Precision powder feeding

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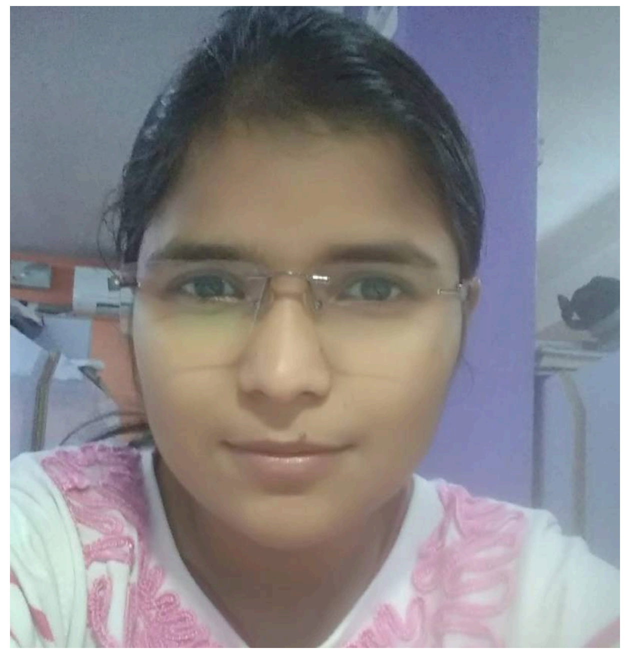

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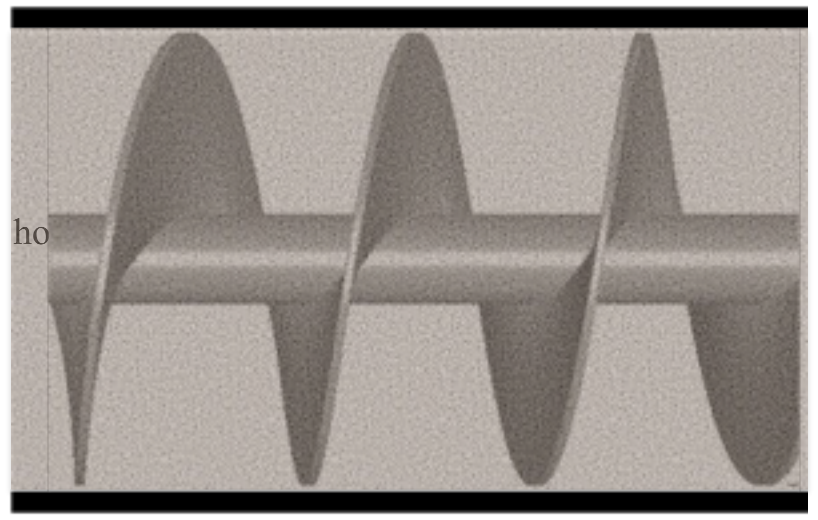
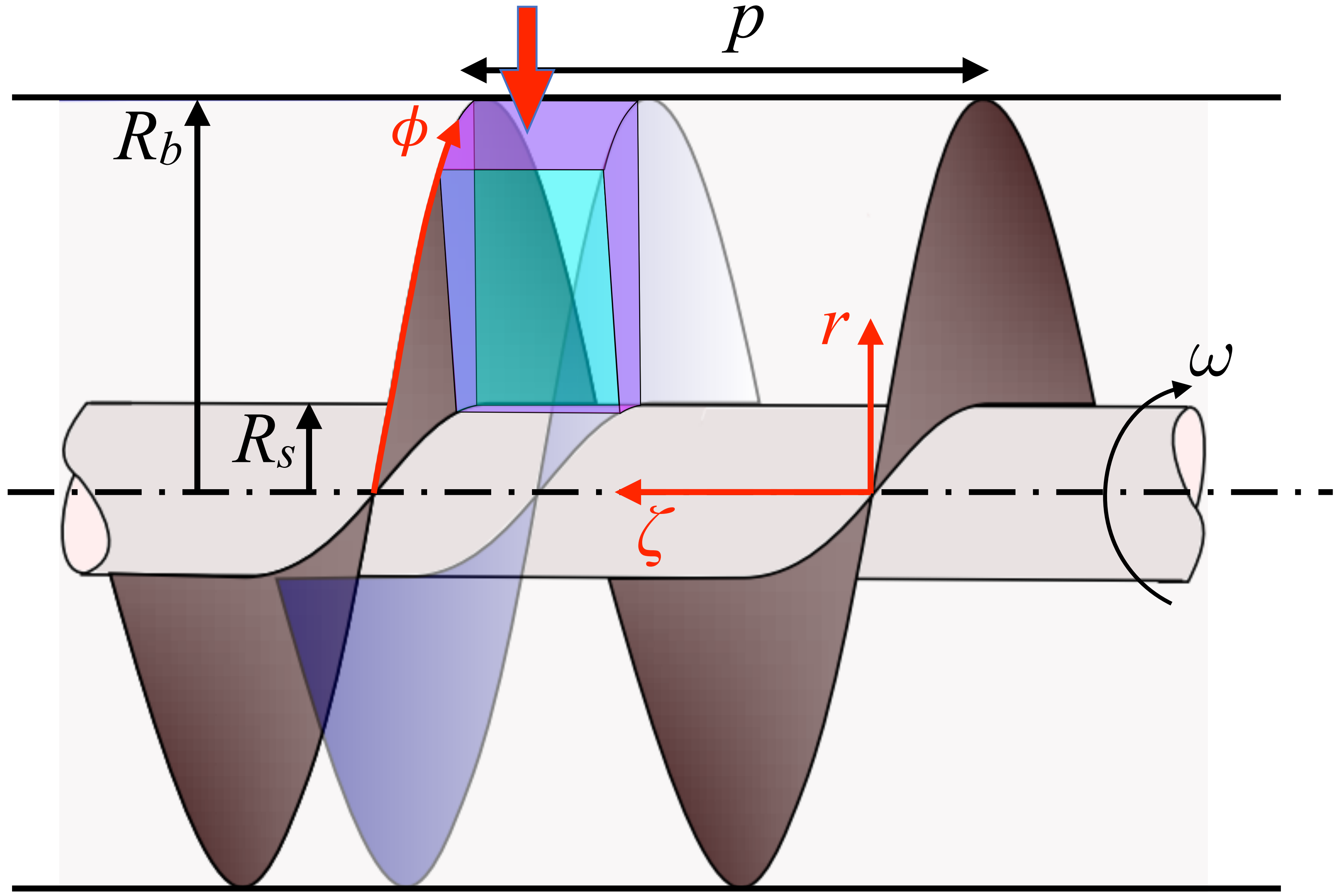


Objectives

- Develop a theoretical model of flow through screw feeders
- Test model predictions against experimental measurements
- Conduct DEM simulations to validate and refine theory and guide further experiments.

Final year of project

Simple macroscopic balance to determine feed rate

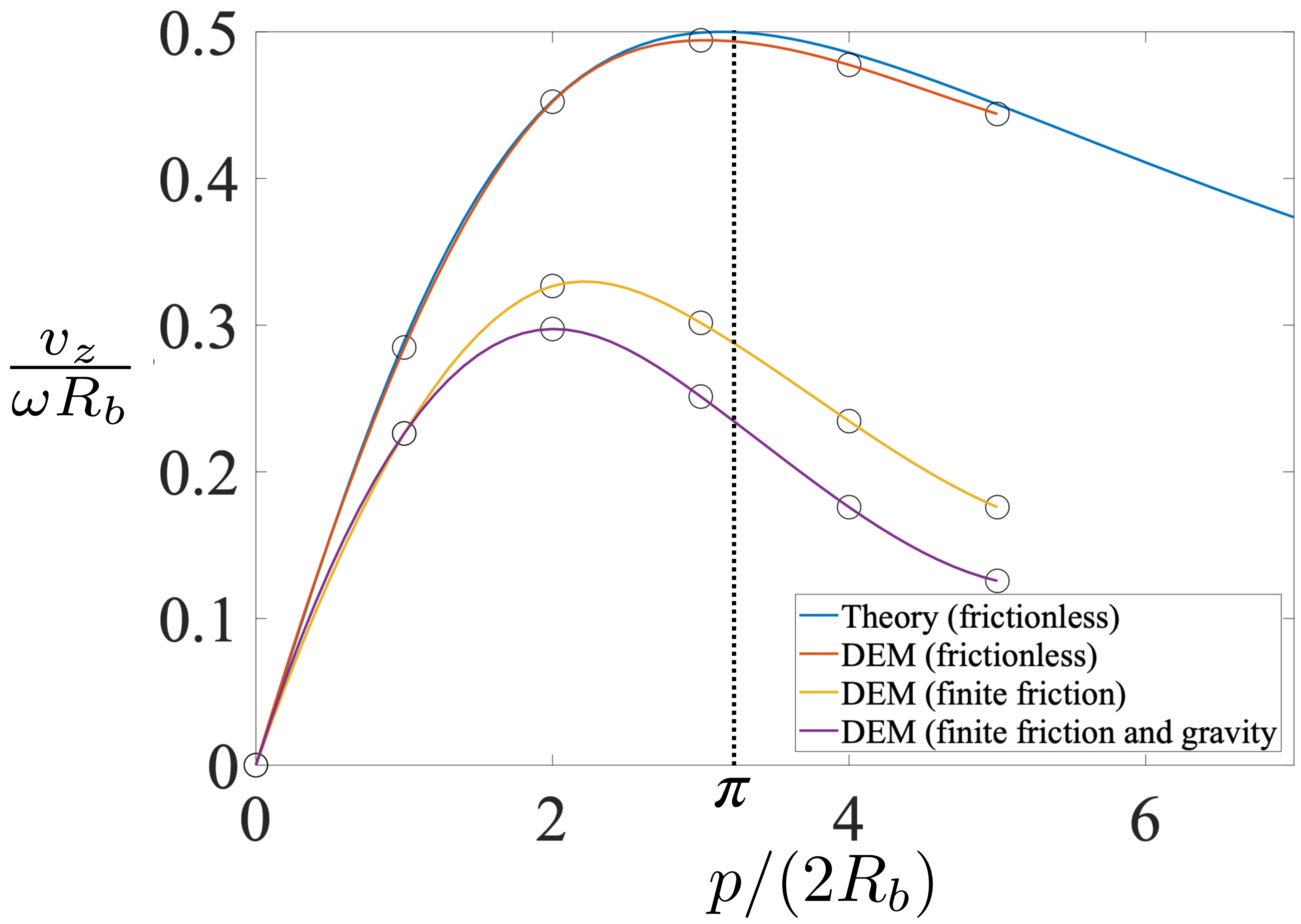


Force balance on a differential wedge in ϕ (helical) direction

For *frictionless* screw and shaft, force balance gives zero shear stress $\sigma_{r\phi}$ on all surfaces of constant r. Assuming flow is plug-like, the feed rate can be readily shown to be

$$v_z = \omega R_b \frac{\pi\, p/(2R_b)}{\pi^2 + p^2/(2R_b)^2}$$

Comparison of prediction with DEM simulations



- Excellent agreement for frictionless shaft and screw without gravity
- Same qualitative trend for frictional surfaces, and with gravity
- An optimum value of p/R_b at which discharge rate is maximum

Continuum models for slow granular flow

Classical plasticity – ‘critical state plasticity’ model

Dsouza & Nott (*J. Fluid Mech.* 2020)

$$\boldsymbol{\sigma} = -p\,\boldsymbol{\delta} + \underbrace{\frac{2\mu\, p_c \hat{Y}}{\dot{\gamma}}}_{\text{`viscosity'}} \boldsymbol{D}', \qquad p = p_c\left(1 - \frac{\mu_b}{\dot{\gamma}} \boldsymbol{\nabla\cdot u}\right)$$

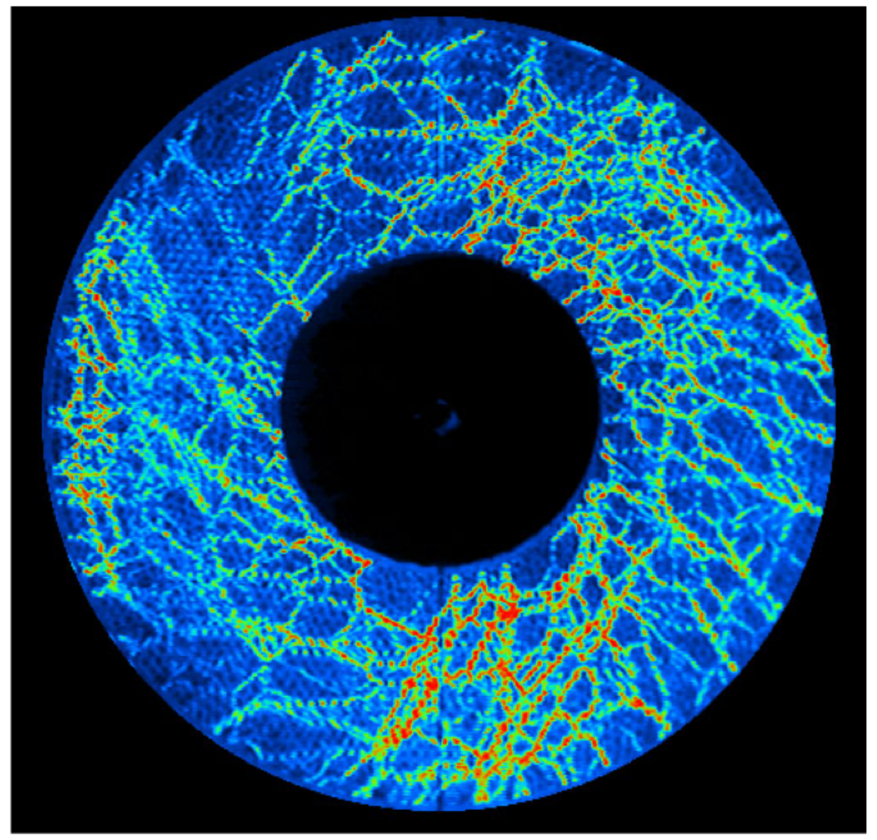
μ – shear friction
μ_b – volumetric friction

$$\dot{\gamma} = (2\boldsymbol{D}'\!:\!\boldsymbol{D}')^{1/2}, \qquad p_c = \Pi(\phi), \qquad \hat{Y} = f(p/p_c)$$

