Evolution and Breakup of Polymeric Filaments

Nasser Ashgriz and Siyu Chen

IFPRI — International Fine Particle Research Institute

Mechanical & Industrial Engineering — UNIVERSITY OF TORONTO
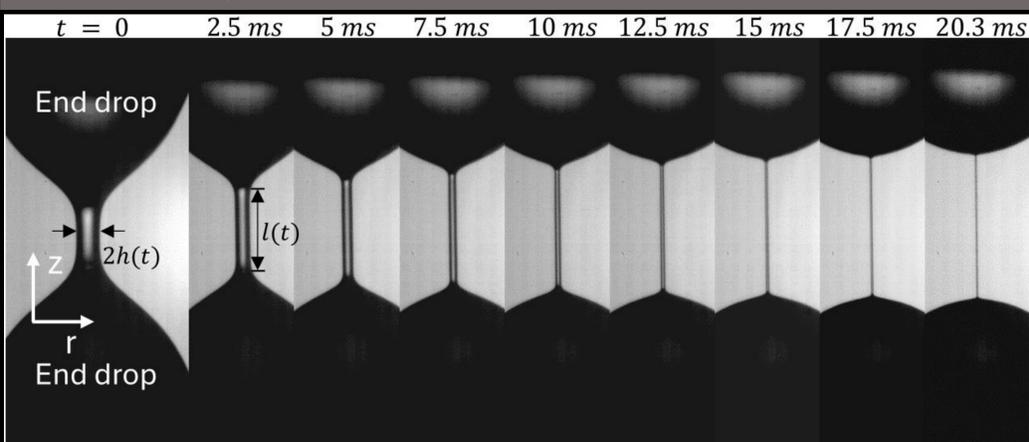
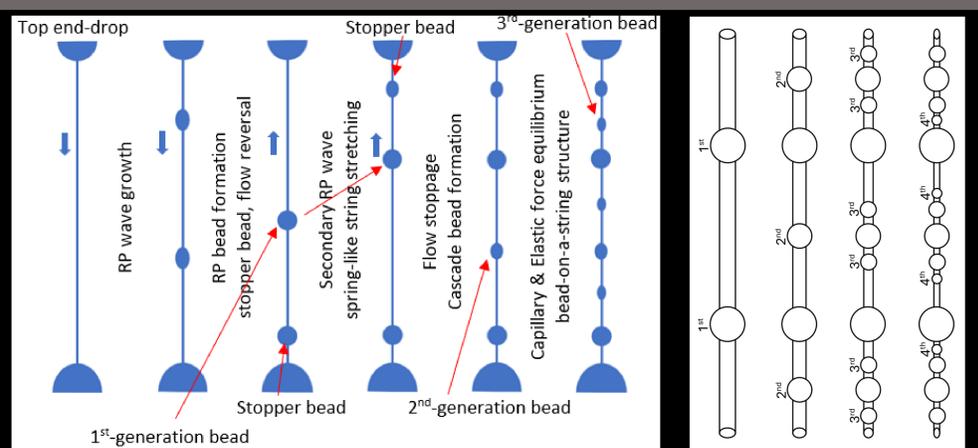## 1. Filament formation in polymeric sprays
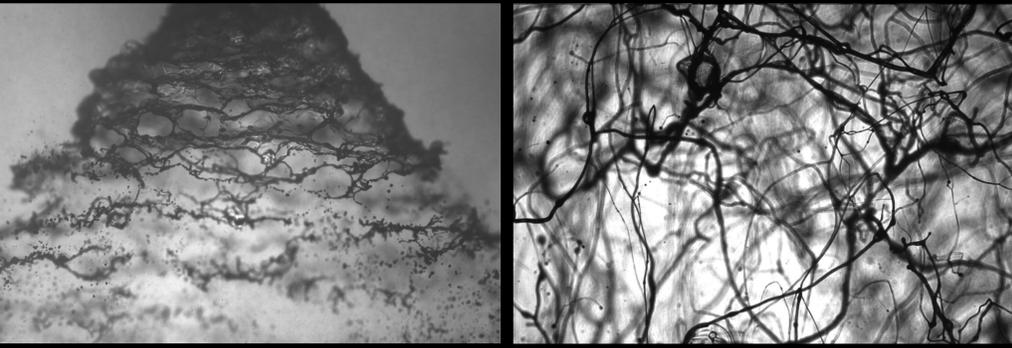
2. Objectives

