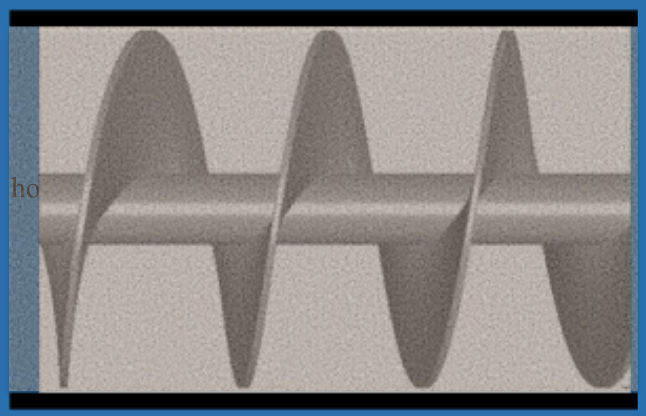


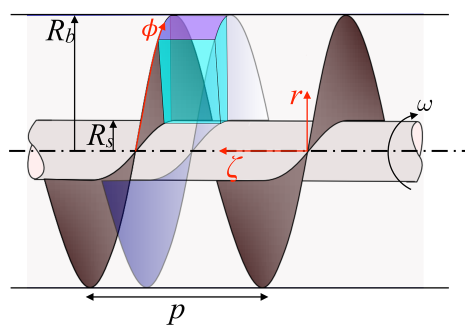
Precision powder feeding

Prabhu Nott

Sanyogita, Gautam Vatsa

Indian Institute of Science, Bangalore

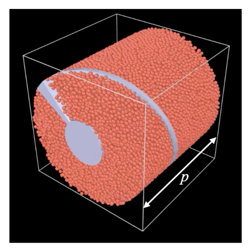
Simple macroscopic balance to determine feed rate



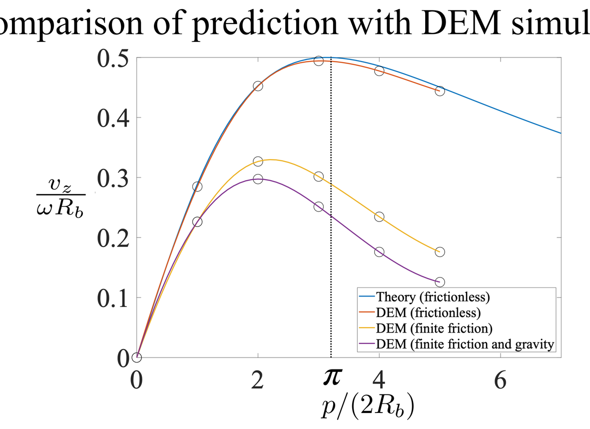
Force balance on a differential wedge in the helical direction

For *frictionless* screw and shaft, force balance gives zero shear stress $\sigma_{r\phi}$ on all surfaces of constant r. Assuming flow is plug-like, the feed rate can be readily shown to be

$$v_z = \omega R_b \frac{\pi\, p/(2R_b)}{\pi^2 + p^2/(2R_b)^2}$$



Comparison of prediction with DEM simulations



- Excellent agreement for frictionless screw without gravity
- Same qualitative trend for frictional surfaces, and with gravity
- Optimum value of p/R_b at which feed rate is maximum

Continuum model for slow granular flow

Classical critical state plasticity model:

$$\boldsymbol{\sigma} = -p\boldsymbol{\delta} + \frac{2\mu p_c \hat{Y}}{\dot\gamma}\mathbf{D}', \quad p = p_c\left(1 - \frac{\mu_b}{\dot\gamma}\nabla\cdot\mathbf{u}\right)$$

μ – shear friction
μ_b – volumetric friction

$$\dot\gamma = (2\mathbf{D}':\mathbf{D}')^{1/2}, \quad p_c = \Pi(\phi), \quad \hat{Y} = f(p/p_c)$$

critical state pressure

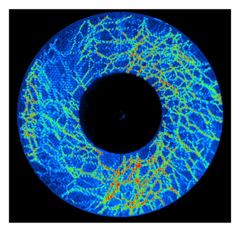
Model suffers from several deficiencies:
- Mathematically ill-posed
- Kinematically indeterminate
- Does not incorporate dilatancy

Previous extensions of classical plasticity address kinematic indeterminacy, but assume medium to be incompressible – do not incorporate dilatancy.