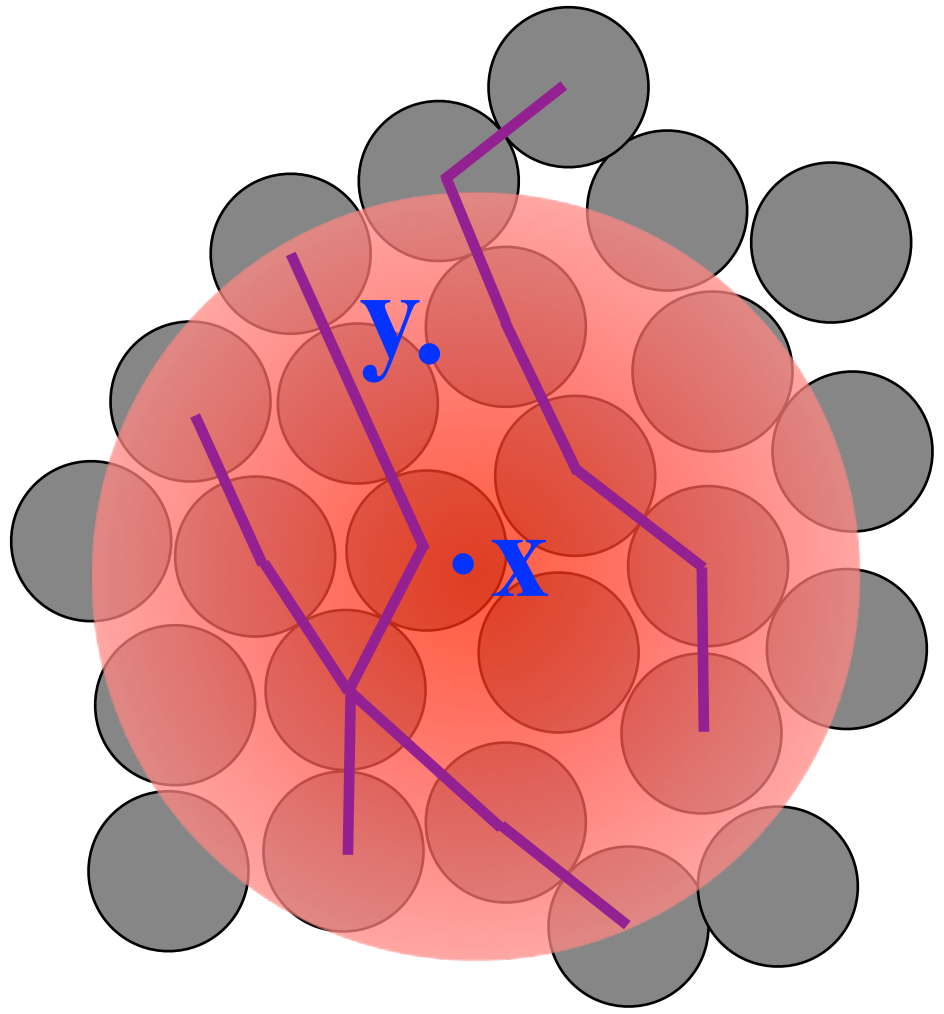
critical state pressure

- Mathematically ill-posed
- Kinematically indeterminate
- Does not incorporate dilatancy

A systematic non-local extension of classical plasticity



Howell *et al* (2000)



Dsouza & Nott (*J. Fluid Mech.* 2020)

Local flow rule $D_{ij}(\boldsymbol{x}) = \dot{\lambda}\frac{\partial F(\boldsymbol{x})}{\partial \sigma_{ji}}$ F: Yield function

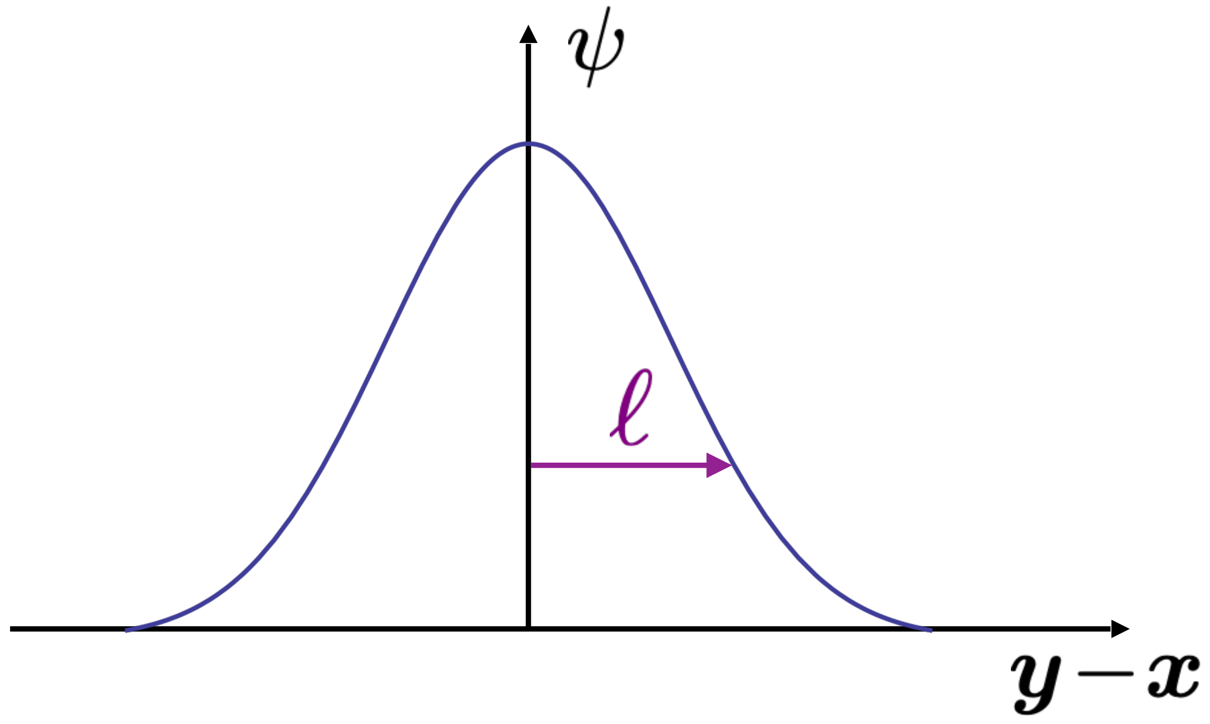
is replaced by a nonlocal flow rule,

$$D_{ij} = \int_y \dot{\lambda}\frac{\partial F(\boldsymbol{y})}{\partial \sigma_{ij}}\psi(\boldsymbol{y}-\boldsymbol{x})\,\mathrm{d}\boldsymbol{y}$$

$$\int_y \psi(\boldsymbol{y}-\boldsymbol{x}))d\boldsymbol{y} = 1$$

$$\int_y \psi(\boldsymbol{y}-\boldsymbol{x}))(\boldsymbol{y}-\boldsymbol{x})^2 d\boldsymbol{y} = \ell^2$$

where $\psi(\mathbf{y}-\mathbf{x})$ is a weight function with these properties.

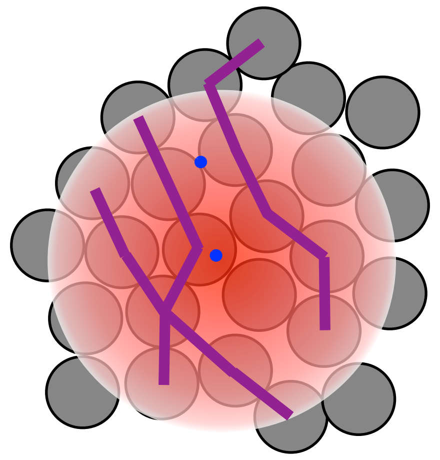


Similarly, the local relation for density, $\phi = \Pi^{-1}(p_c)$ is replaced by a nonlocal relation

$$\phi = \int_{\boldsymbol{y}} \Pi^{-1}(p_c(\boldsymbol{y})\,\psi(\boldsymbol{y}-\boldsymbol{x})\,\mathrm{d}\boldsymbol{y}$$

Using the extended von Neumann yield condition and after some algebra …

Non-local model that resolves kinematic indeterminacy and incorporates dilatancy

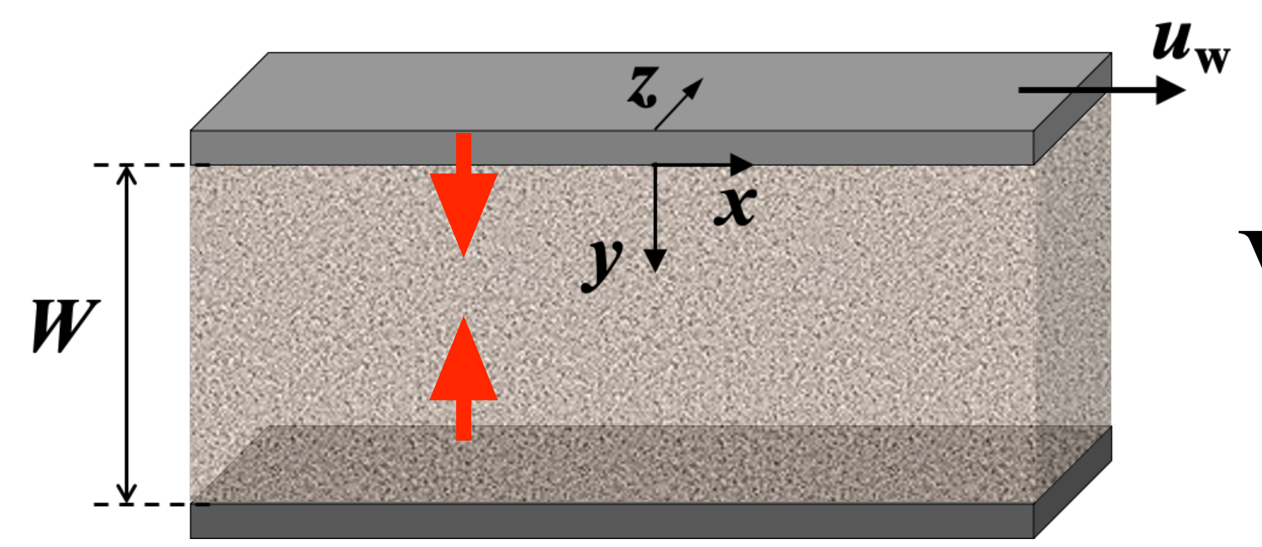


$$\boldsymbol{\sigma} = -p\,\boldsymbol{\delta} + \frac{2\mu}{\dot\gamma}\left(p_\mathrm{c}\,\boldsymbol{D}' - \underline{\ell^2\,\Pi\,\nabla^2\boldsymbol{D}'}\right), \qquad p = p_\mathrm{c}\left(1 - \frac{\mu_\mathrm{b}}{\dot\gamma}\boldsymbol{\nabla}\cdot\boldsymbol{u}\right) - \underline{\ell^2\,\Pi\,\frac{\mu_\mathrm{b}}{\dot\gamma}\nabla^2\boldsymbol{\nabla}\cdot\boldsymbol{u}}$$

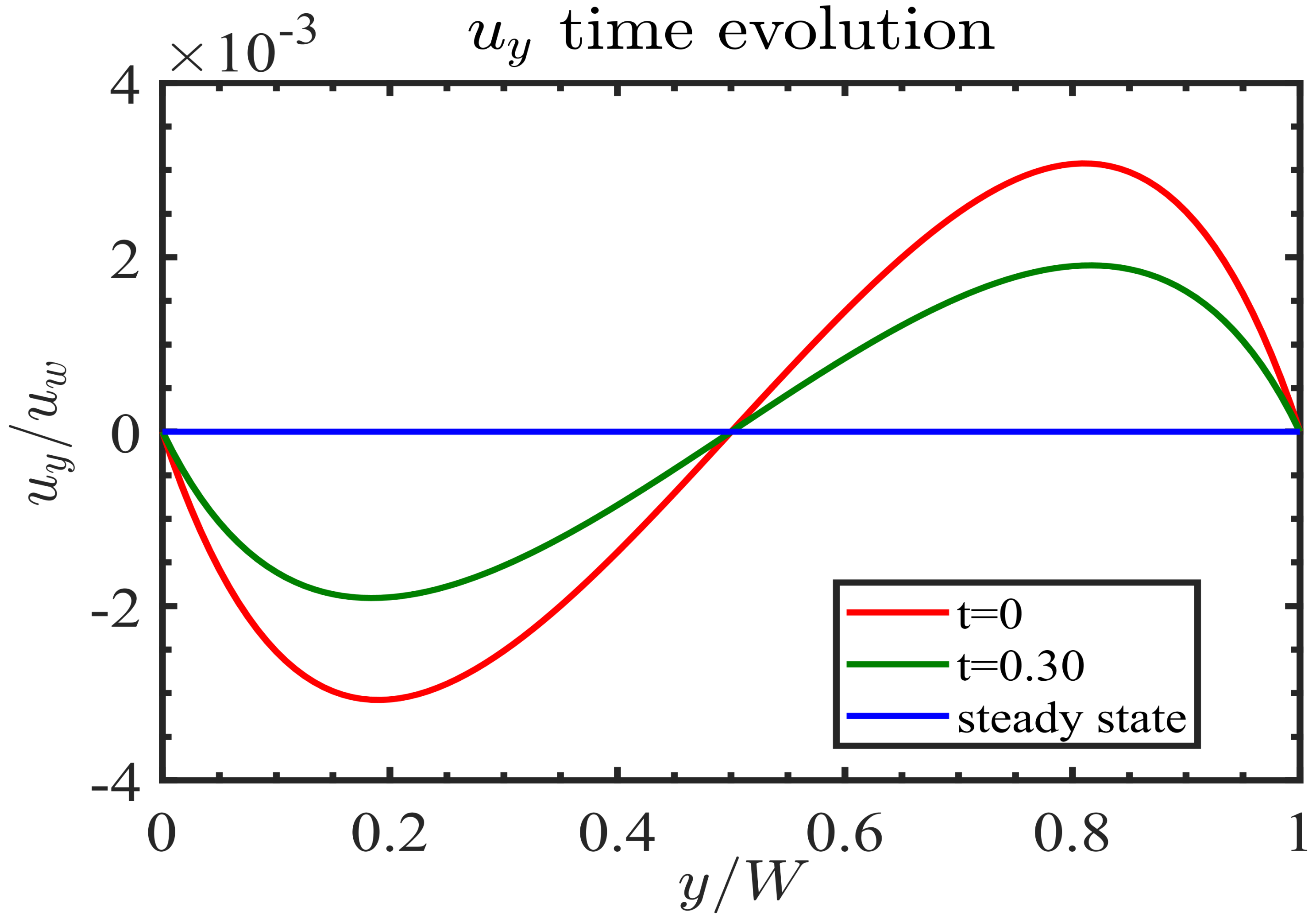
$$p_\mathrm{c} = \Pi - \underline{\ell^2\,\frac{\mathrm{d}\Pi}{\mathrm{d}\phi}\,\nabla^2\phi},$$

