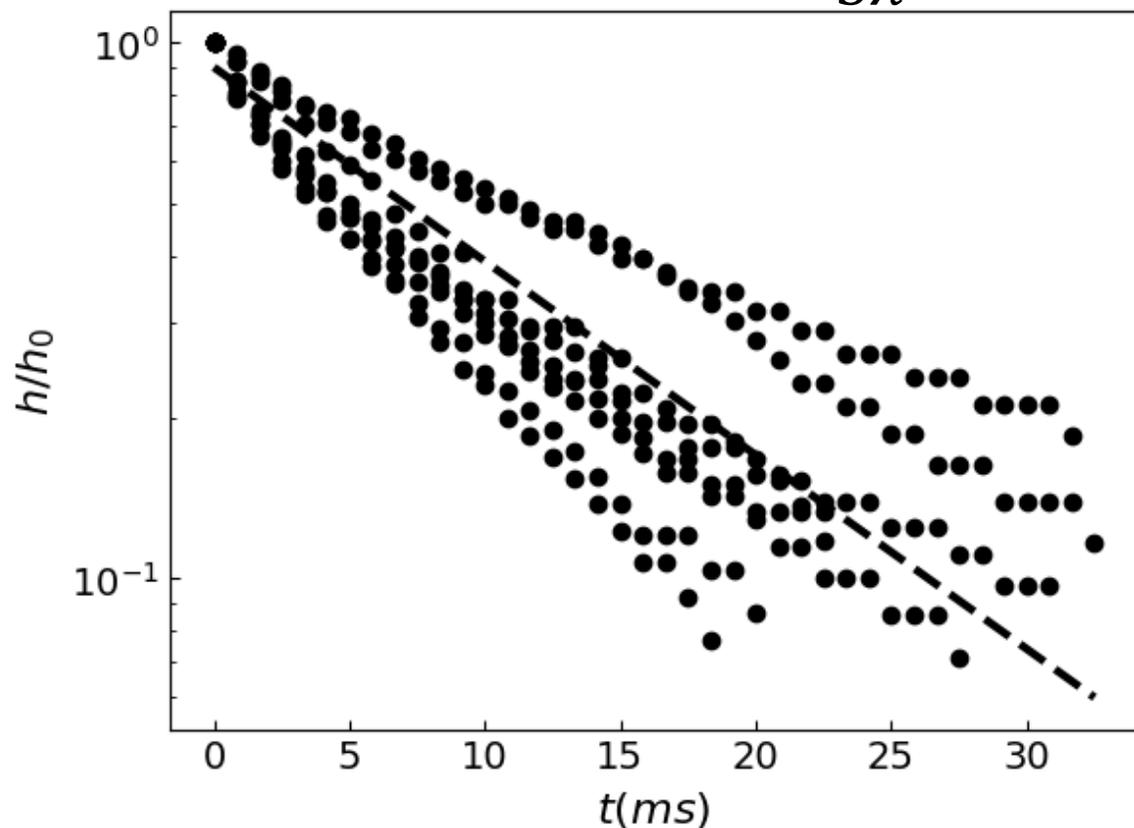# 1. Initial Thinning

- **How is the filament thinning?**

$$h(t) = h_0 \exp(-\beta_h t)$$

$$\beta_h = \frac{1}{3\lambda}, \qquad \lambda = 4 \pm 1 \; ms \; (relaxation \; time)$$

Filament length increases

Filament Volume decreases due to draining

1. Initial Thinning
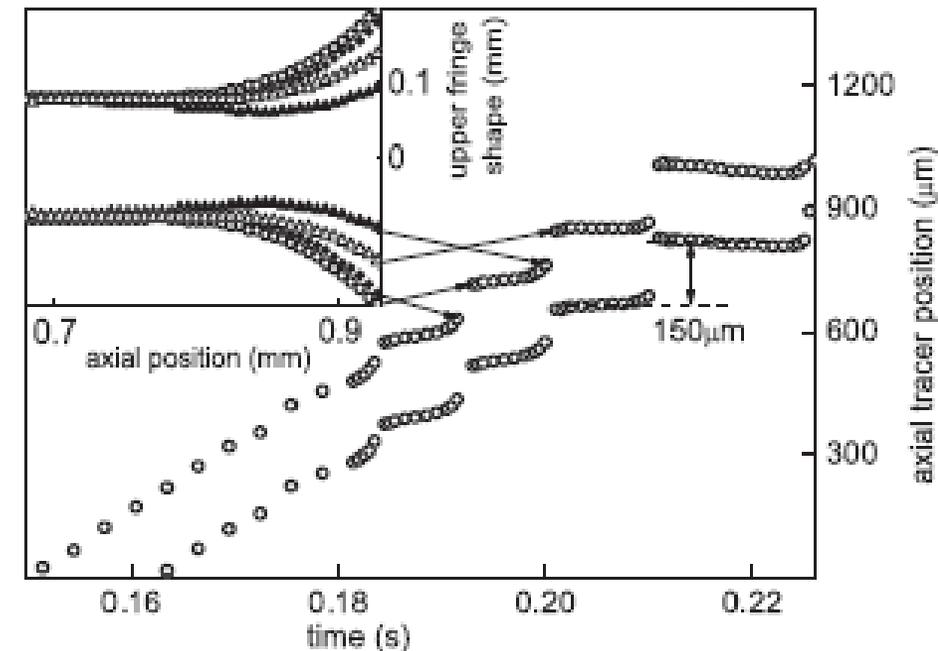
- **How thin will it be to generate beads?**
  - The filament radius reached a plateau at the end of thinning process.
  - The internal flow stopped at the end of initial thinning.
  - A static solution of filament dynamics gives