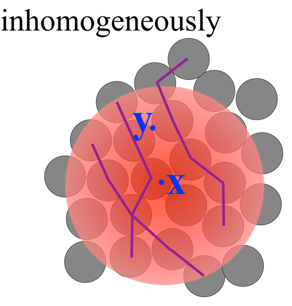
A systematic non-local extension of classical plasticity

Dsouza & Nott (*J. Fluid Mech.* 2020)

Stress transmitted inhomogeneously



Howell *et al* (2000)



Local flow rule $D_{ij}(\mathbf{x}) = \dot\lambda \dfrac{\partial F(\mathbf{x})}{\partial \sigma_{ji}}$

is replaced by a nonlocal flow rule,

$$D_{ij} = \int_{\mathbf{y}} \dot\lambda \frac{\partial F(\mathbf{y})}{\partial \sigma_{ij}} \psi(\mathbf{y}-\mathbf{x})\,\mathrm{d}\mathbf{y}$$

Similarly, the local relation for density,

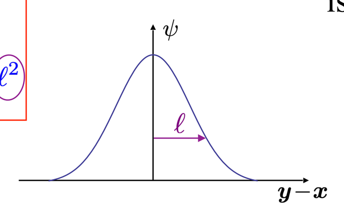
$$\phi = \Pi^{-1}(p_c)$$

is replaced by a nonlocal relation

$$\phi = \int_{\mathbf{y}} \Pi^{-1}(p_c(\mathbf{y}))\,\psi(\mathbf{y}-\mathbf{x})\,\mathrm{d}\mathbf{y}$$

weight function $\psi(\mathbf{y}-\mathbf{x})$ has properties:

$$\int_{\mathbf{y}} \psi(\mathbf{y}-\mathbf{x})\,d\mathbf{y} = 1$$
$$\int_{\mathbf{y}} \psi(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{x})^2 d\mathbf{y} = \ell^2$$

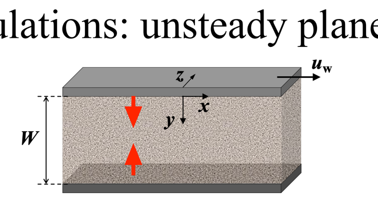


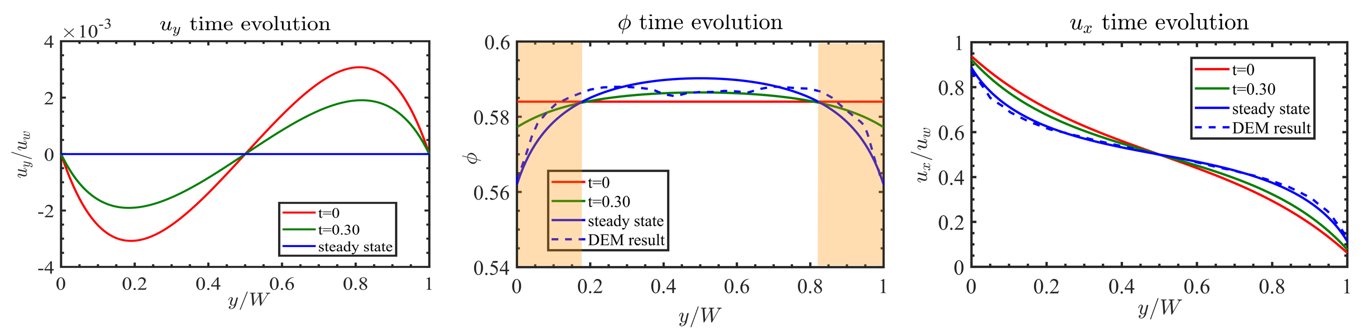
Using the extended von Neumann yield condition and after some algebra, we get …