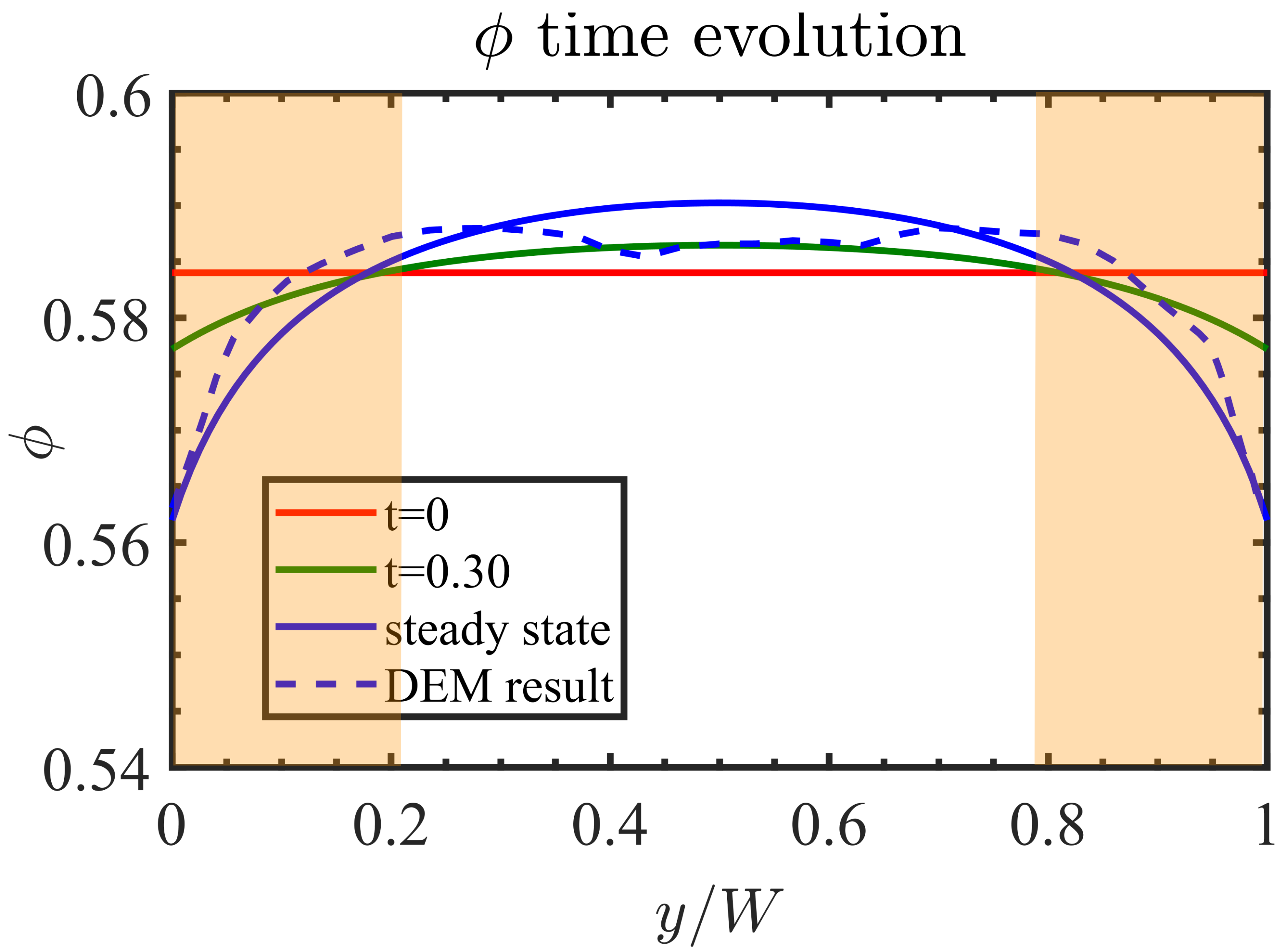
Model unconditionally well-posed

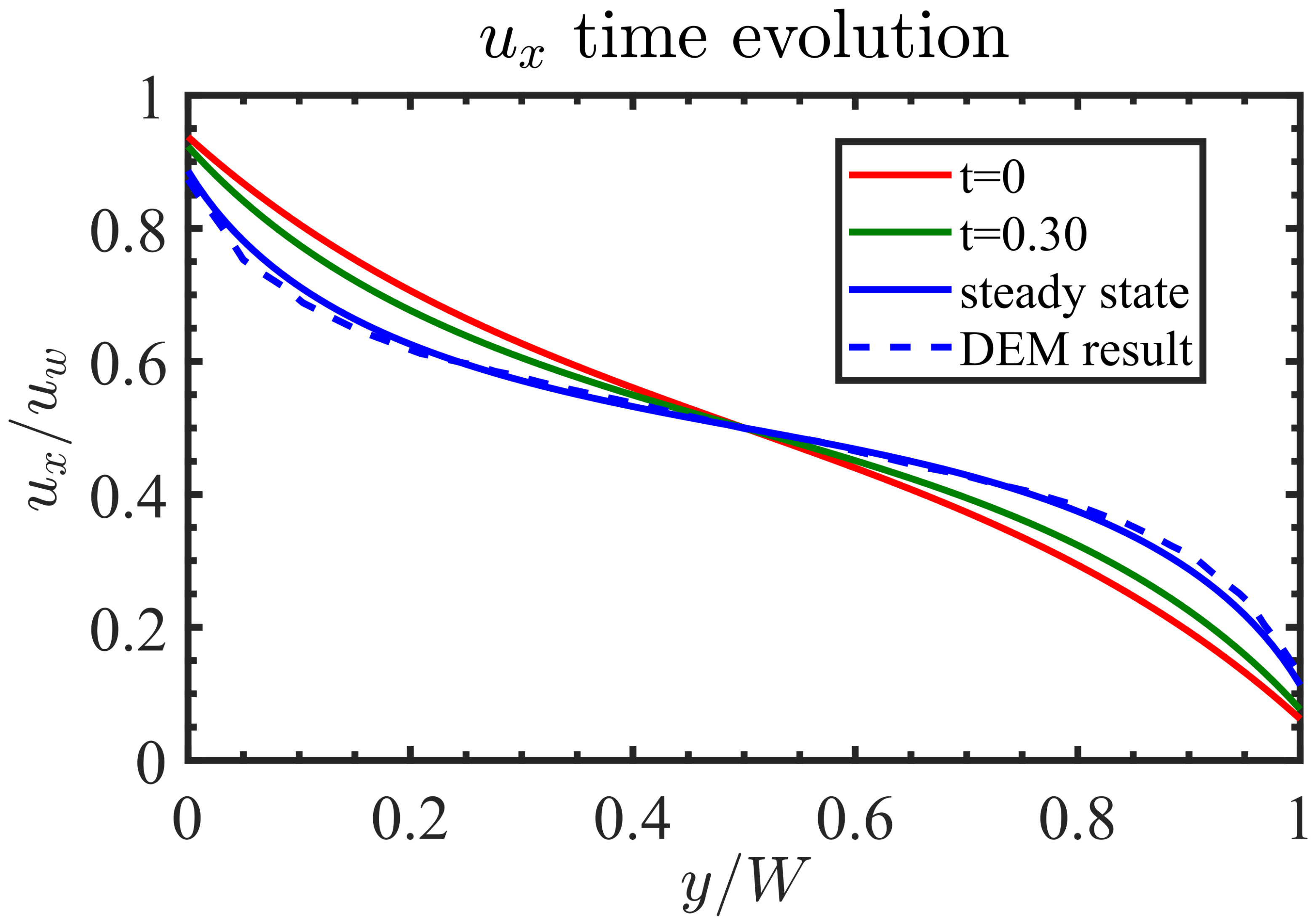
Dsouza & Nott (*J. Fluid Mech.* 2020)



Validation: unsteady plane shear in the absence of gravity





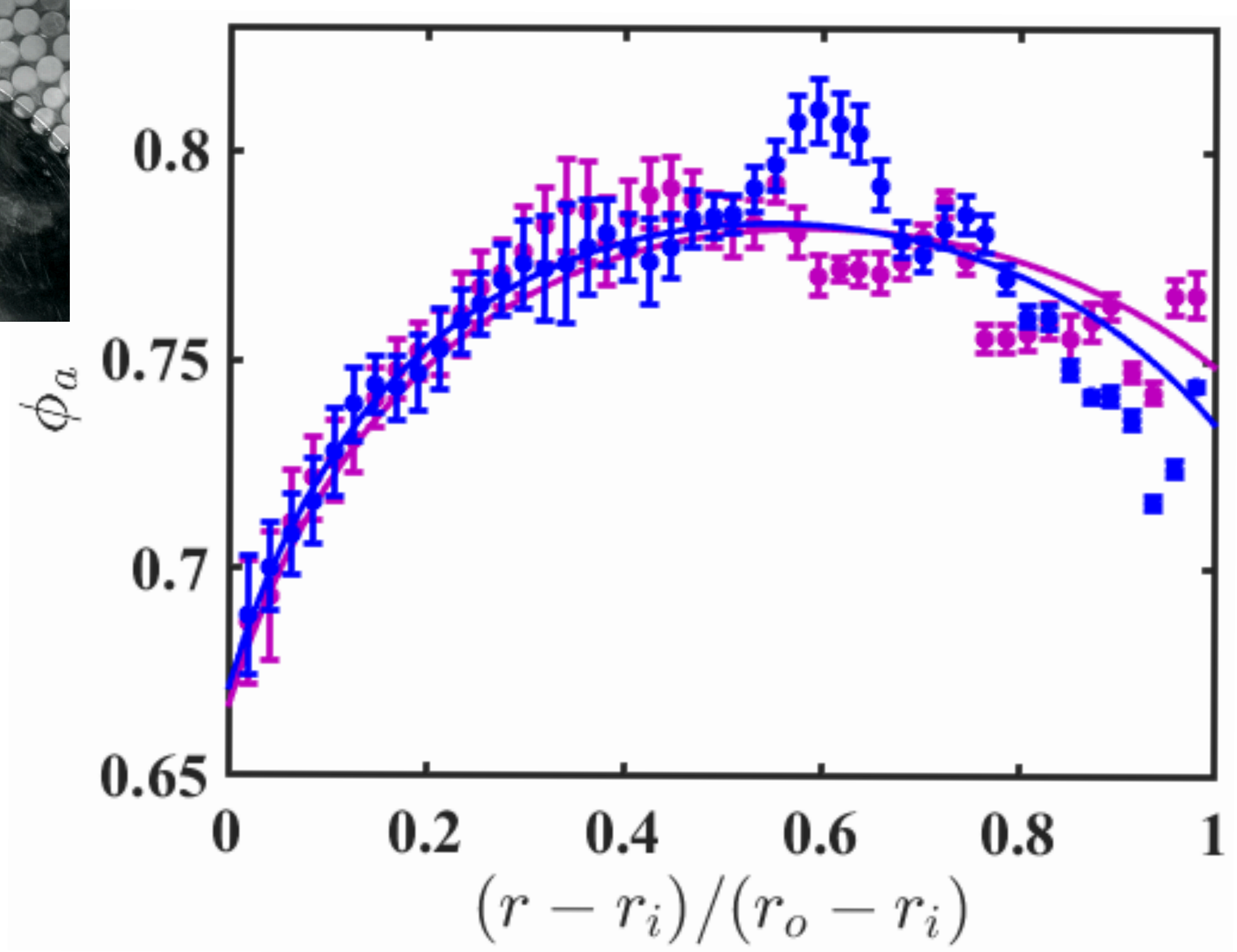


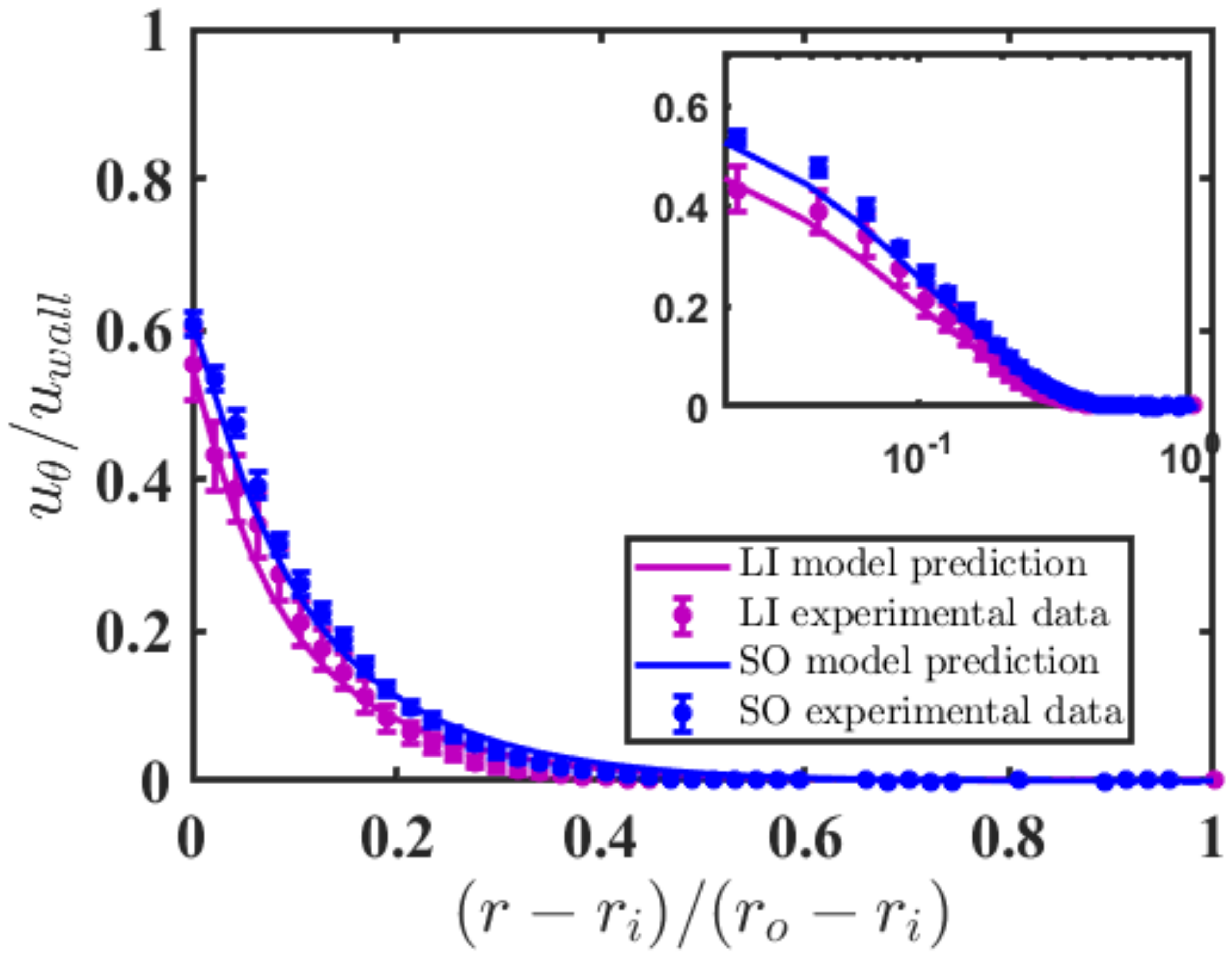
Excellent agreement with DEM results at steady state