$$\frac{h_f}{h_0} = \left(\frac{G}{2}\right)^{\frac{1}{3}}, G = \frac{\mu_p h_0}{\lambda \gamma}$$
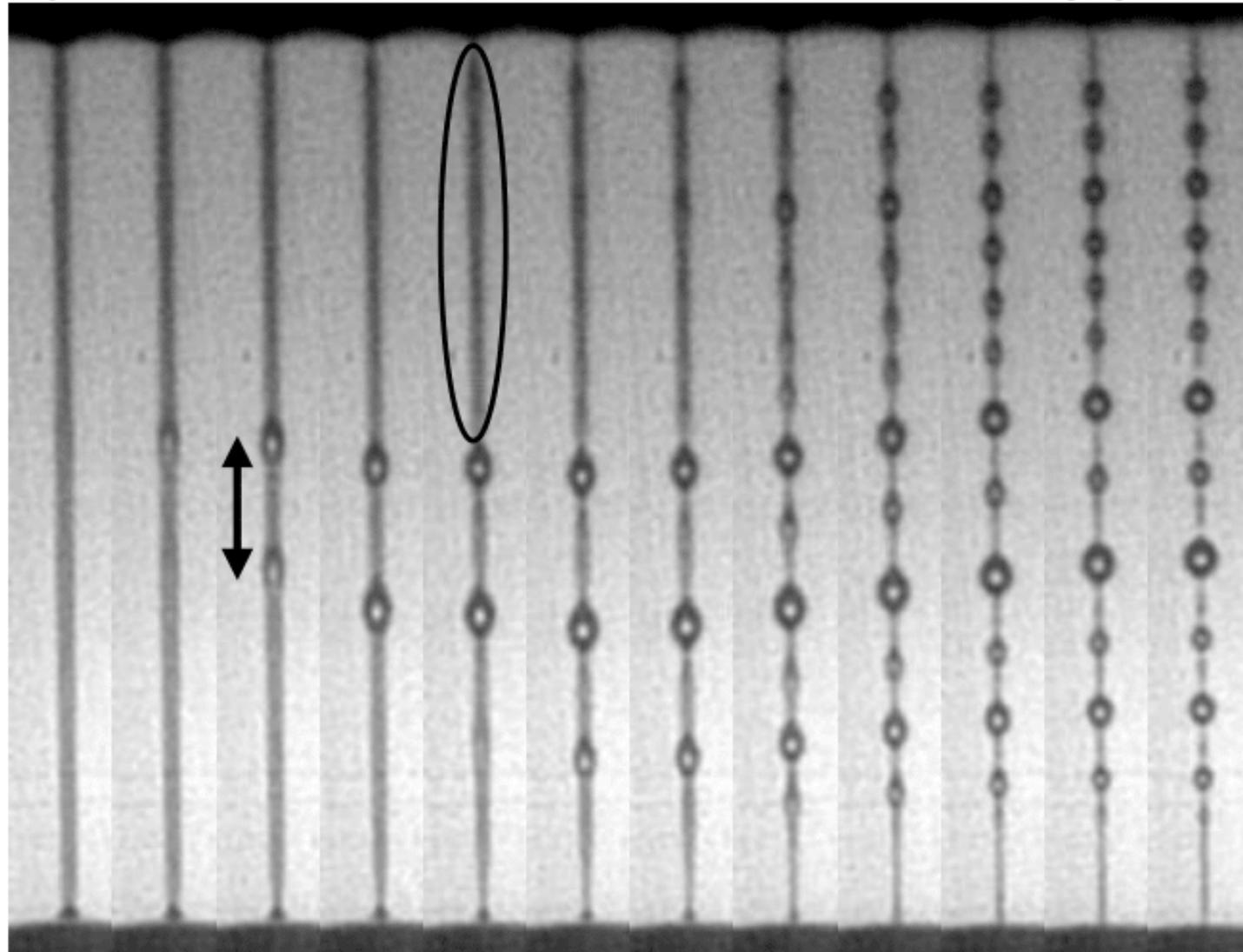
- **How long will it take?**