A non-local model that resolves kinematic indeterminacy, incorporates dilatancy, and is unconditionally well posed

$$\boldsymbol{\sigma} = -p\boldsymbol{\delta} + \frac{2\mu}{\dot\gamma}\left(p_c\,\mathbf{D}' - \ell^2\,\Pi\,\nabla^2\mathbf{D}'\right), \quad p = p_c\left(1 - \frac{\mu_b}{\dot\gamma}\nabla\cdot\mathbf{u}\right) - \ell^2\,\Pi\,\frac{\mu_b}{\dot\gamma}\nabla^2\nabla\cdot\mathbf{u}$$
$$p_c = \Pi - \ell^2\frac{\mathrm{d}\Pi}{\mathrm{d}\phi}\nabla^2\phi,$$

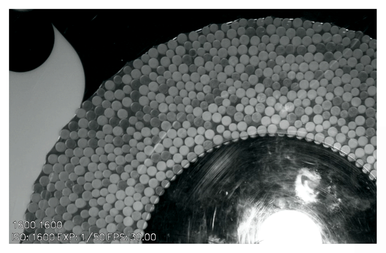
Validation against DEM simulations: unsteady plane shear in the absence of gravity

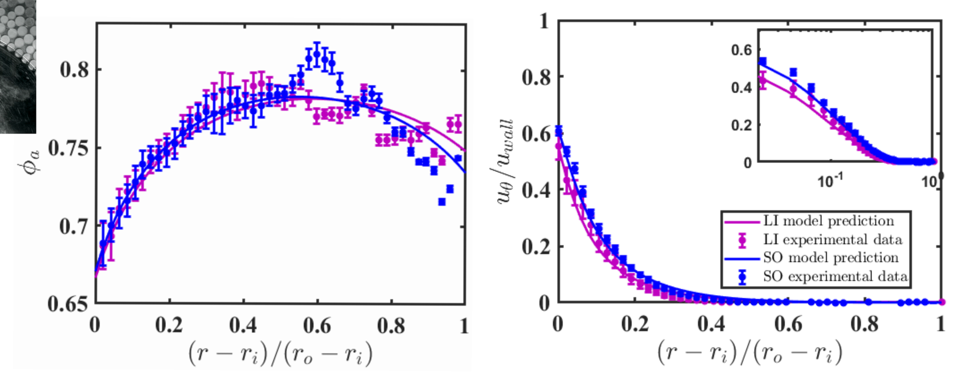




Excellent agreement with DEM results at steady state

Experimental validation in a 2D Couette device – IISc-NCSU collaboration



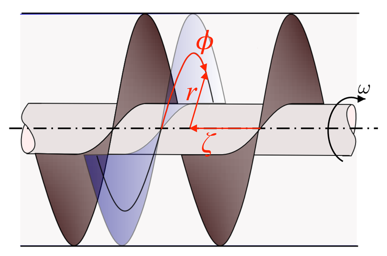


Strong coupling between ϕ and u_θ profiles

Excellent agreement of model predictions with experimental data

See IISc-NCSU collaboration poster for more details and results

Application of the nonlocal model to the screw feeder



Equations of motion in non-orthogonal helical coordinates (r, ϕ, ζ)

r momentum balance

$$\frac{\partial\sigma^{rr}}{\partial r} + \frac{1}{\sqrt{r^2+\frac{P^2}{(2\pi)^2}}}\frac{\partial\sigma^{\phi r}}{\partial\phi} + \frac{\partial\sigma^{\zeta r}}{\partial\zeta} + \frac{\sigma^{rr}}{r} - \frac{r}{r^2+\frac{P^2}{(2\pi)^2}}\sigma^{\phi\phi} = 0$$

ϕ momentum balance

For fully developed flow, $\partial\xi/\partial\phi = 0$ (for all ξ)