Experimental validation of nonlocal model: Experiments on a 2D cylindrical Couette device – IISc-NCSU collaboration

•, • Experimental data for two ϕ profiles (with same average)

—, — predictions of non-local model



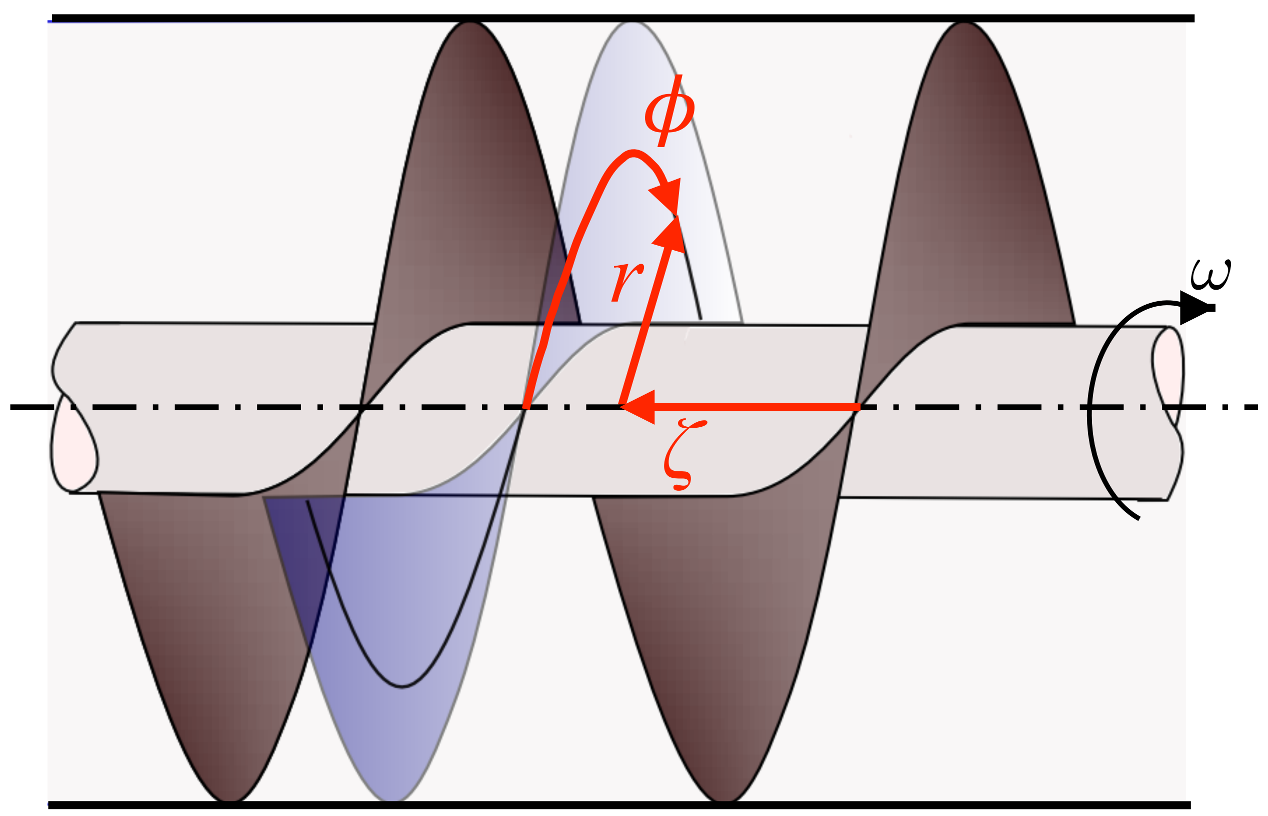


Strong coupling between ϕ and u_θ profiles

Excellent agreement of model predictions with experimental data

See IISc-NCSU collaboration presentation for more details and results

Application of the nonlocal model to the screw feeder



Equations of motion in non-orthogonal helical coordinates (r, ϕ, ζ)

r momentum balance

$$\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}} \cancel{\frac{\partial \sigma^{\phi\zeta}}{\partial \phi}} + \frac{\partial \sigma^{\zeta r}}{\partial \zeta} + \frac{\sigma^{rr}}{r} - \frac{r}{r^2 + \frac{P^2}{(2\pi)^2}} \sigma^{\phi\phi} = 0$$

ϕ momentum balance

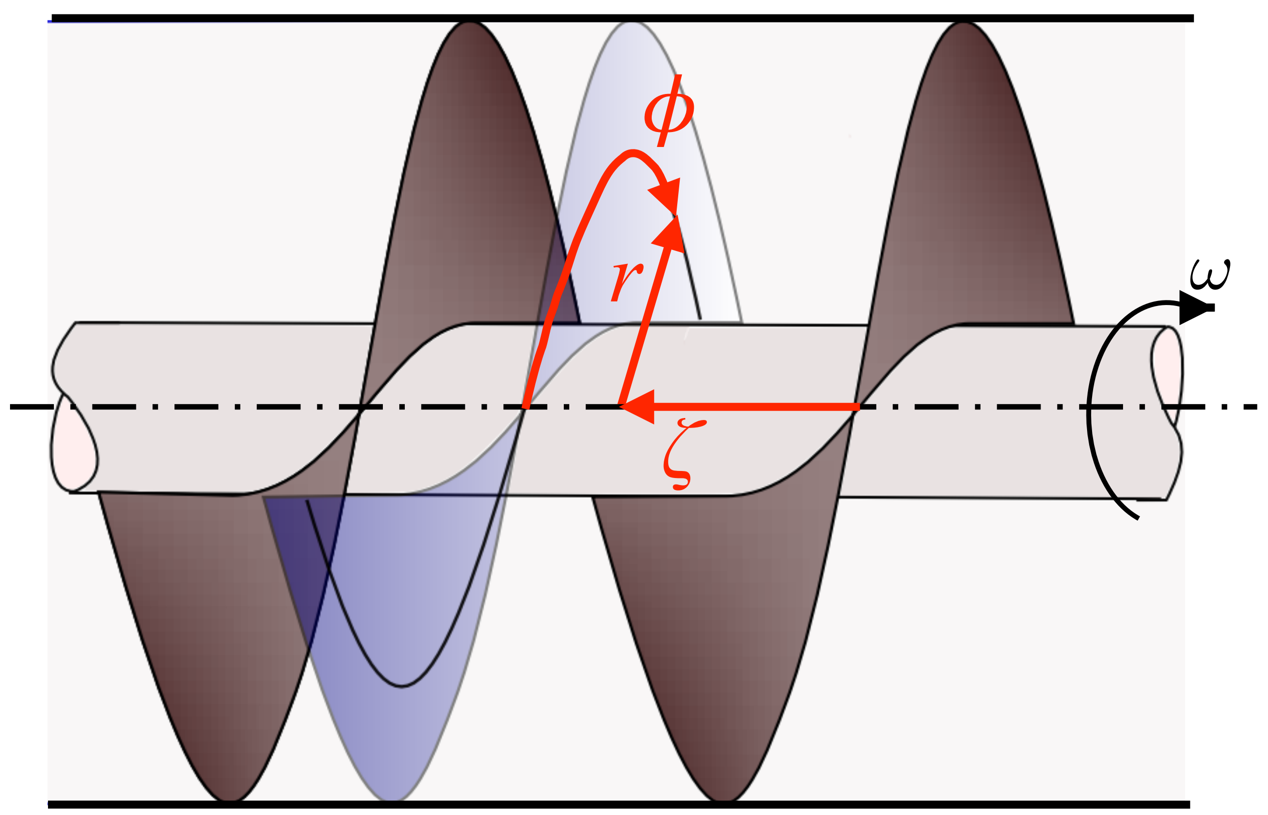
$$\frac{\partial}{\partial r}\left(\frac{\sigma^{r\phi}}{\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}}\right) + \frac{1}{r^2 + \frac{P^2}{(2\pi)^2}} \cancel{\frac{\partial \sigma^{\phi\phi}}{\partial \phi}} + \frac{1}{\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}} \frac{\partial \sigma^{\zeta\phi}}{\partial \zeta} + \frac{3\sigma^{r\phi}}{r\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}} = 0$$

For fully developed flow, $\partial\xi/\partial\phi = 0$ (for all variables ξ)

ζ momentum balance

$$\frac{\partial \sigma^{r\zeta}}{\partial r} + \frac{1}{\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}} \cancel{\frac{\partial \sigma^{\phi\zeta}}{\partial \phi}} + \frac{\partial \sigma^{\zeta\zeta}}{\partial \zeta} + \frac{\sigma^{r\zeta}}{r} - 2\left(\frac{P}{2\pi r}\right) \frac{\sigma^{r\phi}}{\sqrt{r^2 + \frac{P^2}{(2\pi)^2}}} = 0$$

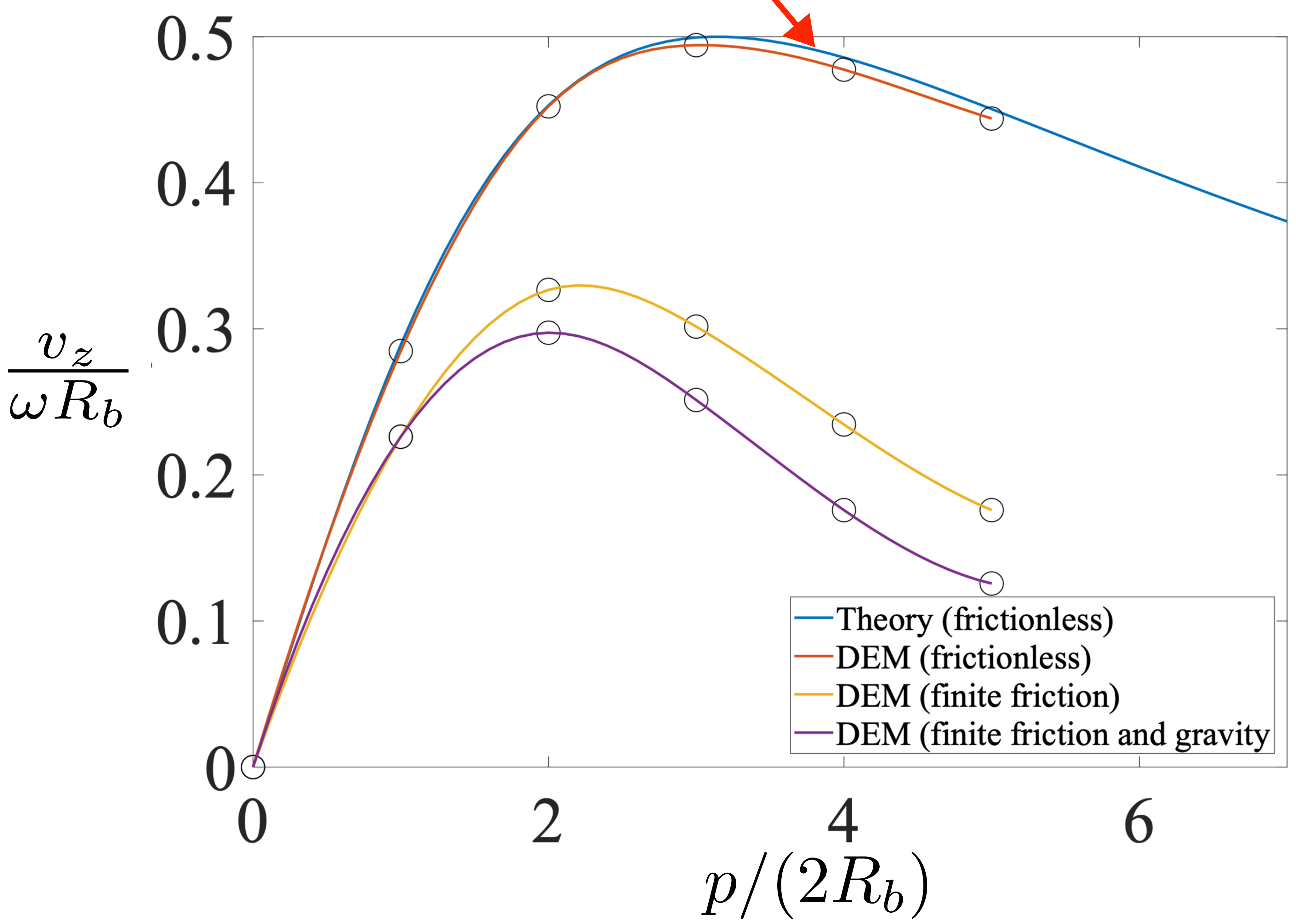
Solution for the velocity and stress



When feeder is fully filled and gravity is absent, flow is fully developed: $u_r = 0$, $\partial u_\phi / \partial \phi = 0$.

Therefore $u_\phi = u_\phi(r,\zeta)$

For frictionless screw and shaft u_ϕ is constant, and we recover the plug flow solution obtained earlier!