$$t = -\lambda \ln\left(\frac{G}{2}\right)$$

Trajectory of bubbles in the filament showing the internal flow (Sattler et al. 2008)
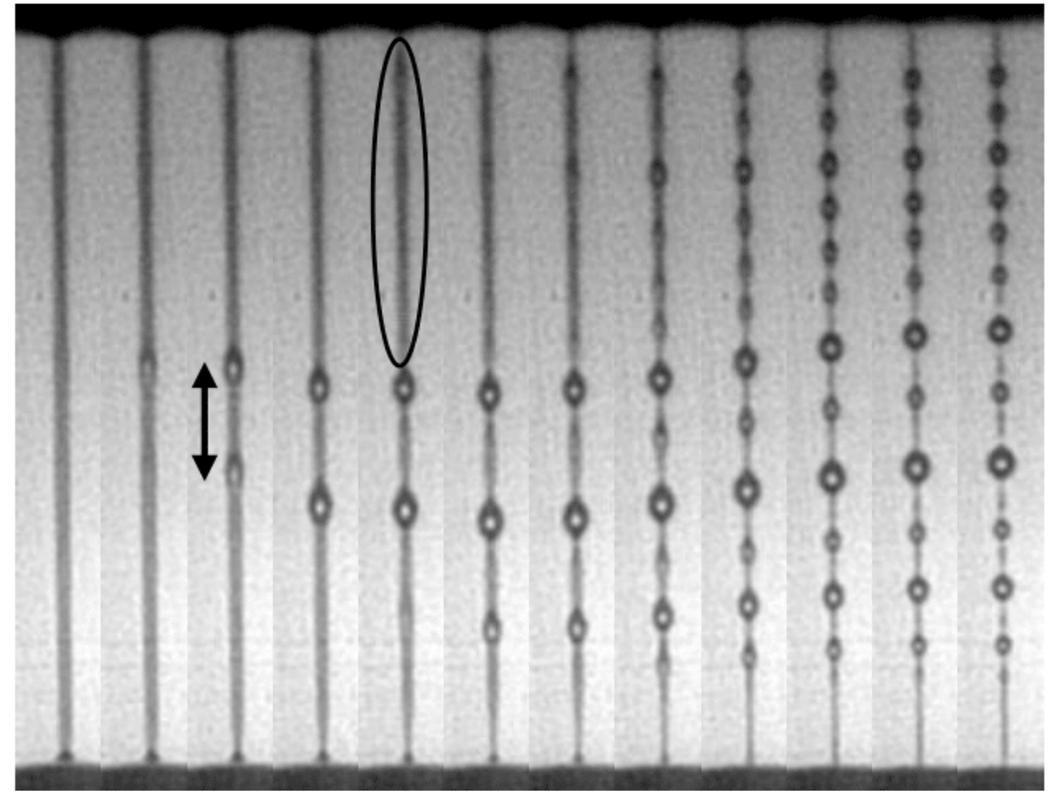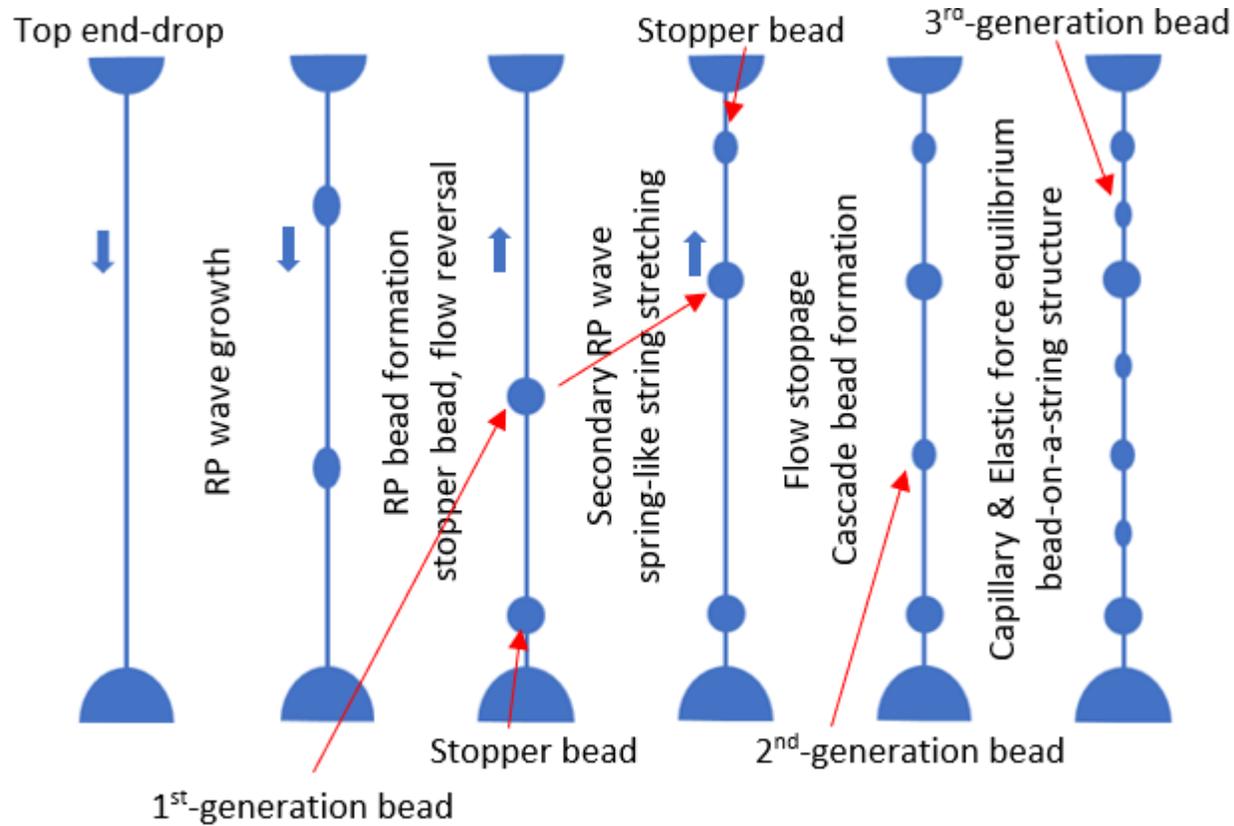
2. Formation of Beads-on-string Structure

$\Delta t \; (ms) \; = \;$ 0 0.5 1 1.5 2.2 2.5 2.8 3.5 4.2 4.8 5.5 6.2

2. Formation of Beads-on-string Structure

- **How does the beads form?**

2. Formation of Beads-on-string Structure

- **What are the sizes of beads formed? (The first-generation beads)**

- Filament generates surface waves due to Rayleigh-Plateau instability.

$$k_{max} = \left(3\boldsymbol{\mu_0} h_f \sqrt{\frac{2h_f}{\gamma\rho}} + 2h_f^2 \right)^{-1/2} , \omega_{max} = \frac{\sqrt{\gamma/2\rho h_f^3}}{2 + \frac{3\boldsymbol{\mu_0}}{\rho}\sqrt{2\rho/\gamma h_f}}$$

- The filament shown in the image has $2h_f = 10\mu m$ and the predicted wavelength is