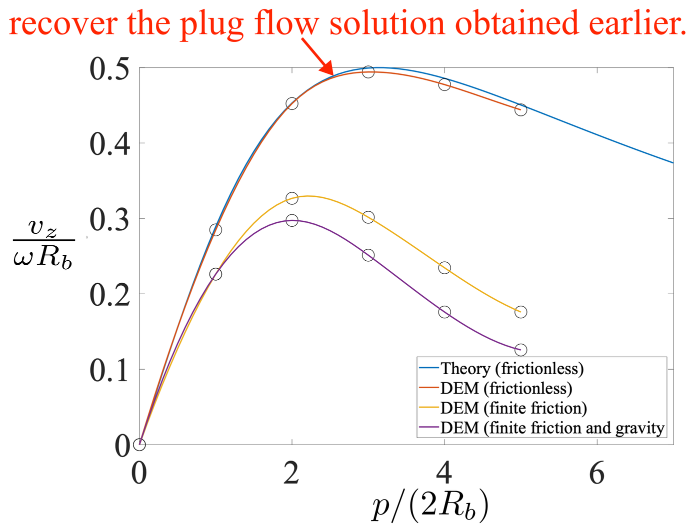
$$\frac{\partial}{\partial r}\left(\frac{\sigma^{r\phi}}{\sqrt{r^2+\frac{P^2}{(2\pi)^2}}}\right) + \frac{1}{r^2+\frac{P^2}{(2\pi)^2}}\frac{\partial\sigma^{\phi\phi}}{\partial\phi} + \frac{1}{\sqrt{r^2+\frac{P^2}{(2\pi)^2}}}\frac{\partial\sigma^{\zeta\phi}}{\partial\zeta} + \frac{3\sigma^{r\phi}}{r\sqrt{r^2+\frac{P^2}{(2\pi)^2}}} = 0$$

ζ momentum balance

$$\frac{\partial\sigma^{r\zeta}}{\partial r} + \frac{1}{\sqrt{r^2+\frac{P^2}{(2\pi)^2}}}\frac{\partial\sigma^{\phi\zeta}}{\partial\phi} + \frac{\partial\sigma^{\zeta\zeta}}{\partial\zeta} + \frac{\sigma^{r\zeta}}{r} - 2\left(\frac{P}{2\pi r}\right)\frac{\sigma^{r\phi}}{\sqrt{r^2+\frac{P^2}{(2\pi)^2}}} = 0$$

Flow fully developed when feeder is fully filled and gravity is absent: $u_r = 0$, $\partial u_\phi/\partial\phi = 0$. Therefore $u_\phi = u_\phi(r,\zeta)$

For frictionless screw and shaft u_ϕ = constant: we recover the plug flow solution obtained earlier.



The model also yields the stress on the barrel,

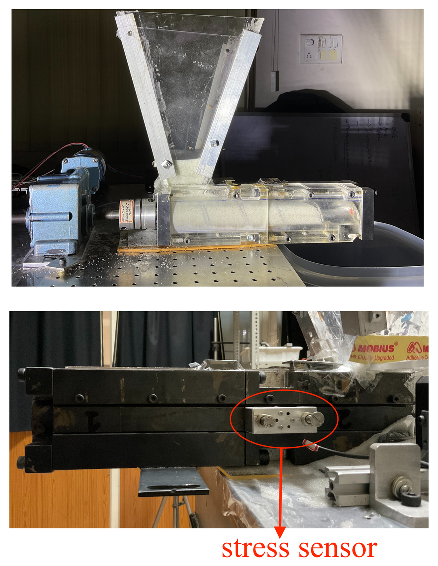
$$\sigma_{rr} = \sigma_{rr0}e^{-K\zeta}$$

Exponential decay has the same physical origin as the Janssen saturation of the stress in a silo

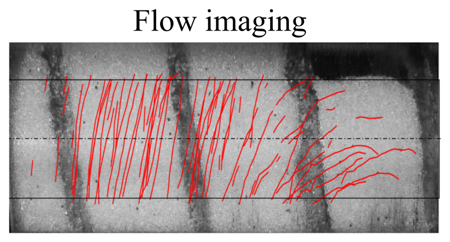
Solutions for frictional screw & shaft involve solution of PDEs – in progress