The model also yields the stress on the barrel,

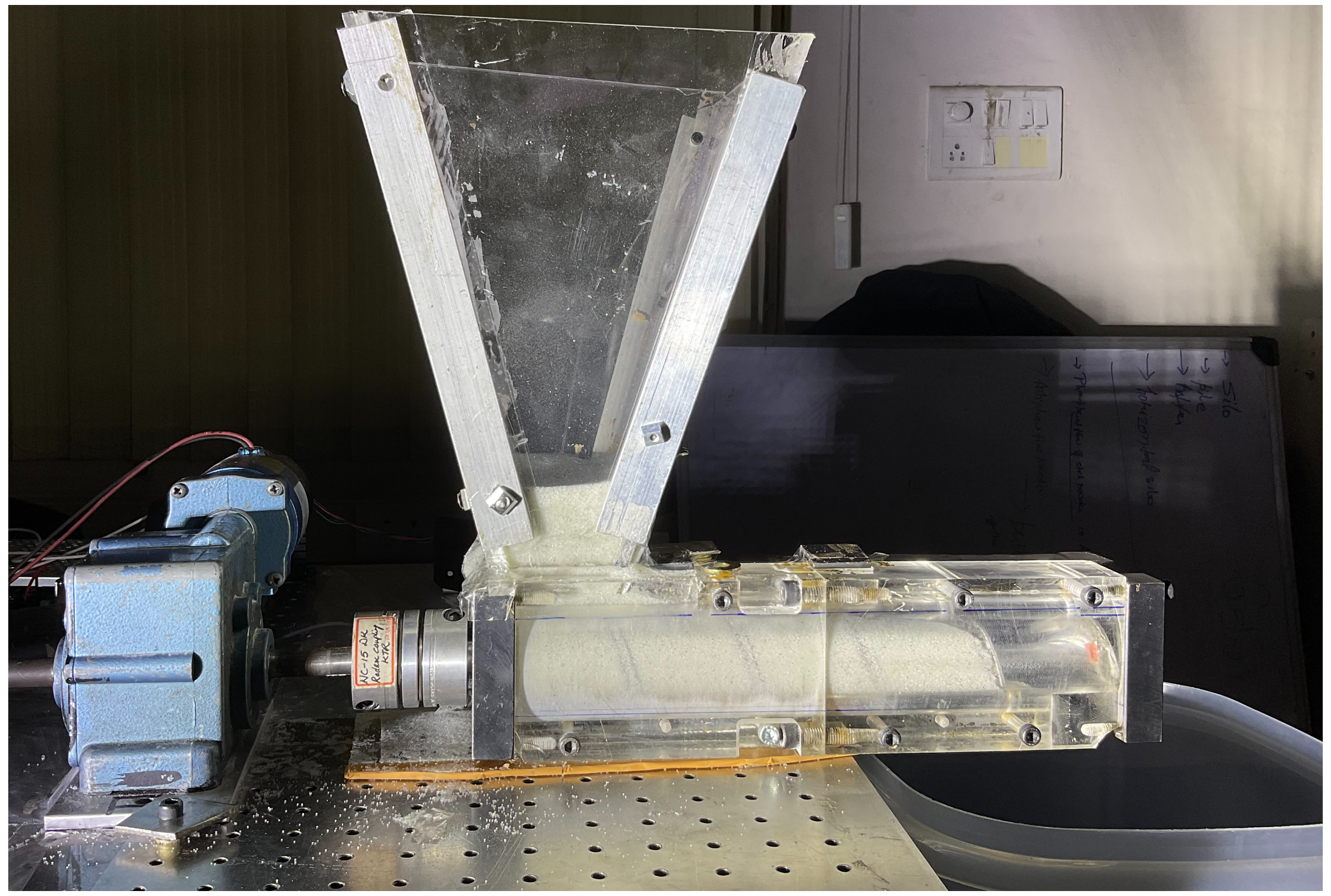
$$\sigma_{rr} = \sigma_{rr0} e^{-K\zeta}$$

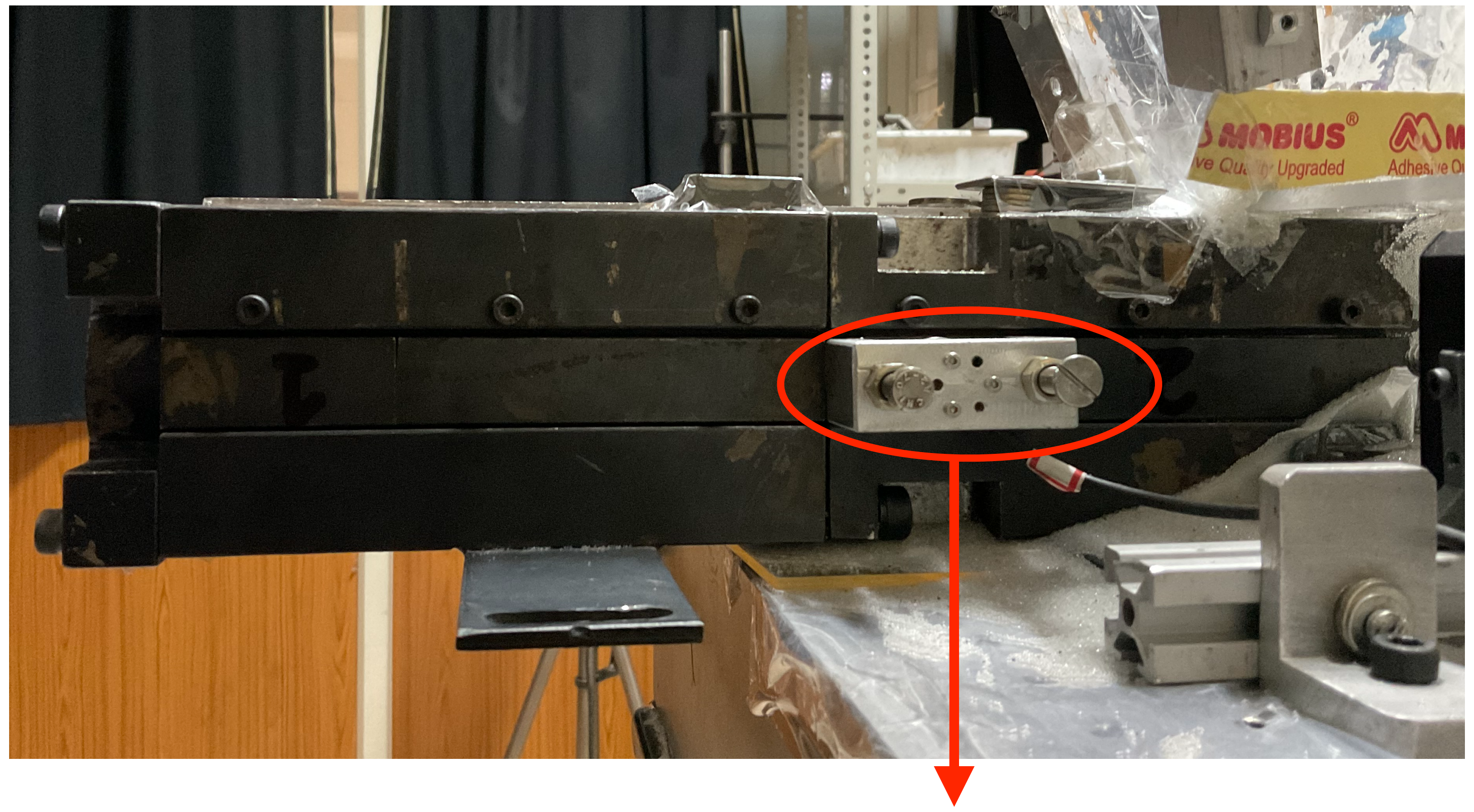
Exponential decay has the same physical origin as the Janssen saturation of the stress in a silo