$$\lambda_{RP} = \frac{2\pi}{k_{max}} \approx 12h_f = 60\mu m$$

- The arrow between the initial two bulges is $70\mu m$, which shows good agreement with theory.
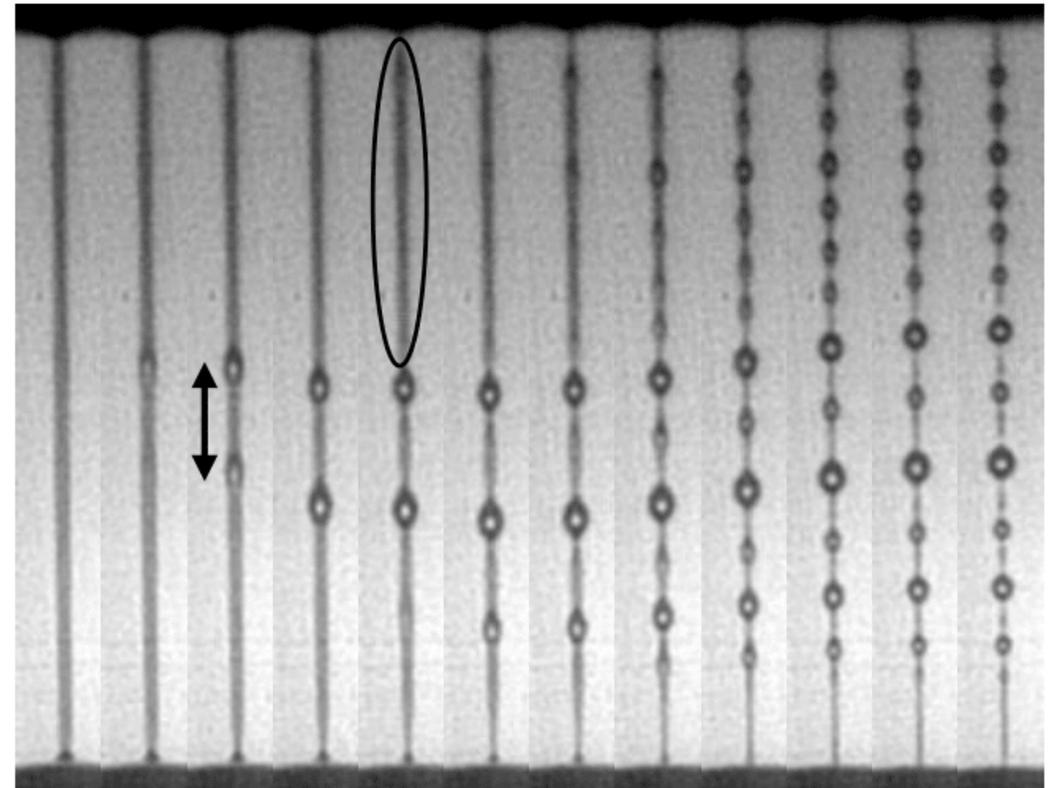
2. Formation of Beads-on-string Structure

- **What are the sizes of beads formed? (Higher generation beads)**

- The size of the first beads can be estimated as

$$d_1 = \sqrt[3]{6h_f^2 \lambda_{RP}}$$

- Higher generation beads have similar size as the filament thickness at the time they form
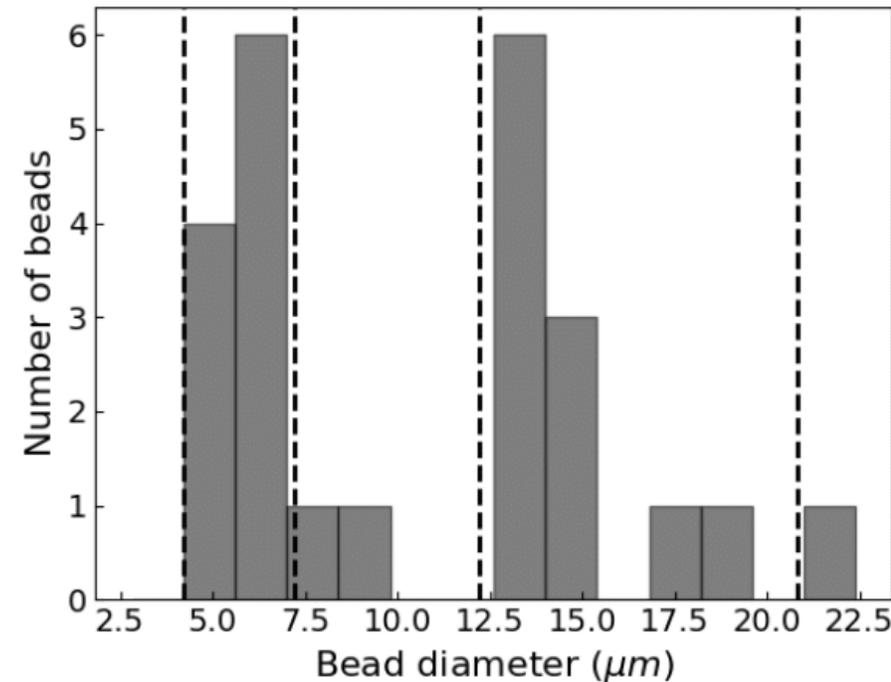
$$\frac{d_n}{d_{n+1}} = \frac{h(t = t_n)}{h(t = t_{n+1})}$$