Experiments: screw feeder assembly

- Transparent front face for imaging
- Stress measured on back face

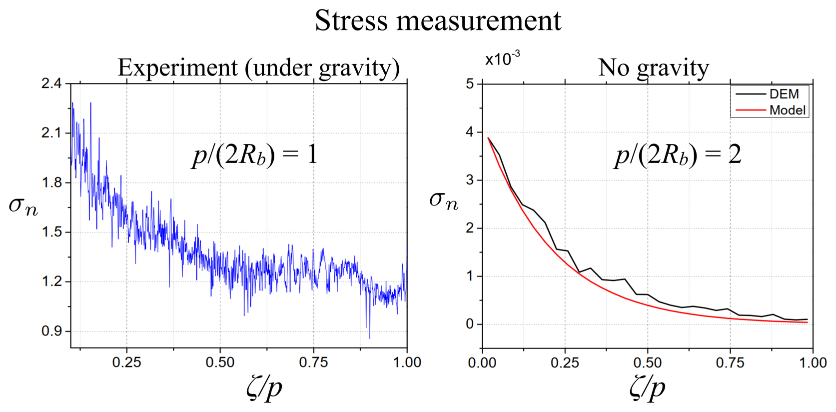


Flow imaging



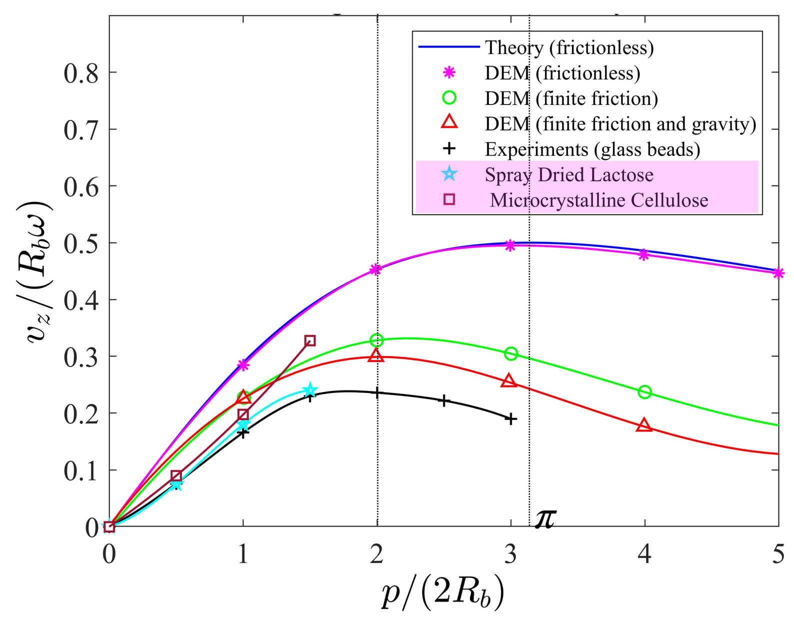
Combination of helical and axial flow, with considerable slip at screw

Stress measurement



Experiments too show the exponential decay in the stress with ζ, as predicted by the model

Feed rate versus $p/(2R_b)$ for cohesionless glass beads and cohesive powders



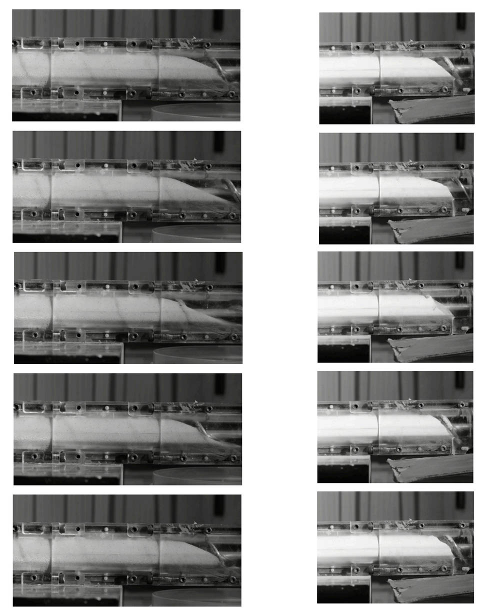
Spray dried lactose

Microcrystalline cellulose

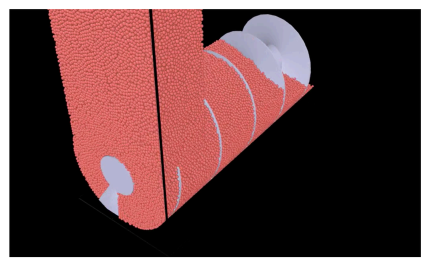
Experiments in qualitative agreement with model: feed rate maximum at optimum value of $p/(2R_b)$