Solutions for frictional screw & shaft involve solution of PDEs – in progress

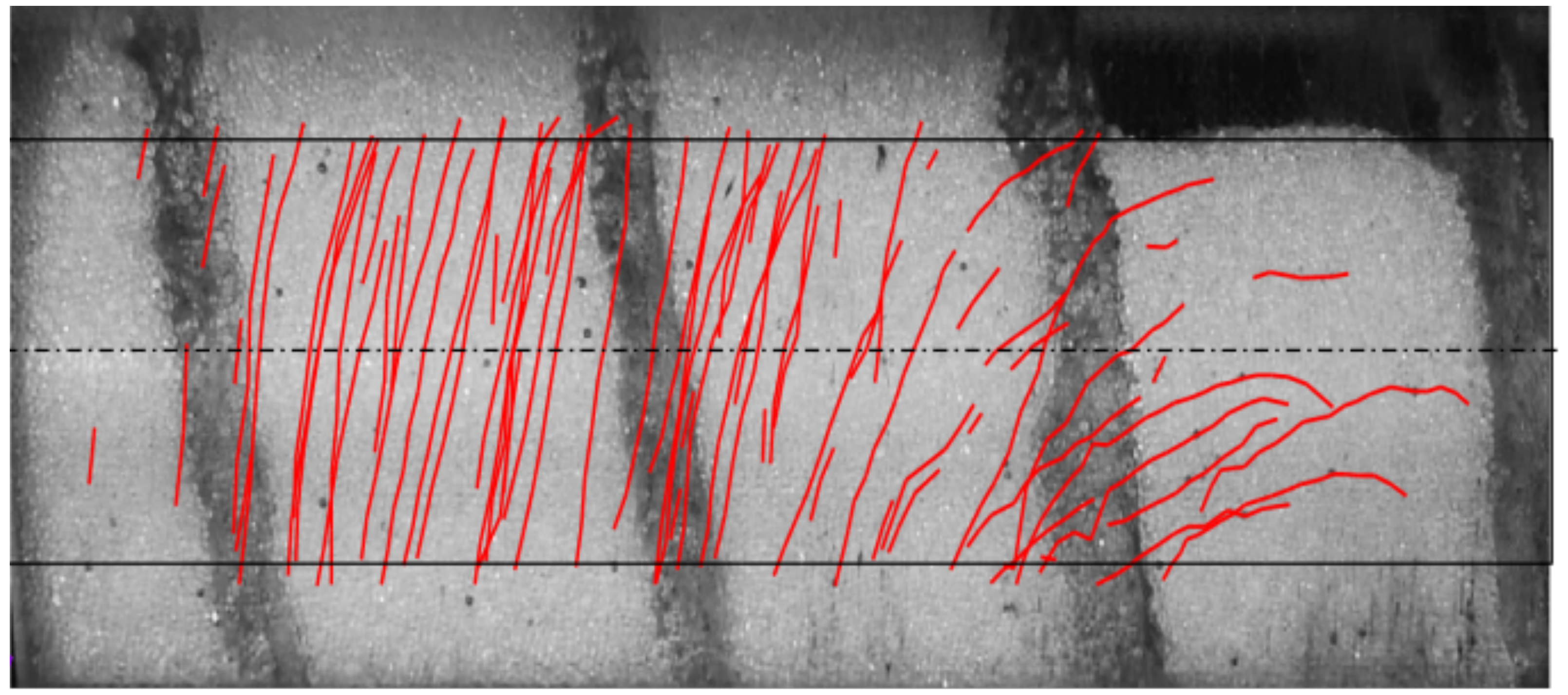
Experimental screw feeder assembly

- Transparent front face for imaging
- Stress measured on back face

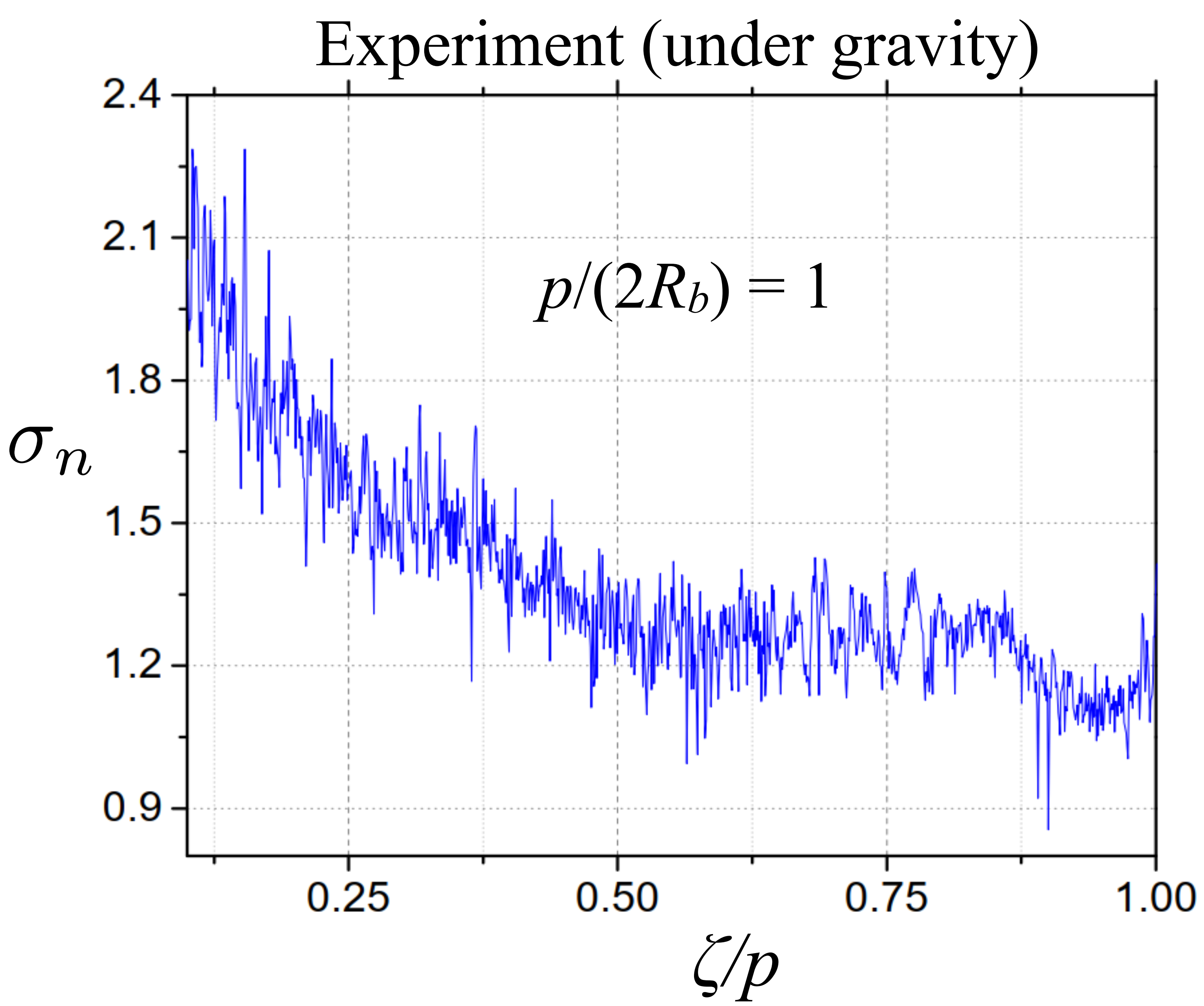


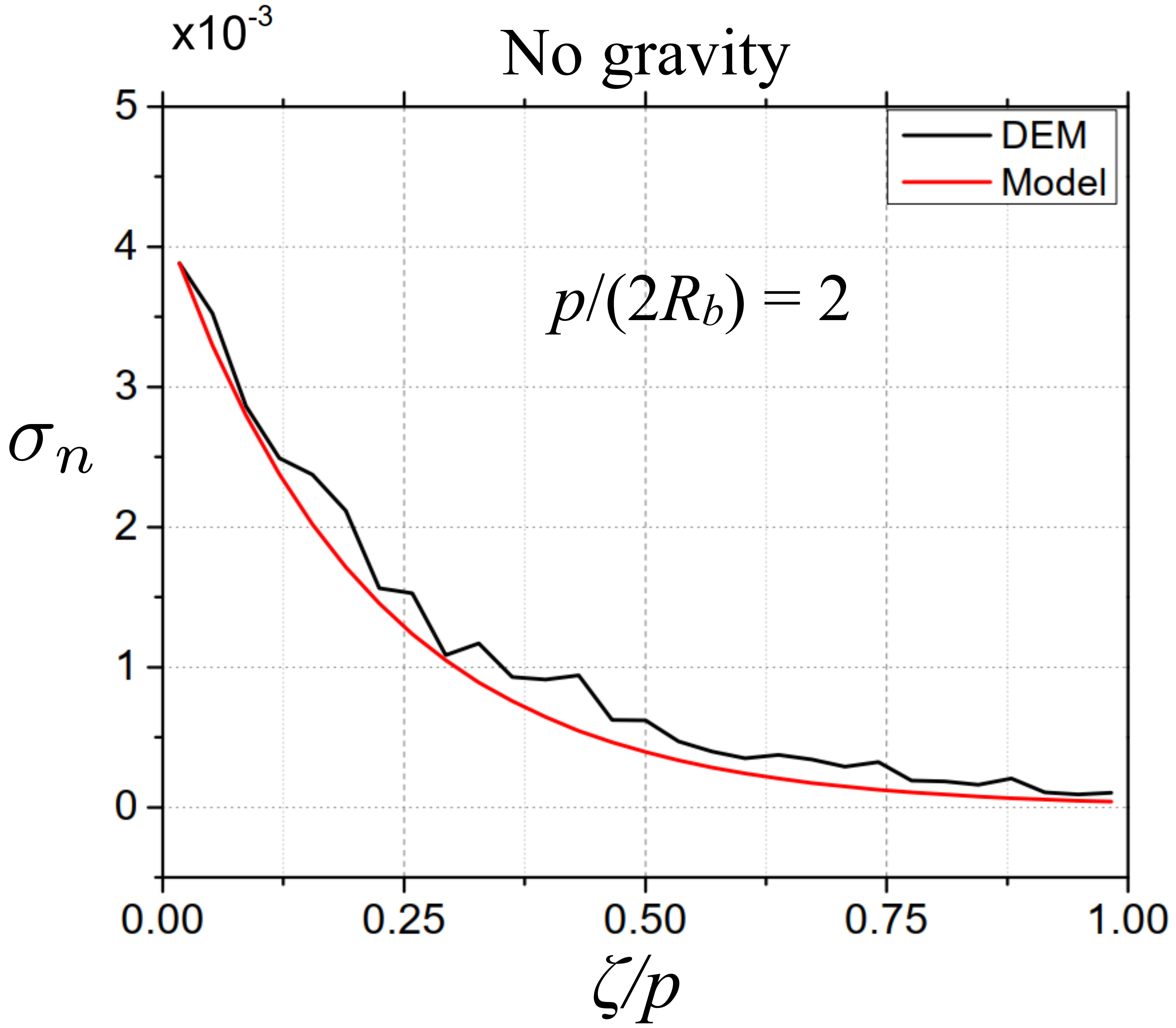


stress sensor



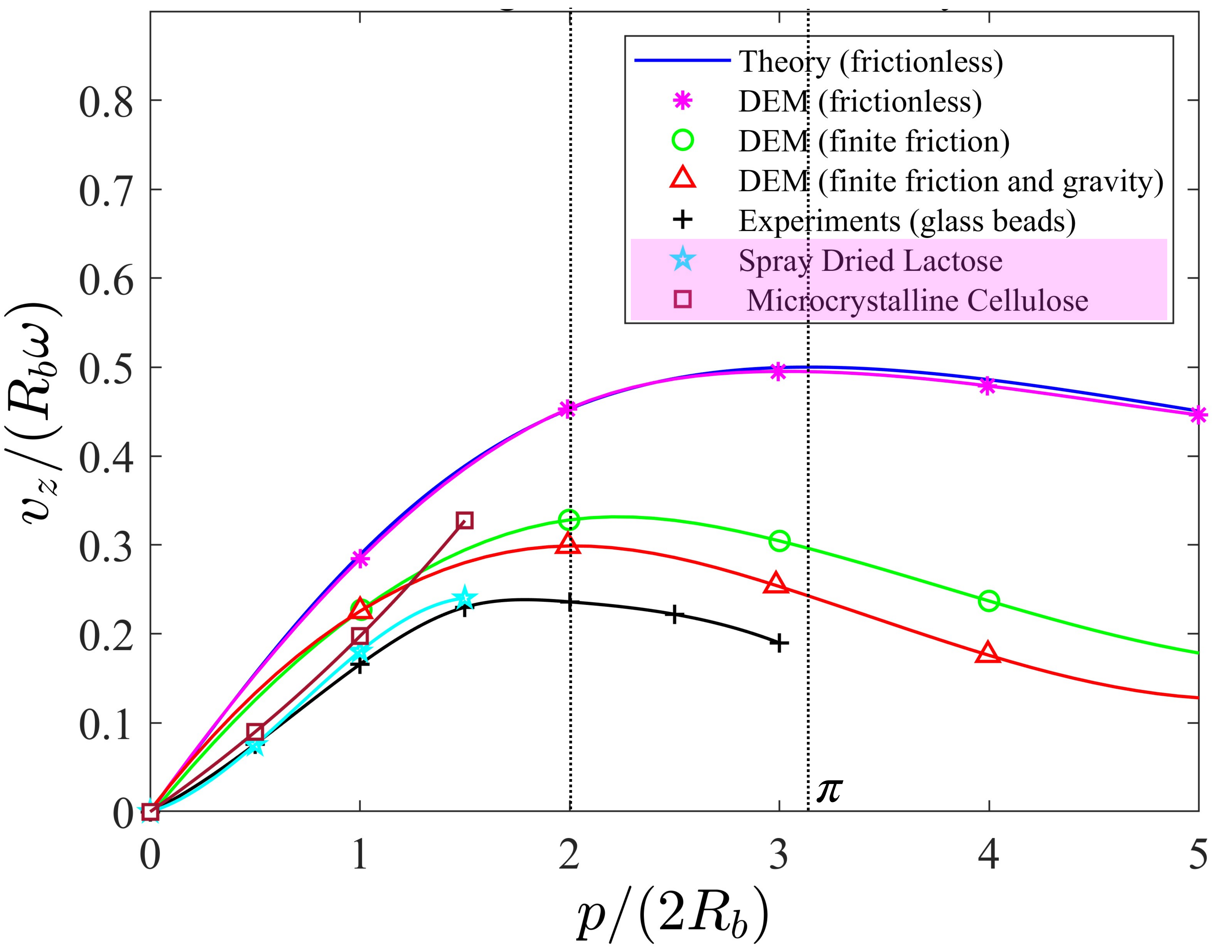
Combination of helical and axial flow, with considerable slip at screw



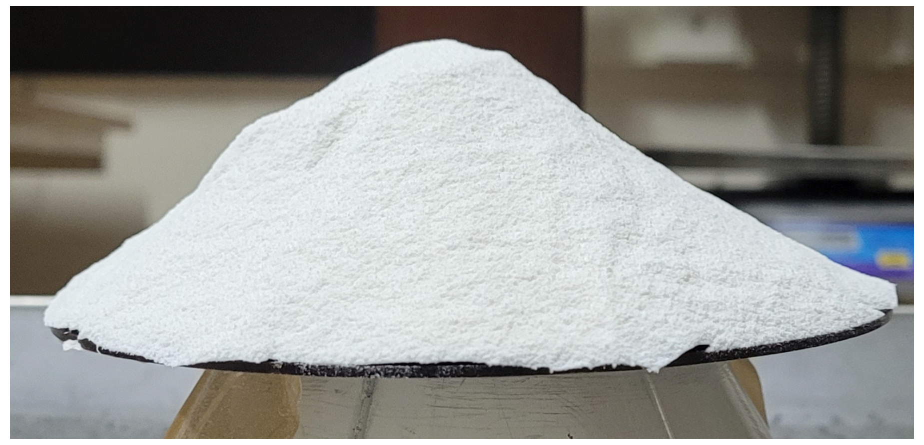


Experiments too show the exponential decay in the stress with ζ, as predicted by the model

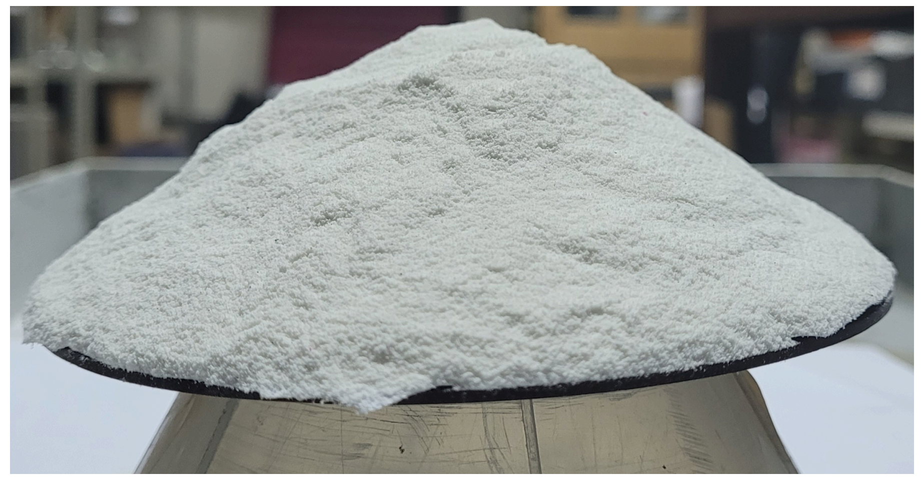
Flow rate versus $p/(2R_b)$ for cohesionless glass beads and cohesive powders