- Considering the thinning rate of the filament

$$\frac{d_n}{d_{n+1}} = \sqrt{2 + \frac{3}{\sqrt{2}}Oh}$$

- For the case shown, $\frac{d_n}{d_{n+1}} \approx 1.7$

- $d_1 = 20\mu m, d_2 = 11.8\mu m, d_3 = 6.9\mu m, d_4 = 4\mu m$

2. Formation of Beads-on-string Structure

- **What are the number of beads with different sizes?**

- For n first-generation beads:
  - There are n+1 second generation beads
  - 2n+2 third generation beads
  - 2n+2 fourth (and higher) generation beads due to asymmetry of bead spacing
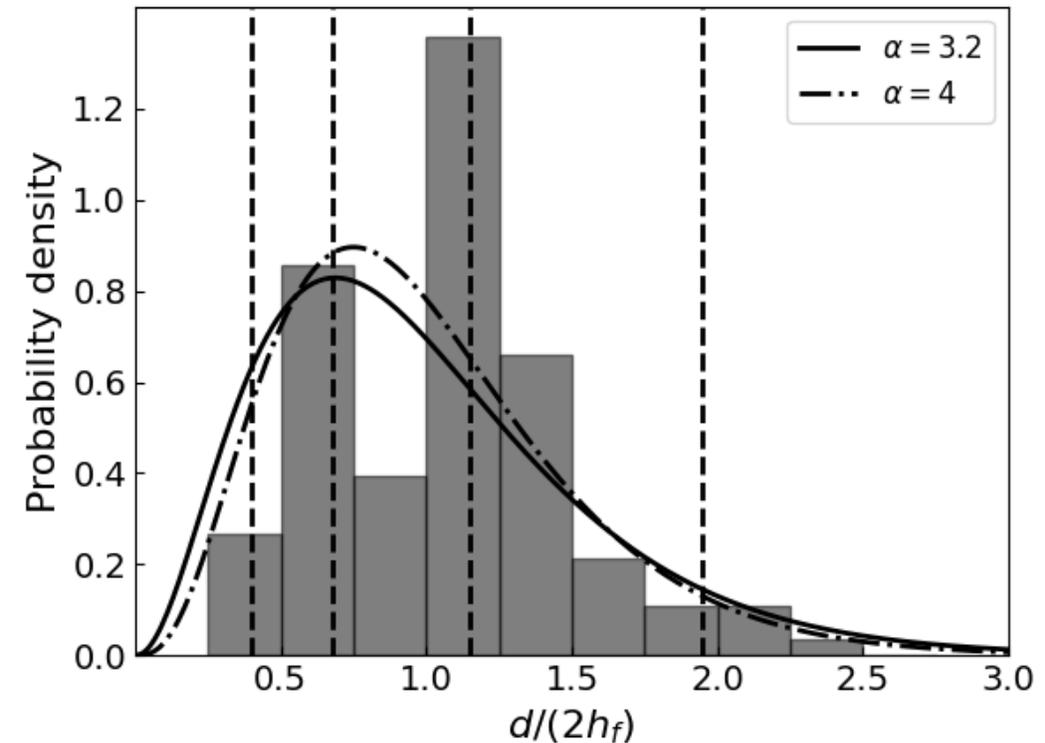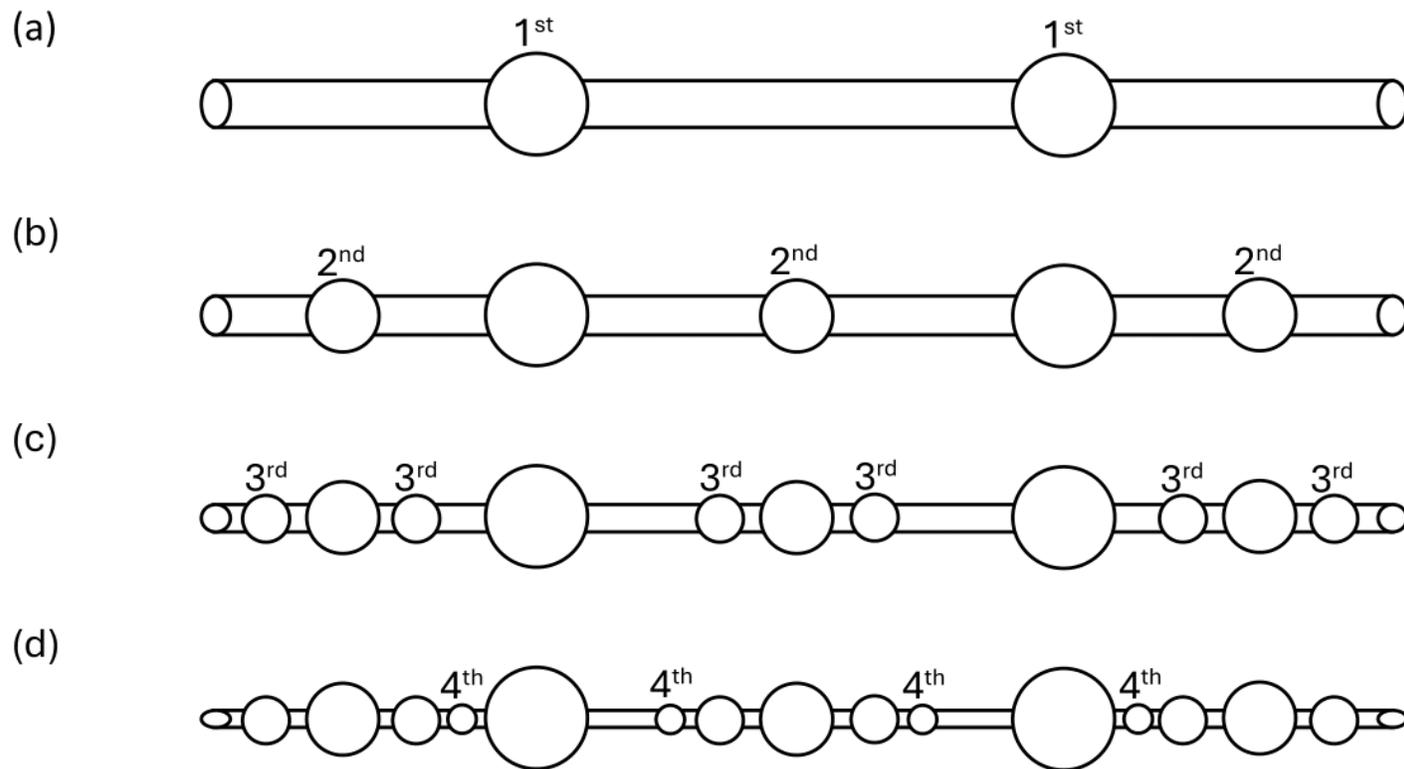
3. Coalesce and Breakup of Filament

