Response to comments of IFPRI liaison committee

"Modelling powder flow through screw feeders"

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Comments and suggestions communicated by the IFPRI liaison members are paraphrased below in blue italicized text. My response to the comments follow.

Liaison 1

- *He is doing good science.*
- *I like all the things he's planning on looking into for his renewal (cohesion, hopper feeder interface, scaling rule, updated experiment system).*
- *However what is missing is the application of all the science to understanding the weight variability of the feeder. In his proposal he mentioned that the scaling relation can be used to understand average feed rate and fluctuations. If he can go into more detail on how that will be accomplished that would be very helpful on the proposal.*

Liaison 2

- *Variability is difficult to solve, and I think he has begun to address it.*

Liaison 3

- *No apparent issues. Strong renewal proposal that addresses variability head on. "Bonus" is the development of a general rheology for cohesive powder flow.*

If is clear from the comments that the only point of concern (for one of the liaison members) is the issue of weight variability. Below, I first provide background on what are the potential causes for variability (§1), followed by the conceptual and experimental methods (§2 and §3) we will employ to understand variability.

1 Causes of feed rate variability

At the broadest level, if the feeder is functioning according to design, i.e., at constant rotation rate of the screw, etc., the feed rate variability (which leads to weight variability of the product) could be due to two factors: (i) fluctuations in the state of the material within the screw feeder that leads to fluctuation in the feed rate, and (ii) fluctuations in the rate of material fed to the screw feeder from the inlet hopper.

We first consider the first of the above factors. For non-cohesive materials such as glass beads, we have shown that the material flows smoothly from the hopper into the feeder, and there are no fluctuations in the state of the material wihin the screw – at each axial position, the fill level and the stress are independent of time. The only observable fluctuations are at the exit of the feeder, where the material avalanches out – the angle of the free surface fluctuates periodically in time, thereby causing periodic fluctuations in the rate at which powder exits the feeder. For slightly cohesive powders such as microcrystalline cellulose, the fluctuations of the free surface angle and therefore the feed rate are erratic, rather than periodic. These results were shown in my presentation in the 2022 AGM and appear in Fig. 2(b) of the renewal proposal. Movies of the flow in the two cases may be viewed by clicking this link.

It is thus clear that for powders that are non-cohesive or slightly cohesive, the feeder is fully filled, and flow variability is entirely due to fluctuations in the free surface slope at the exit. We have found that the variability may be greatly reduced if the exit slot is moved to the top, which is achieved by placing an end cap with an opening at the top. A comparison of the feed rate fluctuations for exit slot at the bottom and top is shown in Fig. 1. The same behaviour is seen for the more cohesive powders too, but with the difference that the torque required to effect the same feed rate is higher. We have so far not encountered a case where the powder flows smoothly into the feeder, but gets 'plugged' within the feeder due to densification or any other reason. Indeed, we find that the screw efficiently transports material flowing into the feeder to the exit. It is very possible that having a constriction at the end of the feeder can result in arching, thereby causing fluctuations. However, this is a matter of design of the exit, rather than a problem with flow in the feeder.

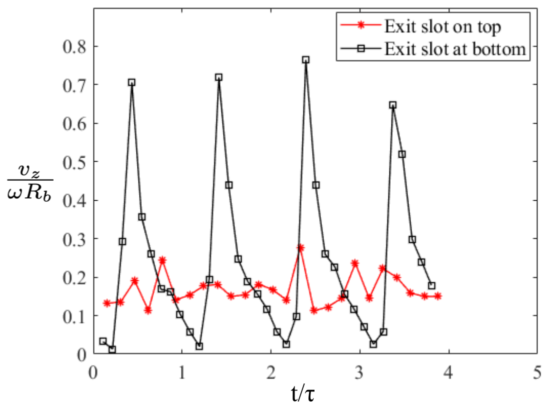


Figure 1: Fluctuations in the feed rate v_z as a function of time t of non-cohesive glass beads when the exit slot is placed at the top and bottom of the cross section of the feeder. Here, ω is the angular speed of the screw, $\tau \equiv 2\pi/\omega$ is the period of revolution, and R_b is the radius of the barrel.