Spray dried lactose

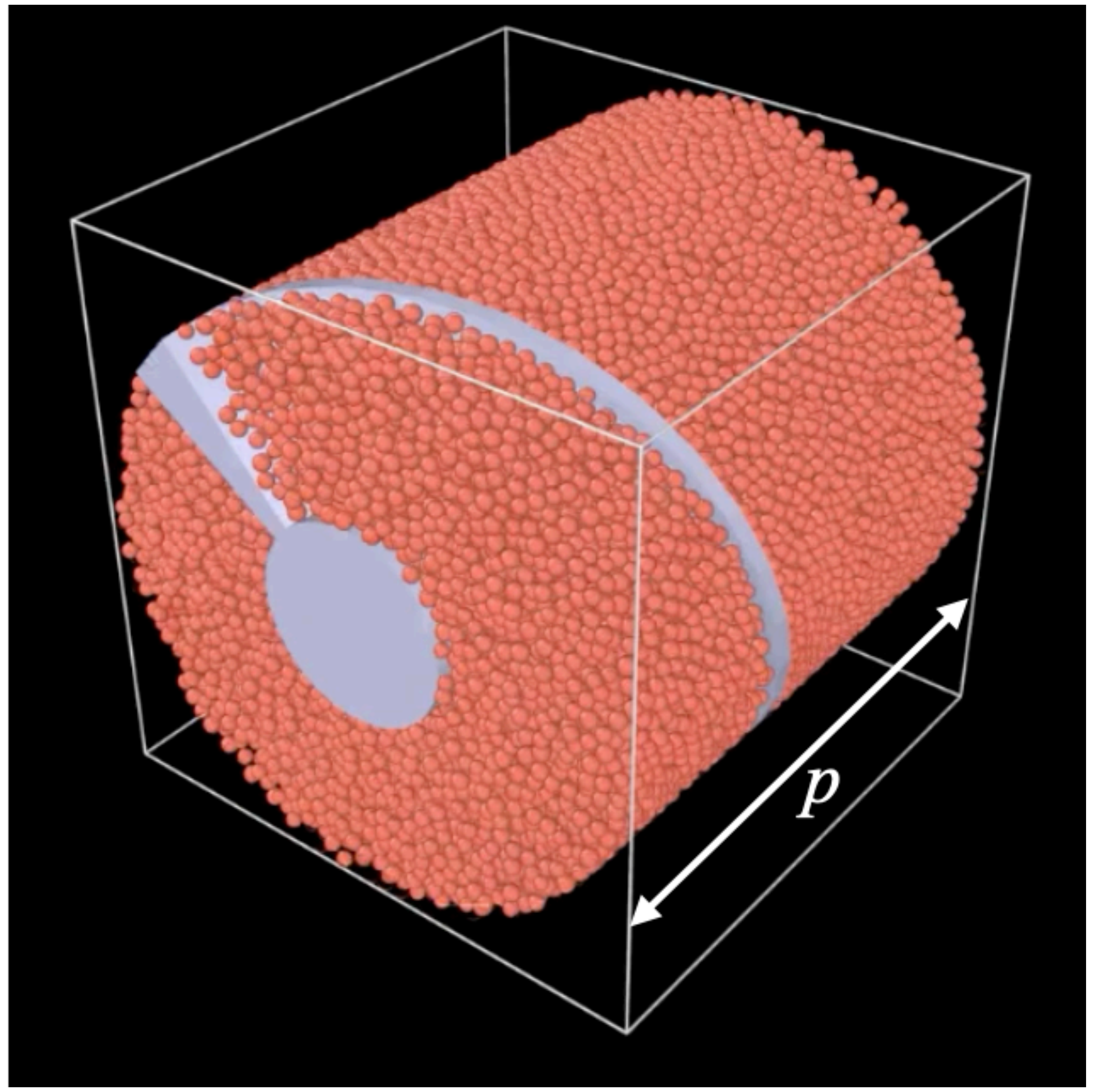


Microcrystalline cellulose

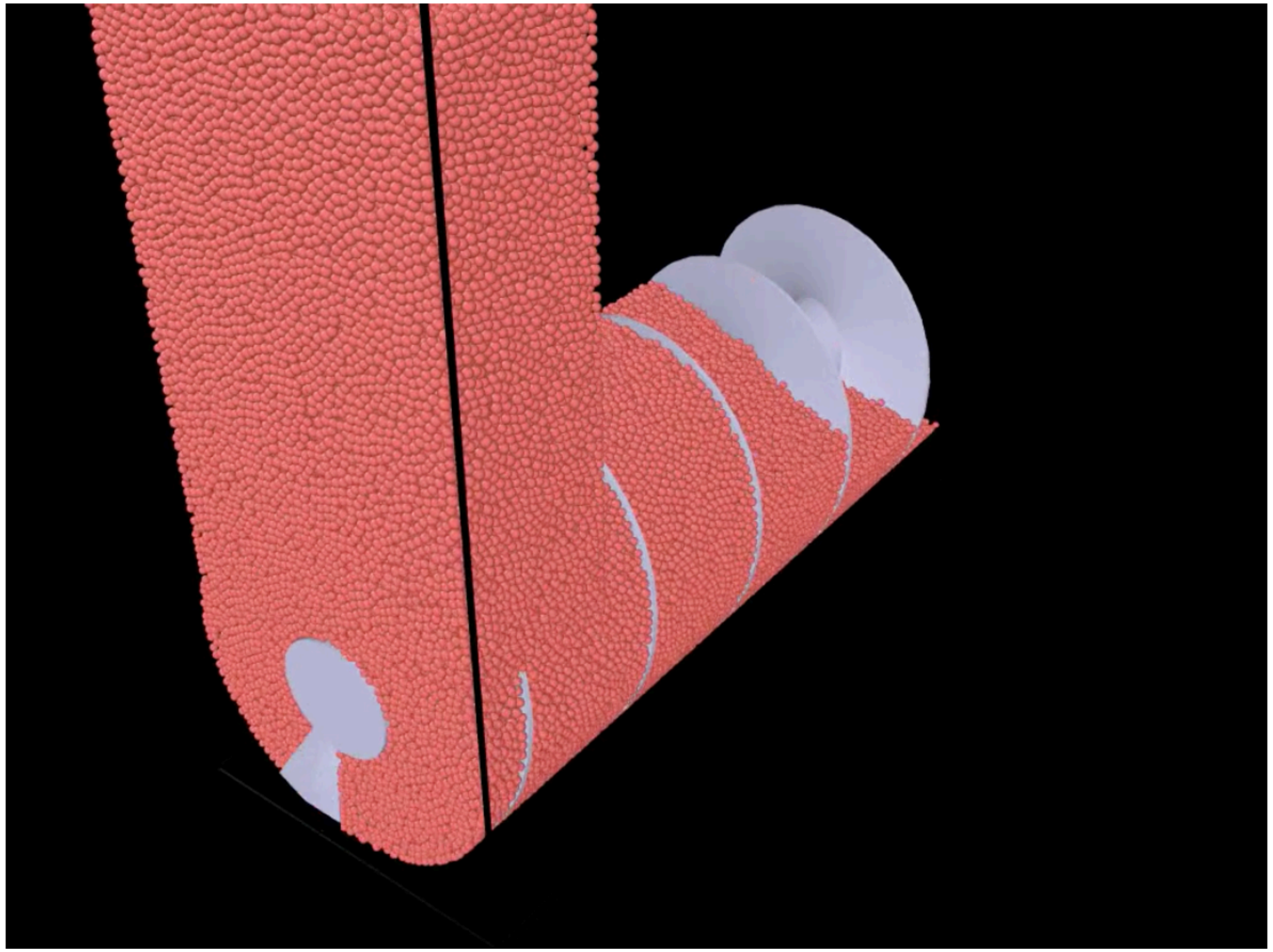


DEM simulation of inlet hopper coupled with screw feeder

Initial DEM simulations with periodic boundary conditions without gravity



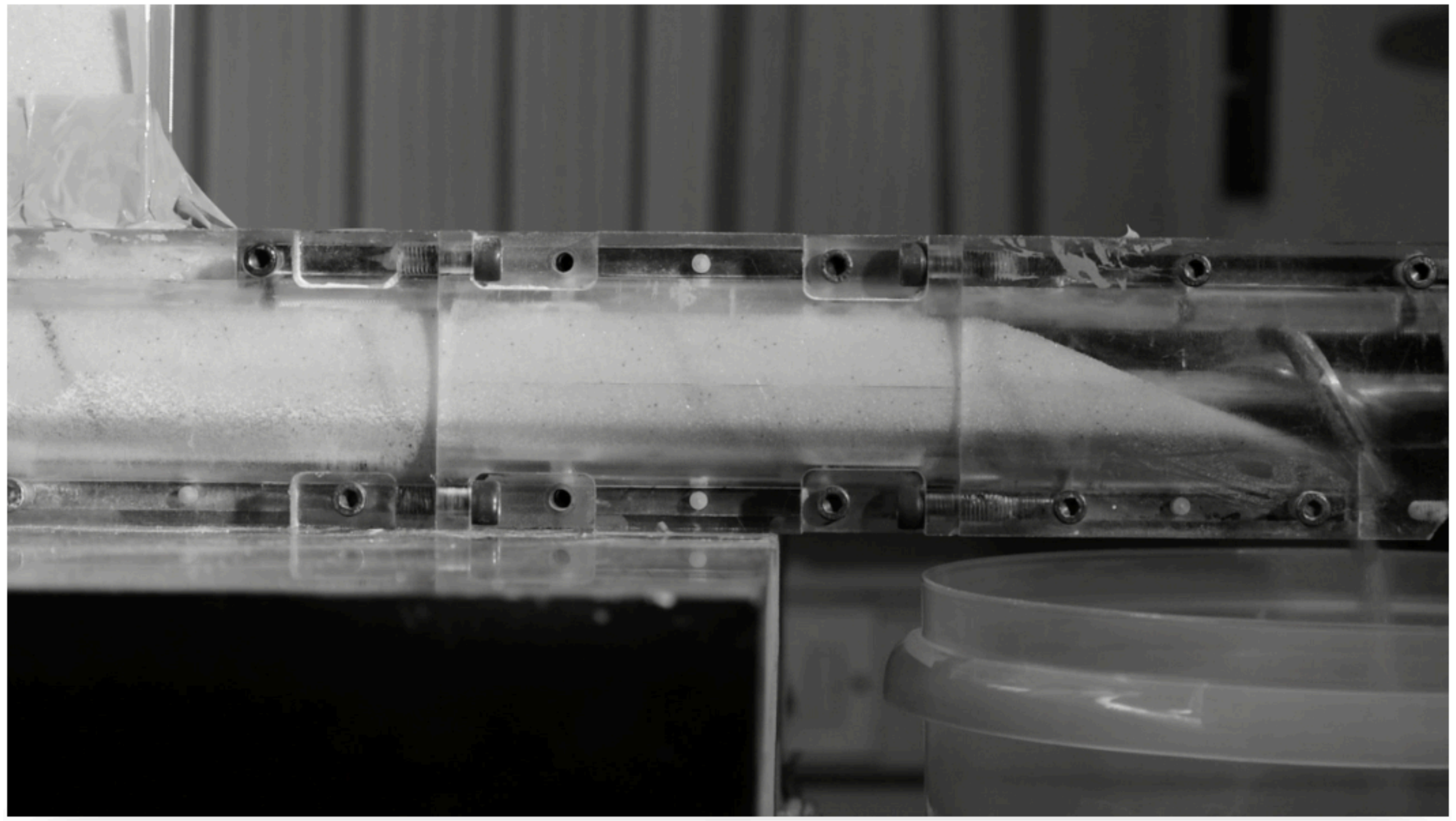
Does not throw light on the variation from inlet to outlet



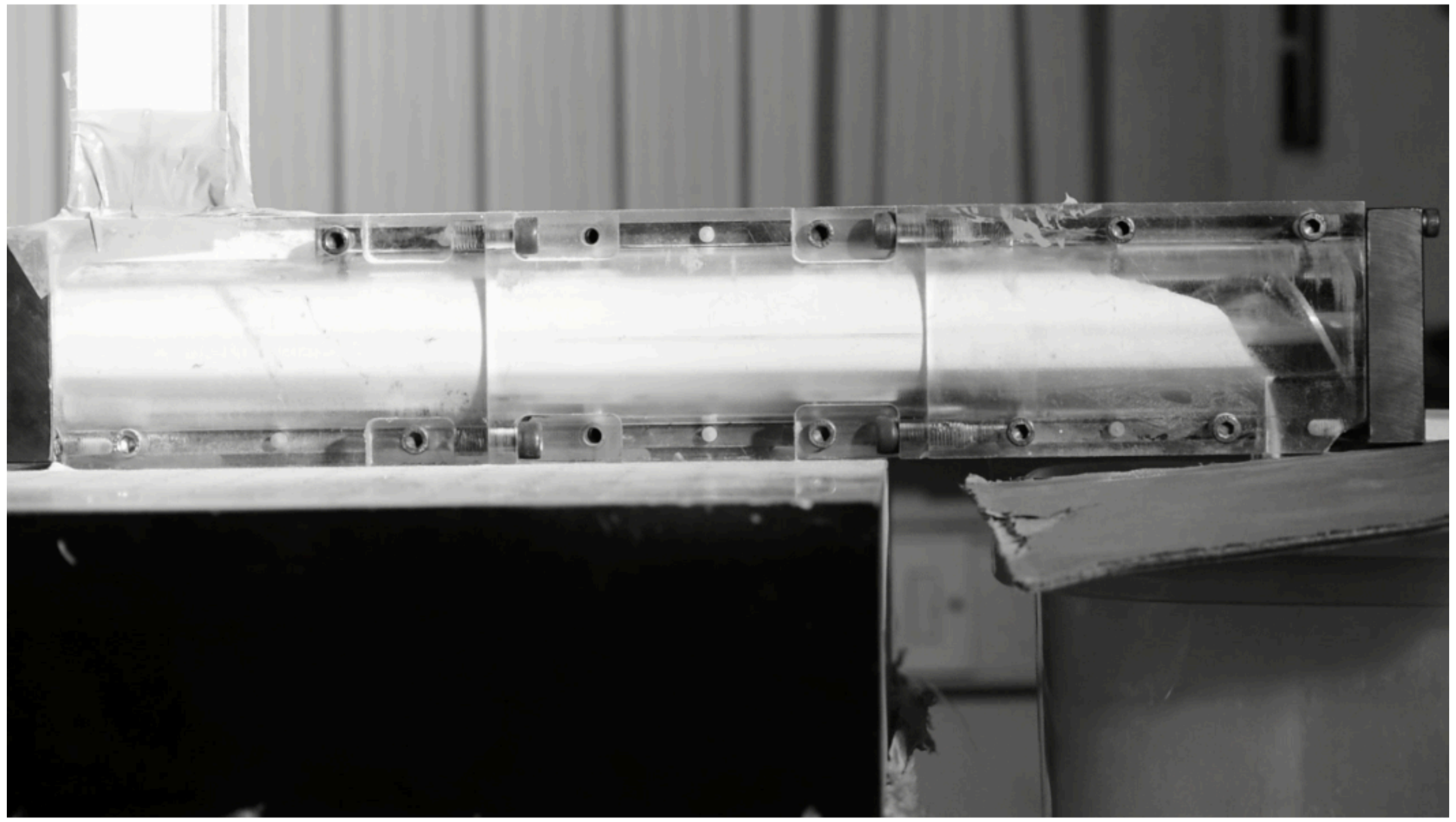
Full hopper-feeder simulation computationally intensive, but throws light on variation in the axial direction.