$\Delta t(ms) =$ 0   2   4   5.5   6.3   8   9.6   10.5   11   12.6   13.5   14.6   15.8   17.8   19.1   21.3   22.1   23   23.5
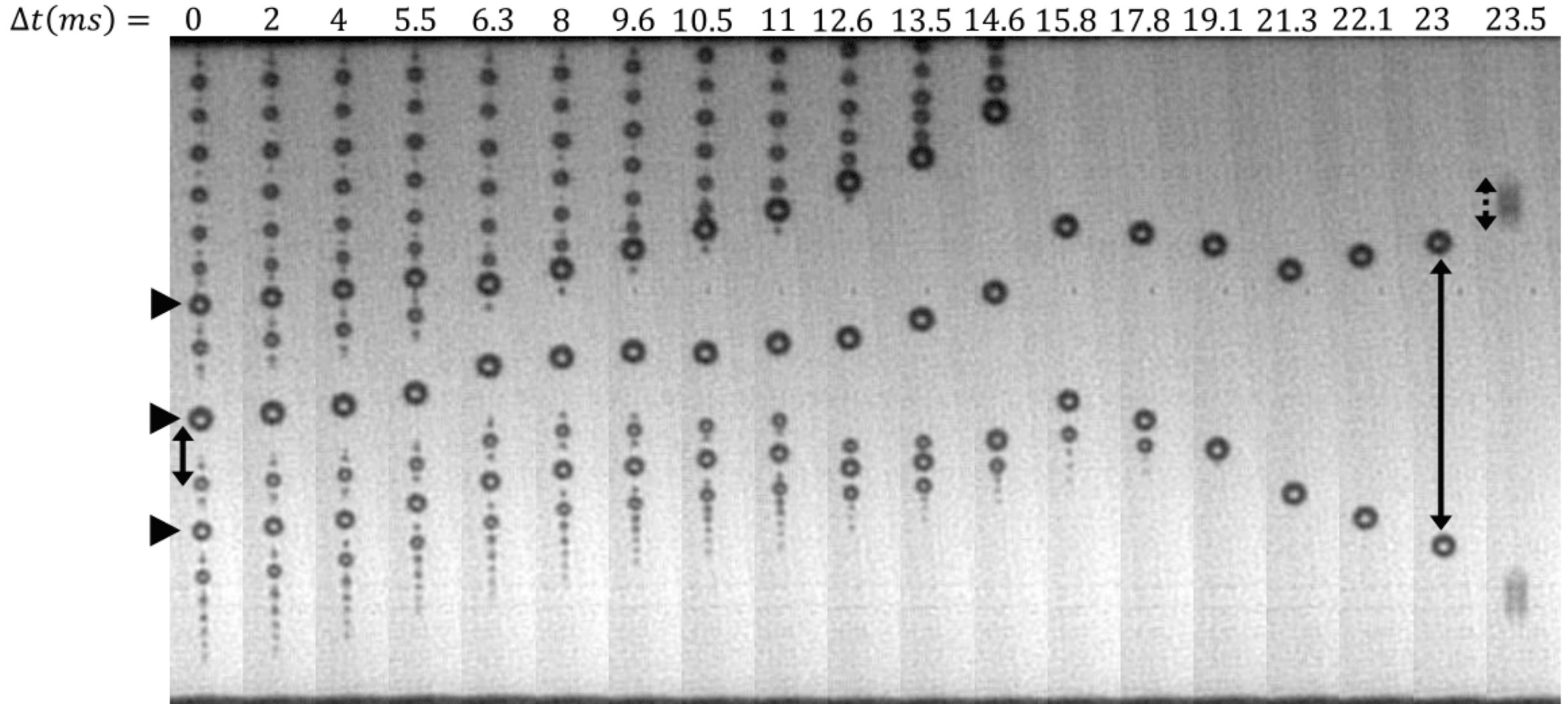
3. Coalesce and Breakup of Filament

- **How does the structure break into droplets?**

- Elastic force starts to relax after the beads-on-string structure formed.

- Smaller beads will move towards larger ones due to pressure difference
  - They can merge into end drops depends on the spacing

- This process stretches the string between beads and increase the strain energy in the string.