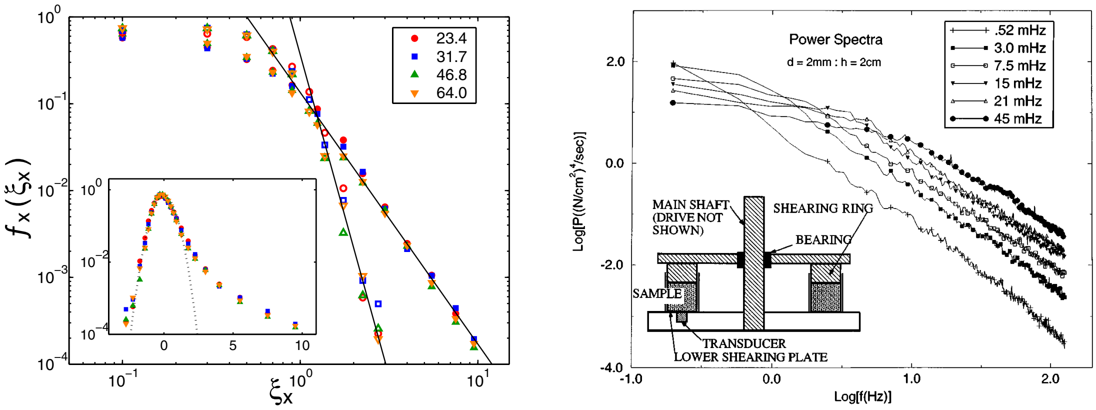
We now come to powders that are more cohesive, for which feed rate variability arises from the second factor mentioned at the beginning of this section, namely fluctuations in the rate of material fed to the screw feeder from the inlet hopper. We have found this to be the reason for feed rate variability for anhydrous lactose – the powder forms large arches at the hopper-feeder interface, which break up intermittently. Once a mass of material falls into the feeder, it is transmitted to the exit, though there could be a 'smoothing out' of fluctuations along the feeder, discussed in more detail in §2. So our experiments so far indicate that flow variability of very cohesive powders is primarily due to intermittent flow at the hopper-feeder interface. Understanding the flow of cohesive powders at the hopper-feeder interface is one of the thrusts of the renewal proposal. The tools that we will use to study this problem are discussed at some length in the two following sections.

2 Analysis of fluctuations in powders

Fluctuations in flowing fluids, such as in turbulent flow at high Reynolds number, are well understood as there is a firm theoretical basis – the Navier-Stokes equations are known to represent the flow exceedingly well. As a result, fluctuations can be computed by direct numerical simulation/solution (DNS) of the equations. The existing solvers for DNS are so well developed that modern aircraft and structures (such as tall buildings, submarine rigs, etc.) are largely designed by DNS, requiring very few wind tunnel tests. There are also highly developed statistical theories of turbulence, based on intuitive arguments, that predict the probability distribution of fluctuations in certain flows to a good degree of accuracy.

Continuum models for granular flow are comparatively less developed, because the media are much more complex and also because they have been under development only for the past ~ 30 years. While there has been substantial improvement in models in recent years – the non-local model we have developed [1] being an example – they are not in a position to predict rapid fluctuations, certainly not catastrophic events such as arching and breakup of cohesive powders. Indeed, the extension of the models to cohesive powders is a major challenge, and it is one of the key thrusts of our renewal proposal. As a corollary, the description of fluctuations is even more rudimentary, though some physical processes that cause them have been identified, such as arching and 'jamming'.

Nevertheless, there are conceptual tools we can use to quantify fluctuations, understand them to some extent, and thereby devise strategies to minimize them. It is now well known that in dissipative, far-from-equilibrium systems such as granular media that probability distribution of fluctuations follow a power scaling for large fluctuation amplitudes, i.e. $p(\xi) \sim \xi^{-n}$, where p is the probability density of the fluctuating variable ξ. Similarly, the the power spectrum of fluctuations too decays as a power of the frequency of fluctuations f. Figure 2 shows examples of experimental measurements of the probability distribution of velocity fluctuations and the power spectrum of stress fluctuations for sheared non-cohesive granular media. The power law decay indicates that the probability of large fluctuations and the frequency of their occurrence decays slowly; in other words, large fluctuations are quite prevalent in granular media. There are only a few studies of fluctuations in cohesive powders, and our intention is to conduct experiments and DEM simulations to fill the gap. We will start by studying the avalanche statistics in a rotating drum, shown on Fig. 3(a) of the renewal proposal. The probability distribution of avalanche size and its power spectrum is expected to be closely related to the statistics of unclogging in a hopper, as the two process have the same mechanical origin. Our key question is: what role powder cohesion plays in determining the statistics of fluctuations? Further details of the proposed experiments are given in the following section.