1. Measure droplet size distribution in polymeric sprays.
2. Characterization and Modelling of filament breakup of polymeric fluids.
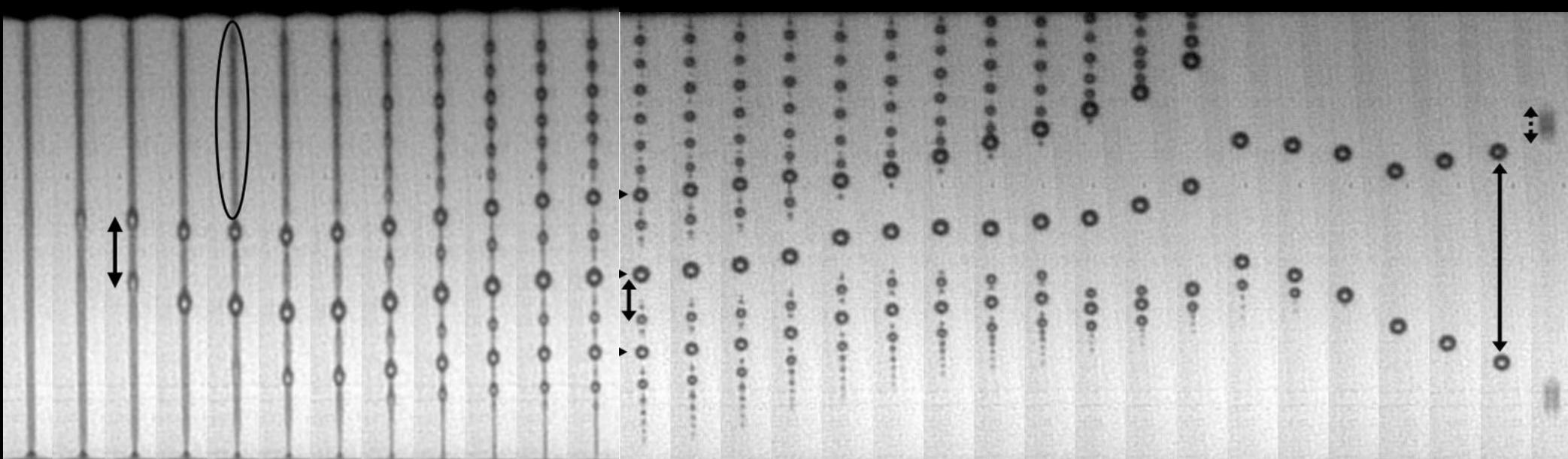3. Modelling the atomization of polymeric fluids.

3. Pulling experiment

$t = 0$ 2.5 ms   5 ms   7.5 ms   10 ms   12.5 ms   15 ms   17.5 ms   20.3 ms

End drop

$2h(t)$

$l(t)$

End drop

4. Number distribution model

Top end-drop — Stopper bead — 3rd-generation bead

RP wave growth — RP bead formation stopper bead, flow reversal — Secondary RP wave spring-like string stretching — Flow stoppage Cascade bead formation — Capillary & Elastic force equilibrium bead-on-a-string structure

1st-generation bead — Stopper bead — 2nd-generation bead

5. Beads-on-string structure formation, beads movement, and filament breakup

6. Model for final filament thickness

$$\frac{\partial h^2}{\partial t} + \frac{\partial (u h^2)}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} = -\frac{\gamma}{\rho}\frac{\partial \kappa}{\partial z} + \frac{3 v_s}{h^2}\frac{\partial}{\partial z}\left(h^2 \frac{\partial u}{\partial z}\right) + \frac{1}{h^2}\frac{\partial}{\partial z}\left(h^2 (\sigma_{zz} - \sigma_{rr})\right)$$

Assume a constant strain rate stretching filament $u(z,t) = \dot{\varepsilon} z$

Assume internal flow stops at the end of the thinning stage $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial z} = 0$

Solve for final filament thickness

$$\frac{h_f}{h_0} = \left(\frac{G}{2}\right)^{\frac{1}{3}}, \qquad G = \frac{\mu_p h_0}{\lambda \gamma}$$

7. Model for string breakup

$$\Delta E_{elastic} + E_{surface} = \sum_{i=top,bottom} \frac{1}{2} m_i v_i^2$$

$$\Delta E_{elastic} = \int_0^{t_b} \sigma \pi h^2 dl = \frac{2}{3}\pi \sigma_1 h_1^2 l_1 \left(exp\left(\frac{t_b}{\lambda}\right) - 1\right)$$

$$E_{surface} = \gamma 2\pi h_b l_b = \gamma 2\pi h_1 l_1 \, exp\left(\frac{t_b}{3\lambda}\right)$$

$$\sum_{i=top,bottom} \frac{1}{2} m_i v_i^2 = 2\pi\gamma h_1 l_1 \left(\frac{1}{3} exp\left(\frac{t_b}{\lambda}\right) - 1 + exp\left(\frac{t_b}{3\lambda}\right)\right)$$

8. Atomization model for polymeric sheets

- A polymeric liquid sheet is broken into filaments due to Kelvin-Helmholtz instability.
- Filaments are stretched to reach a final thickness and breakup into droplets according to the following model:

I. Filament stretches to a final filament thickness

$$\frac{h_f}{h_0} = \left(\frac{G}{2}\right)^{\frac{1}{3}}, \qquad G = \frac{\mu_p h_0}{\lambda \gamma}$$

II. Rayleigh-Plateau instability forms the first-generation beads

$$k_{max} = \left(3\boldsymbol{\mu_0} h_f \sqrt{\frac{2h_f}{\gamma \rho}} + 2h_f^2\right)^{-1/2}$$

III. Nonlinear Formation of higher generation beads

$$d_1 = \sqrt[3]{6h_f^2 \lambda_{RP}}, \qquad \frac{d_n}{d_{n+1}} = \sqrt{2 + \frac{3}{\sqrt{2}} Oh}$$

Number distribution of bead sizes

IV. Coalescence and breakup of filament

Tension force $T \sim 1nN$