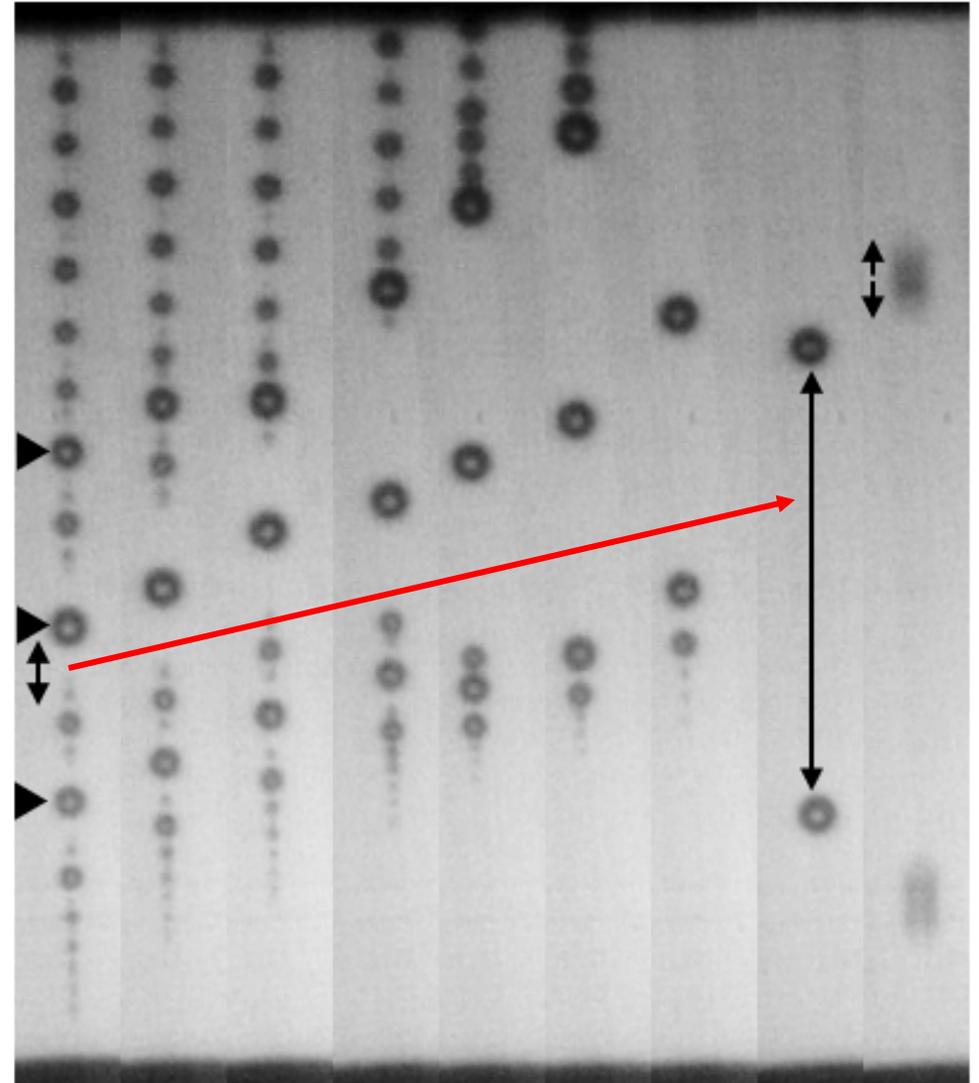# 3. Coalesce and Breakup of Filament

- **How does the structure break into droplets?**

- The string stretches exponentially until it breaks.
- The stretch rate is $\sim 1/(3\lambda)$
- The elastic energy and surface energy converts to the kinetic energy.

$$\Delta E_{elastic} + E_{surface} = \sum_{i=top,bottom} \frac{1}{2} m_i v_i^2$$

$$\Delta E_{elastic} = 2\pi \boldsymbol{\sigma_0} h_0^2 l_0 \left(exp\left(\frac{t_b}{3\lambda}\right) - 1 \right)$$

$$E_{surface} = \gamma 2\pi h_0 l_0 \, exp\left(\frac{t}{3\lambda}\right)$$

- This provides the stress and filament thickness at the breakup.
- At breakup, the filament thickness is $\sim 10 nm$ and tension force in the filament is $\sim 1 nN$.
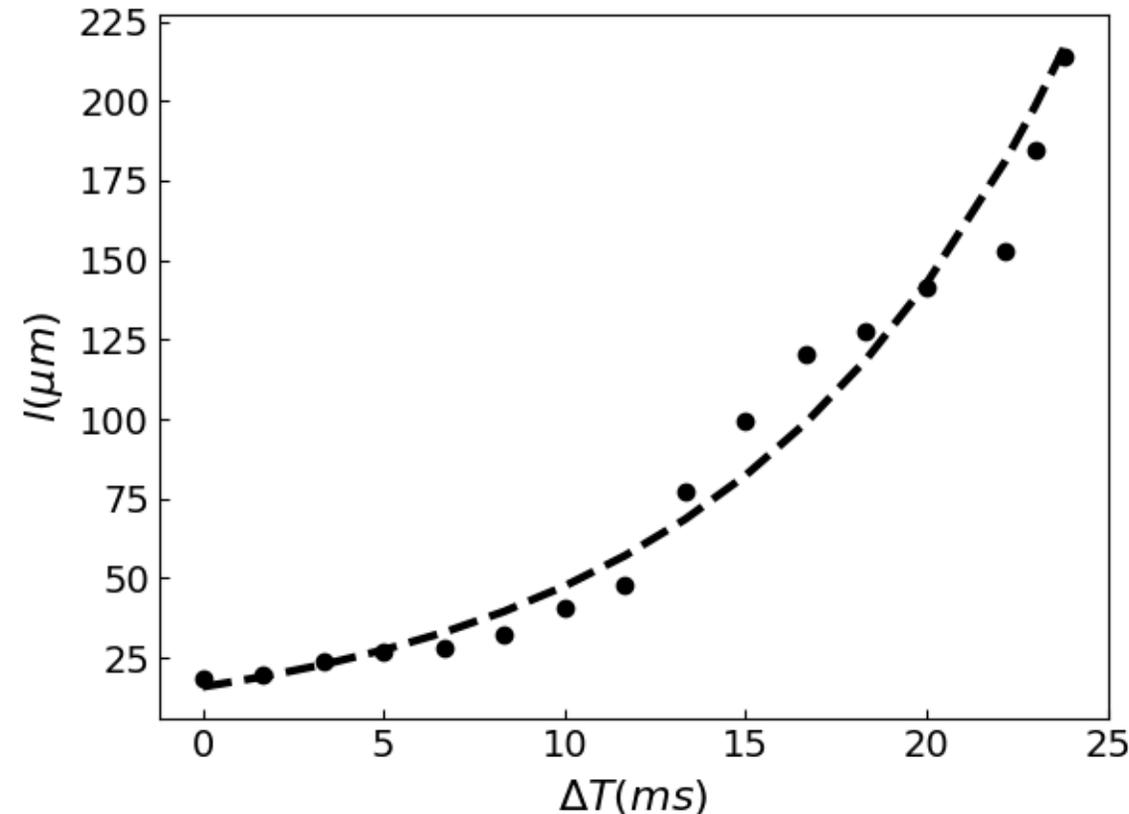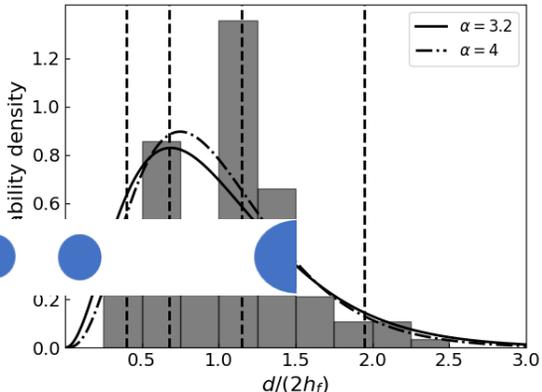
- The evolution of the viscoelastic filament: <u>initial thinning</u>, <u>formation of beads-on-string structure</u>, and <u>coalescence and breakup of beads-on-string structure</u>.
- The number distribution of bead sizes is a Gamma distribution with $\alpha = 3.2$.
- The string stretches at a rate of $\sim 1/(3\lambda)$ until it breaks. The tension force at breakup is of the order of $1nN$. An external force larger than this can result in earlier breakup and generate smaller droplets.