(a) (b)

Figure 2: Probability distribution of the vertical velocity fluctuation (normalized by the standard deviation) ξ_x for glass beads of mean diameter d flowing through a vertical channel; reproduced from [2]. The filled and unfilled symbols represent data for positive and negative fluctuations, respectively, for different channel width W/d. The inset shows the distribution in linear-log axes, the dotted line representing a Gaussian distribution of the same standard deviation. (b) Power spectrum of fluctuations of the normal stress N for glass beads in a ring shear cell; reproduced from [3].

3 Our proposed investigation of fluctuations and agglomeration of cohesive powders

The horizontal rotating drum partially filled with powder is a convenient experimental device to understand the statistics of fluctuations, as the apparatus can be made transparent to image the flow. As the drum is rotated slowly (click here to view movie), the inclination of the free surface increases and avalanches are continuously triggered, thereby allowing us to gather sufficient data to derive statistical distributions. For a non-cohesive powder, the free surface remains close to the kinematic angle of repose and fluctuations are small. The flow becomes increasingly discontinuous as the cohesive force between grains increases and, as mentioned in §2, the probability distribution of avalanche size and their power spectrum tend to follow a power law. The power law index n is expected to depend strongly on the cohesive force. The probability distribution of avalanches can of course be altered, such as by placing inserts in the drum or by vibrating the setup. The point of determining the 'native' distribution is to determine the level intervention required to get a the desired distribution.

As outlined in the renewal proposal, our aim is to use this apparatus to go beyond the statistics of fluctuations. This experiment can also be used to extract the distribution of agglomerate size, and how it affects the flow. As shown in Fig. 3(b) of the renewal proposal, agglomeration of particles tends to increase the 'flowability' of the powder.

Moreover, one can study how the fluctuations of the height $h(x,t)$ (see Fig.3) of the free surface are correlated in time and space. For a non-cohesive powder, the time correlation $\langle h(x,t)h(x,t+\tau)\rangle$, where the angle brackets indicate an average over time t, is expected to decay rapidly (exponentially) with lapse time τ; similarly the spatial correlation $\langle h(x,t)h(x+\delta,t)\rangle$ will decay rapidly with displacement δ.

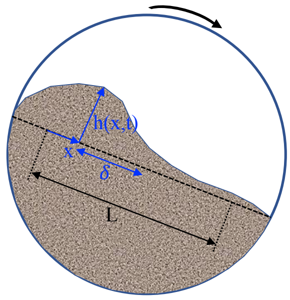


Figure 3: Spatial and temporal fluctuations of the height $h(x,t)$ of the free surface in a rotating horizontal drum partially filled with a powder.

How these correlations are altered by particle cohesion will throw light on how the material deforms as a continuum. For fluids, there are statistical mechanical theories that relate correlations to transport properties: the velocity autocorrelation yields the diffusivity, the stress autocorrelations yields the viscosity, etc. (see, e.g. Ref. [4]). Our intention is to extend these theories to powders to arrive at the flow rule for cohesive powders, as described in §2.1 of the renewal proposal, thereby extending our non-local constitutive model (Equation 2 in the renewal proposal) to cohesive powders.

4 Summary

The work conducted in the past year has already addressed one factor that causes variability in the feed rate, namely the periodic or erratic avalanching at the feeder exit. We have show that the fluctuations can be greatly mitigated by proper design of the exit, such as by placing the exit slot at the top.

The more difficult factor that causes variability is random arching at the hopper-feeder interface, which occurs for quite cohesive powders. We have outlined a conceptual framework for how the statistical distribution of fluctuations will be understood and quantified. Such a study will lead to an understanding of how their effects can be mitigated.

Finally, we have a plan for using the correlation of fluctuations and the statistics of particle agglomeration to determine the flow rule for cohesive powders, which in turn will extend of the non-local rheological model to cohesive powders.

Additional References

[1] P. V. Dsouza, P. R. Nott, *J. Fluid Mech.* **888**, R3 (2020).

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[3] B. Miller, C. O'Hern, R. P. Behringer, *Phys. Rev. Lett.* **77**, 15 (1996).

[4] D. J. Evans, G. M. Morris, *Statistical Mechanics of Non-equilibrium Liquids*, Cambridge University Press (2008).