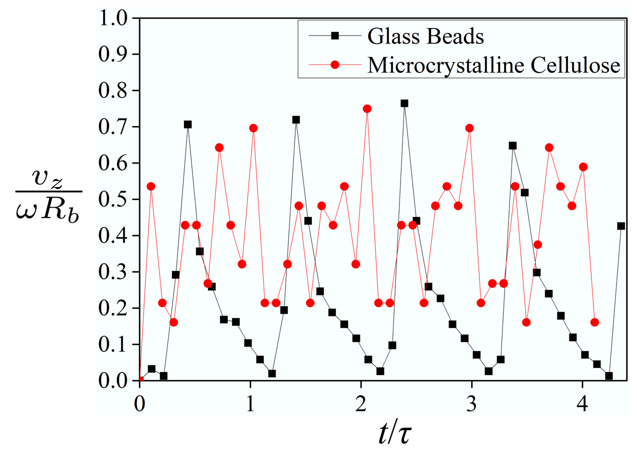
Feed rate fluctuations

Snapshots of the free surface at the exit



glass beads | microcrystalline cellulose



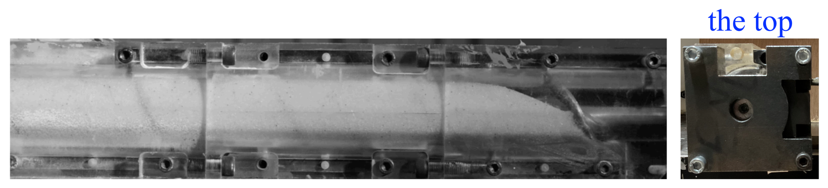


(τ – period of revolution)

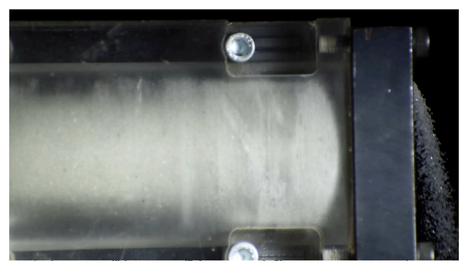
Fluctuations caused by variation of free surface slope at the exit. Periodic for glass beads and erratic for microcrystalline cellulose.

Changing the location of the exit can mitigate feed rate fluctuations

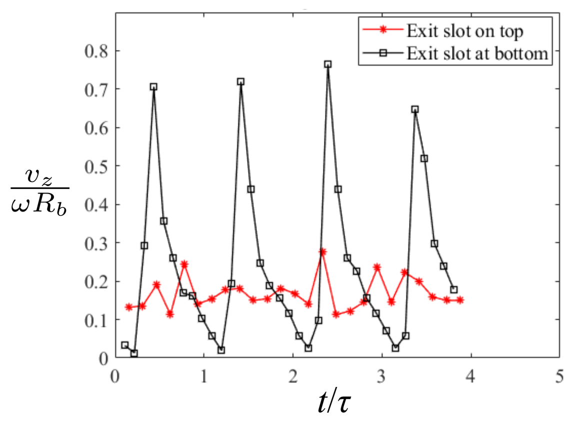
Exit slot at the top



Exit slot at the bottom



Fully filled when exit slot at the top



Conclusions

- Simple model predicts maximum flow rate at optimum value of $p/(2R_b)$. Prediction validated by DEM and experiments.
- Non-local rheological model shows promise. Validated for frictionless screw & shaft. Must be solved for frictional surfaces, partial fill, unsteady flow.
- Experiments conducted for non-cohesive glass beads and cohesive powders. Validate predictions of non-local model and DEM simulations.
- Feed rate fluctuations due to periodic or erratic changes in free surface slope at exit. Fluctuations can be mitigated by proper exit design.
- DEM simulations useful to validate models and motivate experiments.
- Future work: Scaling relations for mean and fluctuations of feed rate; Develop rheological model for cohesive powders that incorporates agglomeration; Mitigate feed rate fluctuations at hopper-feeder junction.