I. Filament stretches to a final filament thickness
$$\frac{h_f}{h_0} = \left(\frac{G}{2}\right)^{\frac{1}{3}}, \qquad G = \frac{\mu_p h_0}{\lambda \gamma}$$
II. Rayleigh-Plateau instability forms the first-genreation beads
$$k_{max} = \left(3\boldsymbol{\mu_0} h_f \sqrt{\frac{2h_f}{\gamma \rho} + 2h_f^2}\right)^{-1/2}$$
III. Nonlinear Formation of higher generation beads
$$d_1 = \sqrt[3]{6h_f^2 \lambda_{RP}}, \qquad \frac{d_n}{d_{n+1}} = \sqrt{2 + \frac{3}{\sqrt{2}} Oh}$$

Number distribution of bead sizes
IV. Coalescence and breakup of filament
Tension force $T \sim 1nN$
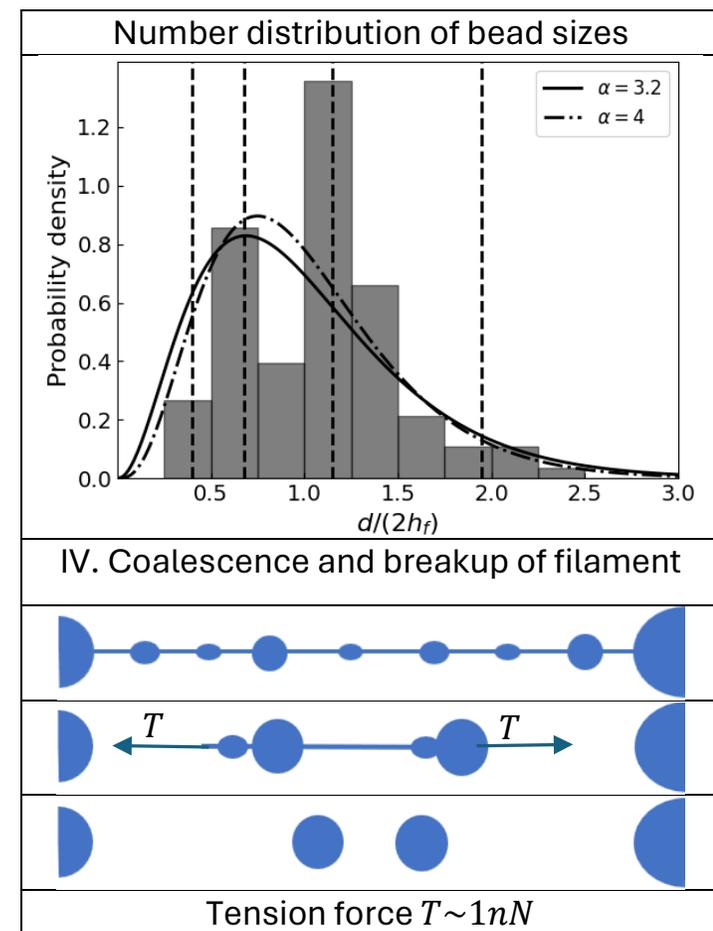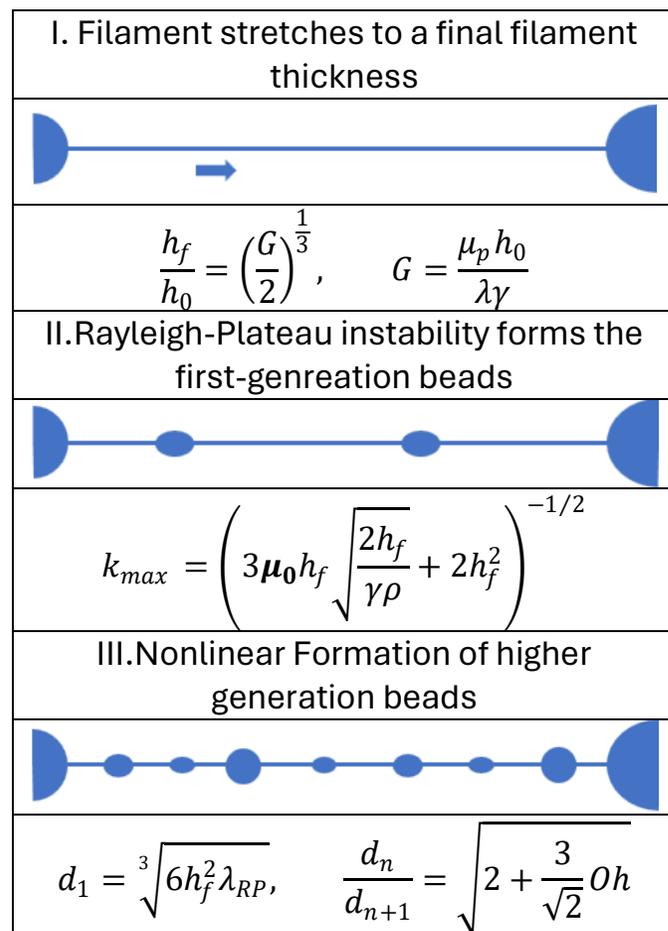
Conclusion

1. The evolution of the viscoelastic filament has three stages: <u>initial thinning,</u> <u>formation of beads-on-string structure</u>, and <u>coalescence and breakup of beads-on-string</u> <u>structure</u>.

2. For a filament with initial thickness $2h_0$, filament decreases until $h_f = (G/2)^{1/3}h_0$ such that surface waves will appear.

3. The size of the first beads is $d_1 = \sqrt[3]{6h_f^2 \lambda_{RP}}$,where $\lambda_{RP} = 2\pi \left(3\mu_0 h_f \sqrt{\frac{2h_f}{\gamma\rho}} + 2h_f^2\right)^{1/2}$ is the dominant RP wavelength.

4. The size of higher generation beads can be estimated by $d_n/d_{n+1} = \sqrt{2 + 3Oh/\sqrt{2}}$

5. The number distribution of bead sizes is a Gamma distribution with $\alpha = 3.2$.

6. The string stretches at a rate of $\sim 1/(3\lambda)$ until it breaks. The tension force at breakup is of the order of $1nN$. An external force larger than this can result in earlier breakup and generate smaller droplets.