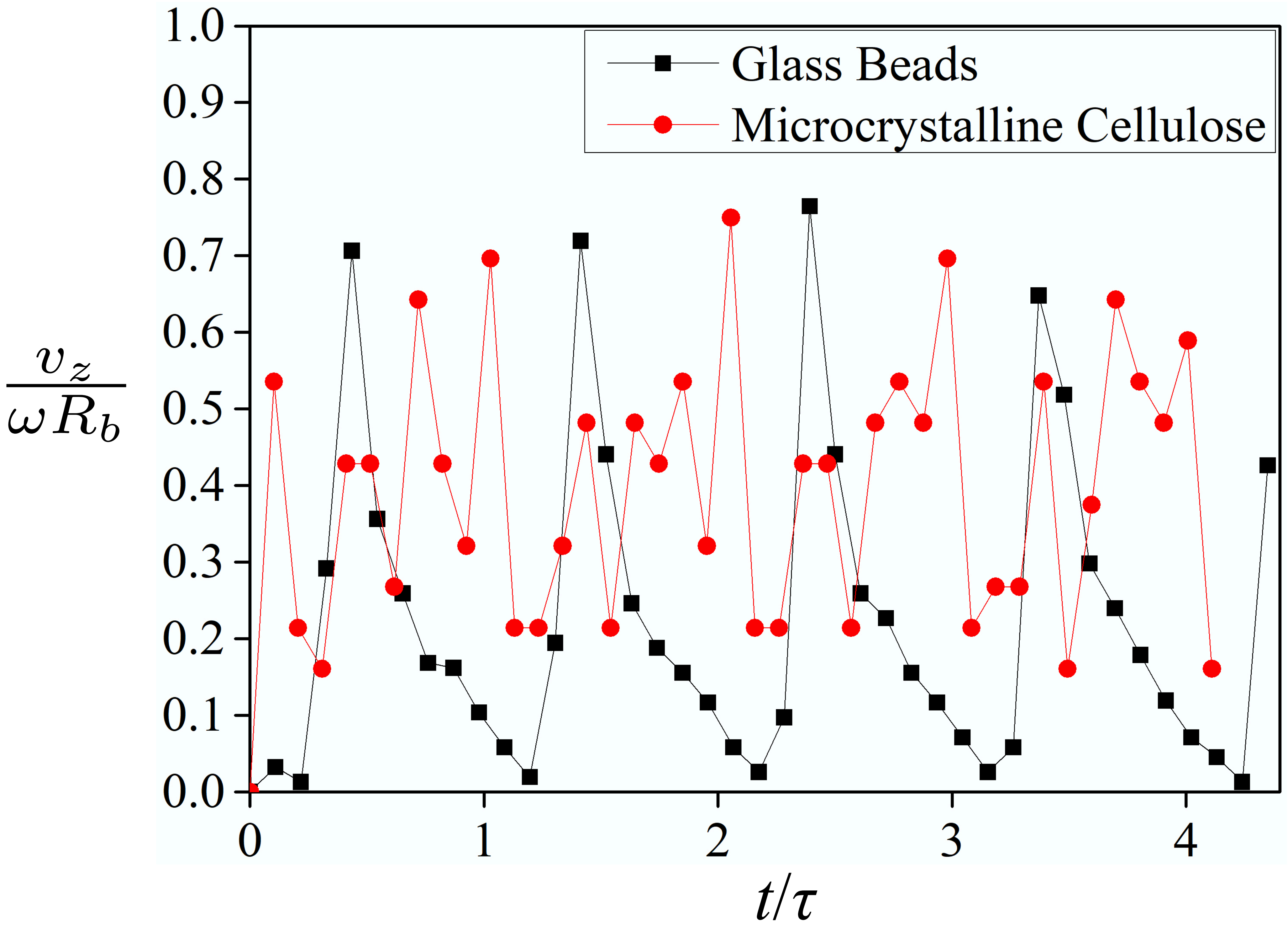
Feed rate fluctuations

glass beads



microcrystalline cellulose

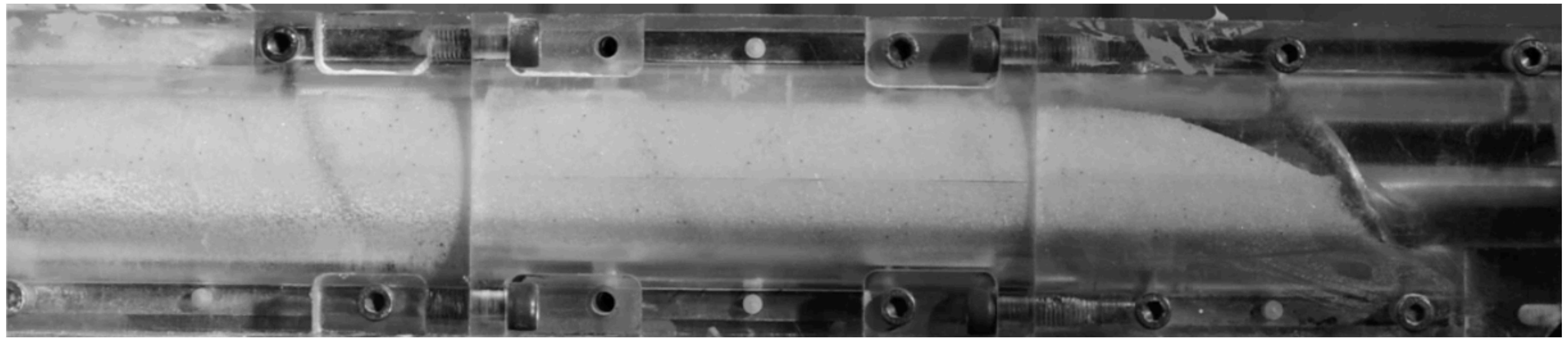




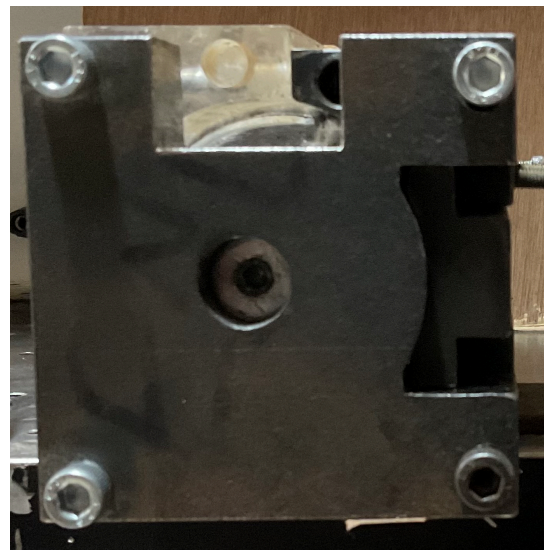
(τ – period of revolution)

Fluctuations due to variation of slope at the exit

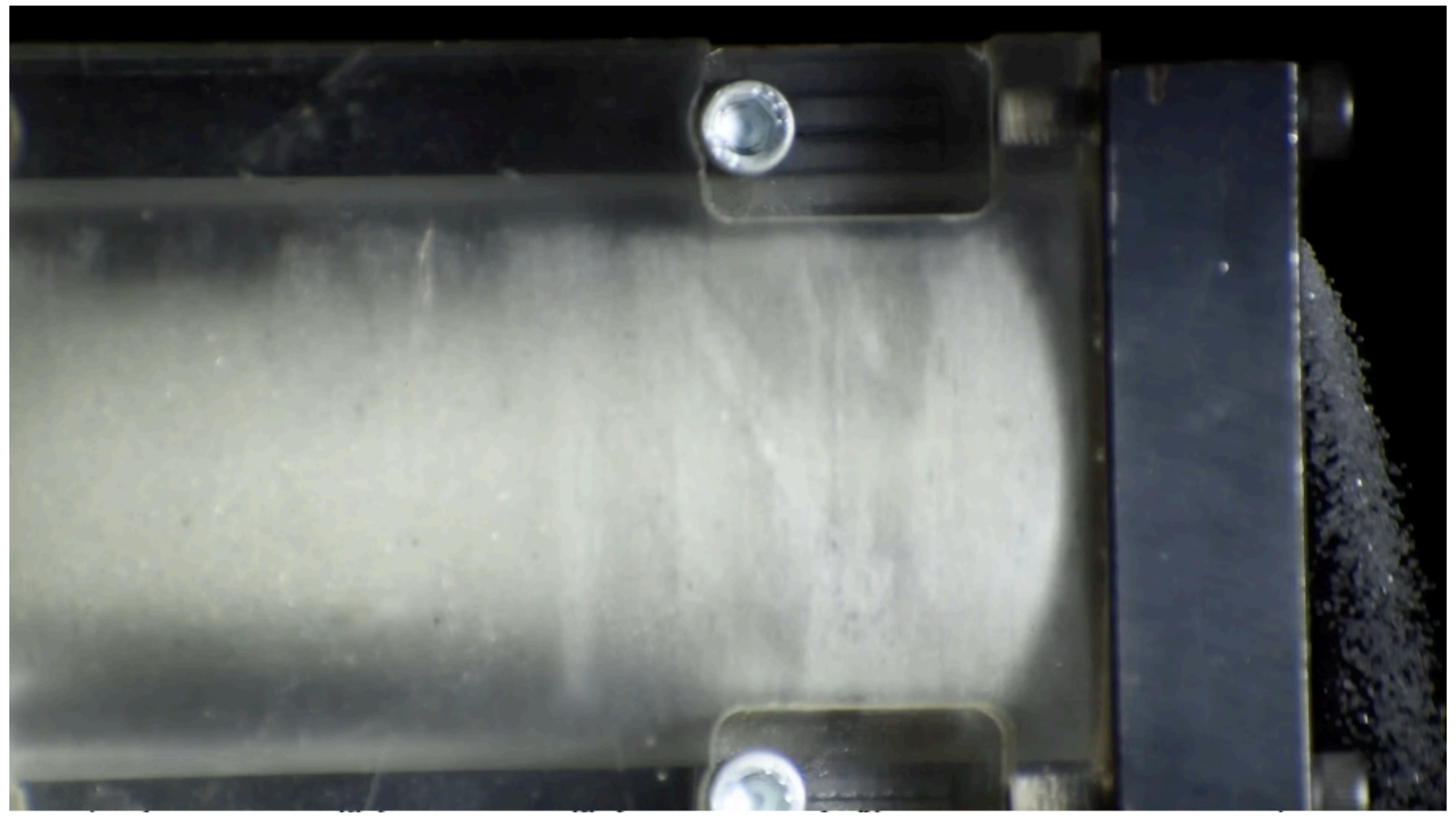
How does changing the location of the exit slot alter the flow?

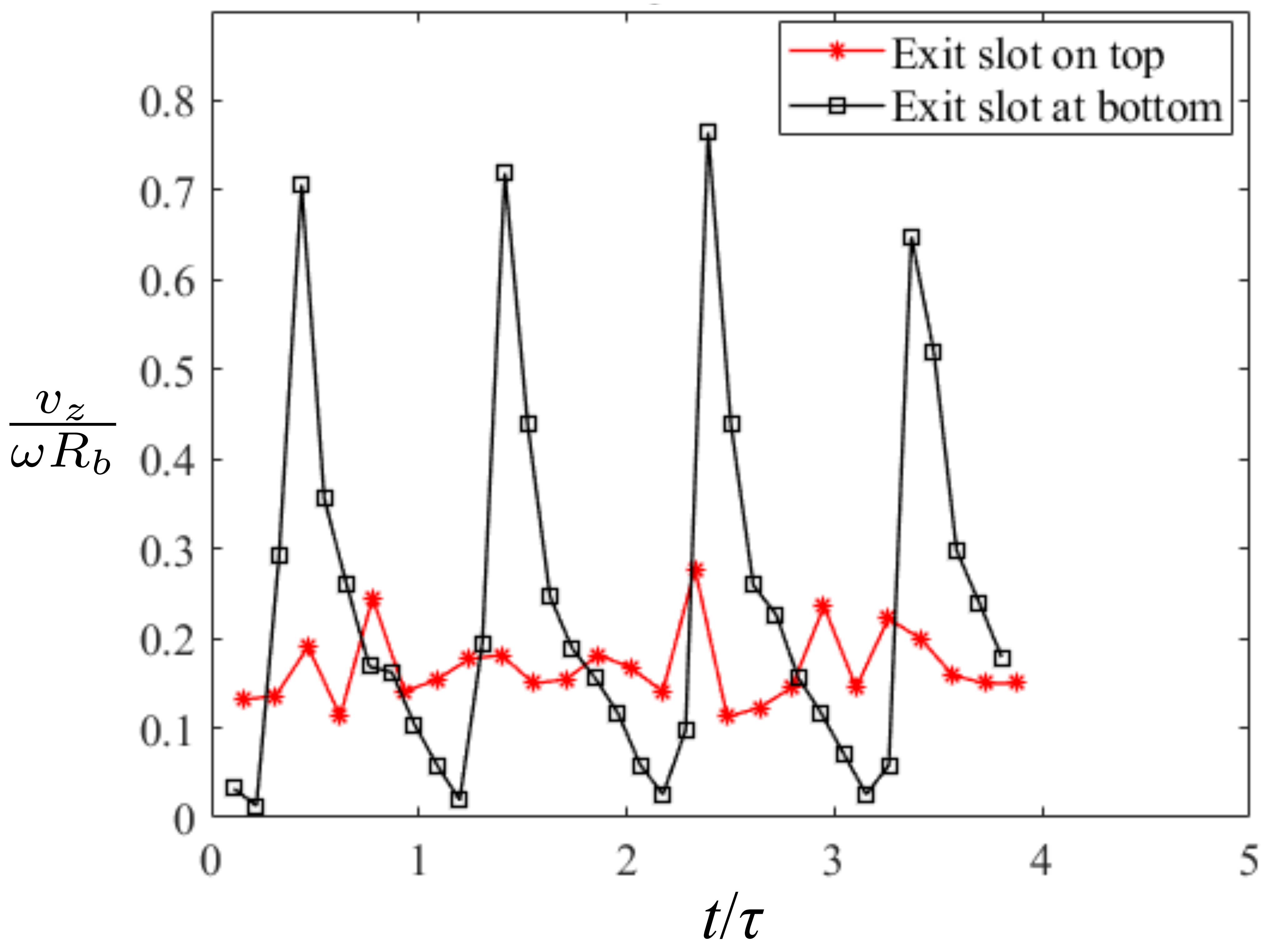


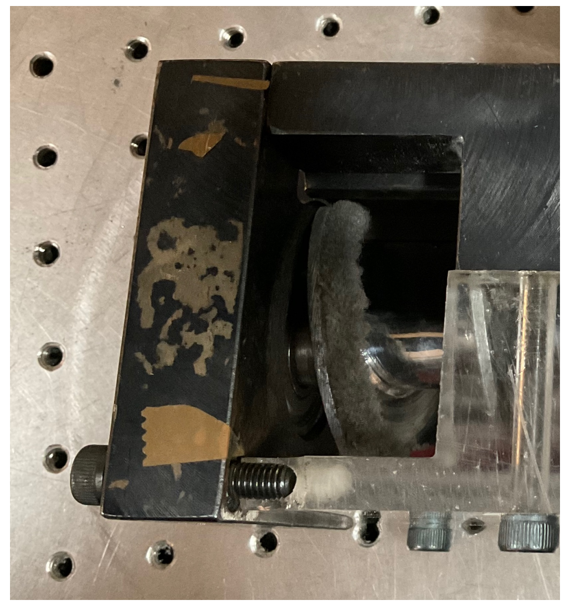
Exit slot at the top



Feeder fully filled when exit slot is at the top







Exit slot at the bottom

Appropriate exit design can mitigate feed rate fluctuations

Conclusion

- Simple mechanics-based model predicts flow rate maximum at an optimum value of $p/(2R_b)$. Prediction validated by DEM simulations and experiments.
- Non-local rheological model shows promise. Validated for frictionless screw & shaft. Needs to be solved for frictional surfaces, partial fill, unsteady flow.
- Experiments conducted for non-cohesive glass beads and cohesive powders. Validates the predictions of non-local model and DEM simulations.
- Feed rate fluctuations due to periodic or erratic changes in free surface slope at exit. Fluctuations can be mitigated by proper exit design.
- DEM simulations were useful to validate models and motivate experiments, e.g. feed rate fluctuations.
- Future work: Scaling relations for mean and fluctuations of feed rate; Develop rheological model for cohesive powders that incorporates agglomeration; Mitigate feed rate fluctuations at hopper-feeder junction.